Repetitive Control of Linear Time Varying Systems with Application to Electronic Cam Motion Control

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Abstract— **This paper addresses repetitive control for linear time varying systems. This problem is motivated by a broad range of applications where periodic disturbances or reference signals are existent and synchronized with a rate varying process variable. In the area of motion control, this is a master-slave type electronic cam follower problem, where the slave axis motion must follow the master axis coordinate (process variable) according to a given cam profile while the master axis motion has a time varying speed. In the real-time domain, the disturbance/reference profile changes as master's speed changes. However, in view of the master axis coordinate, which is called in the angle domain, the disturbance/reference profile is periodic with a fixed period even when the master axis changes speed. To exploit the internal model of repetitive control to achieve asymptotic tracking performance, it is advantageous to design the control system from the angle domain perspective. In the angle domain the slave's time invariant dynamics become time varying and dependent on the mast axis speed. Therefore, the problem of repetitive control for time varying systems arises. In this paper, Model Reference Control is applied to compensate for the linear time varying system, rendering a linear time invariant input/output system. Asymptotic output tracking/regulation performance is achieved by employing repetitive control loop to compensate for the linear time invariant model.**

I. Introduction

A broad range of applications involves compensation of periodic disturbances or reference signals that are synchronized with a process variable. When the rate of change of this process variable is constant, these signals are also periodic in time, but when the rate of change is variable, the signals are no longer periodic with time. In the area of motion control, this represents a master-slave type electronic cam follower problem, where the slave axis motion must follow the master axis coordinate (process variable) according to a given cam profile while the master axis motion has a time varying speed. The electronic cam motion is widely used in the industry. Present industrial programmable multi-axis motion control systems provide the electronic cam function by generating the slave axis's reference signal according to the real-time sensed master axis's position. This open loop type of control between the master axis and the slave axis relies on the servo control performance of the slave axis to follow otherwise arbitrary reference signals. The open loop control does not exploit the unique characteristic that the cam profile is periodic with respect to the master axis's rotational angle. Repetitive control, which achieves asymptotic tracking and disturbance rejection of periodic signals, has been widely used in many applications.The internal model principle (Francis and Wonham[1]) requires that a periodic signal generator be included in the feedback loop in order to track a periodic reference and this periodic signal generator includes a time delay term corresponding to the period. Early work on repetitive control was initiated by Inoue *et.al*[2]. Hara *et.al.*[3] presented the stability analysis of the continuous repetitive control system. Tomizuka et.al.[4] presented the analysis and synthesis of the discrete time repetitive controllers based on Zero Phase Error Tracking compensation of the plant model (Tomizuka,[5]). Tsao and Tomizuka[6] presented a robust repetitive control algorithm by using Q filter. More recently, repetitive control design has been cast into LFT form and solved by mu-synthesis approach ([7]). These fixed period repetitive control can be applied to the electronic cam motion generation problem when the master axis's speed is constant. When the master axis's speed varies sufficiently slow, variable period repetitive control[8] may be applied.

When the master-axis's speed varies periodically in synchronization with exogenous signal's period, the problem can be considered as a periodically linear system with its period synchronized with the exogenous signal period. Omata *et.al.*[9] provided a sufficient condition for stability based on L-1 induced norm and small gain theorem. Hanson and Tsao[10] proposed a gain-scheduling approach analyzed by the lifting method to ensure stability and implemented the control to variable speed non-circular machining. Luo and Manhawan[11] extended the idea to design repetitive controller capable of tracking time varying periodic references. A invertible unitary operator is used to transform the signals between angle-domain and time-domain.However, only bounded tracking error performance can be achieved.

This paper addresses the electronic cam following problem when the master-axis's speed is time varying. The speed variations are not necessarily slow or periodically synchronized with the signal period, as the aforementioned works have addressed. Noting that the reference and disturbance signals are periodic functions of the maser axis's rotational angle, the control problem may be modelled in the master axis "angle domain" to exploit the signal periodicity in the repetitive control design. This "angle domain" aspect was addressed by Tsao et.al.[12], [13] by using spindle angle, instead of time, as the independent variable for repetitive controller design. The validity of this aspect has been demonstrated by experiments (Tsao[14], Hiro[15], She and

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M. Nakano[16])

We will show that this problem entails applying repetitive control to linear time varying systems since the linear time-invariant plant in the time domain is linear time-varying in the angle domain as the master-axis speed varies. The control design involves two steps. In the first step, an inner Model Reference Control developed by[17] is employed to render a linear time invariant input-output map. In the second step, an outer loop repetitive control is employed to render asymptotic output regulation performance.

II. Problem Formulation

A. Plant model in the angle domain

Consider a *SISO* motor plant in the time domain

$$\begin{aligned}\frac{dx(t)}{dt} &= Ax(t)+Bu(t) \\ y(t) &= Cx(t)\end{aligned} \tag{1}$$

which is completely controllable and observable. Converting it into the angle domain yields:

$$\begin{aligned}\frac{dx(\theta)}{d\theta} &= \frac{A}{\tilde{\omega}(\theta)}x(\theta)+\frac{B}{\tilde{\omega}(\theta)}u(\theta) \\ y(\theta) &= Cx(\theta)\end{aligned} \tag{2}$$

where $\tilde{\omega}(\theta)$ is defined as follows by noting the relationship between time and the spindle angle:

$$\tilde{\omega}(\theta) := \omega(t(\theta)) = \omega(t) = \frac{d\theta}{dt}, \qquad t = \int_0^{\theta}\frac{d\theta}{\tilde{\omega}(\theta)}$$

It is clear that the plant becomes time-varying in the angle domain.But the reference and disturbance are periodic in the angle domain.So perfect tracking can be achieved by applying repetitive control to the system.

B. Control Design in the angle domain

The control scheme for the variable speed machining system is shown in Fig. 1. The inner controller C_{mrc} is designed so as to match the I/O characteristics to a nominal linear time-invariant in the angle domain, and the outer controller C_{rc} is the prototype discrete repetitive controller to reject disturbance and provide tracking performance of the plant in the angle domain.

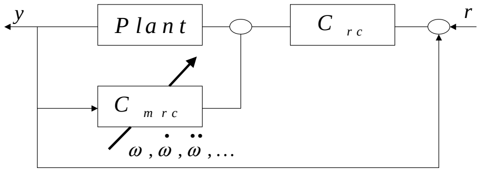

Fig. 1. Controller Scheme

1) **MRC Controller Design:** Since the plant model in the angle domain is linear time-varying, the prototype discrete repetitive control can't be applied directly. The model reference controller design techniques for linear time-varying systems in the I/O operator perspective([17]) is employed to make the system linear time-invariant in the angle domain.

Mathematical Preliminaries:

We first introduce the following definitions which will be used to represent the system model and controller in the I/O operator perspective.

Definition 1: s is defined as the differential operator, $\frac{d}{dt}(.)$. A left polynomial differential operator (*PDO*) of degree n is defined as: $P(s,t) = a_0(t)s^n + a_1(t)s^{n-1} + \ldots + a_n(t)$.Similarly, the right *PDO* is defined as: $P(s,t) = s^n a_0(t) + s^{n-1}a_1(t) + \ldots + a_n(t)$.

And a general system described by

$$y^{(n)} + a_1(t)y^{(n-1)} + \ldots + a_{(n-1)}(t)y^{(1)} + a_n(t)y = b_0u^{(m)} + b_1(t)u^{(m-1)} + \ldots + b_{(n-1)}(t)u^{(1)} + b_n(t)u$$

can be written in the form:

$$P(s,t)[y] = Q(s,t)[u]$$

where $P(s,t) = s^n + a_1(t)s^{n-1} + \ldots + a_n(t)$ and $Q(s,t) = b_0(t)s^m + b_1(t)s^{m-1} + \ldots + b_m(t)$.

In order to describe the above system by fractional representations, we need the following definition to introduce the "inverse" operator corresponding to the *PDO* $P(s,t)$.

Definition 2: A left (right) polynomial integral operator *PIO* of order n is defined as the operator that maps the input u to the zero-state response of the differential equation $P(s,t)[y] = u$, where $P(s,t)$ is the left(right) monic left(right) *PDO*.

So we can write the system as:

$$[y] = P^{-1}(s,t)Q(s,t)[u]$$

To assess the stability of the system given by the above LTV I/O operator, we first define the ES stability concept of *PIO*.

Definition 3: A *PIO*, $P^{-1}(s,t)$, is said to be *ES*(or uniformly asymptotically stable) with rate $-a, a > 0$, if there exist some positive constants k,a such that the state transition matrix associated with the linear differential equation, satisfies $||\phi(t,\tau)|| \leq kexp(-a(t-\tau)), t \geq \tau \geq 0$.

Given the above definition, it is clear that a system described by an I/O operator is ES if all the *PIO*'s in it are ES.

MRC design technique:

Later we are going to treat our systems as I/O operators expressed as a combination of *PDO*'s and *PIO*'s. Such a description leads it way to the fractional representation approach in the analysis and design of stabilizing controllers in the I/O perspective. So now we consider the LTV system described by the following two forms:

$P_R form:$

$$y_p = k_p(t)N_p(s,t)D_p^{-1}(s,t)[u_p] \tag{3}$$

$P_L form:$

$$y_p = D_p^{-1}(s,t)N_p(s,t)k_p(t)[u_p] \tag{4}$$

where $D_p(s,t)$,$N_p(s,t)$ are monic *PDO*'s with *UB* coefficients and of constant degree,denoted by n, m respectively.And in Eq.(3), $D_p(s,t)$,$N_p(s,t)$ are strongly right co-prime while in Eq.(4), $D_p(s,t)$,$N_p(s,t)$ are strongly left co-prime. The need for strong co-primeness will be seen later in the controller design.

And the MRC objective is defined as follows: Determine a control input u_p such that the closed-loop plant is internally stable and the plant output y_p tracks the output y_m of the LTI reference model:

$$y_m = W_m(s)[r] = k_m D_m^{-1}(s) N_m(s)[r] \qquad (5)$$

for any *U*B, piecewise continuous reference input signal.

For this control problem to be feasible we need to make

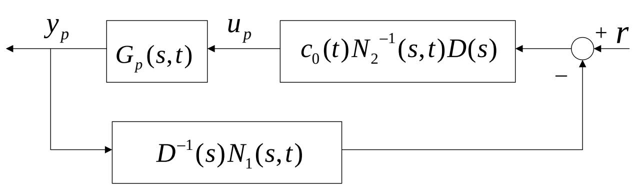

Fig. 2. Block diagram for the MRC controller

the following assumptions: $k_p(t)$ is positive, smooth, *UB* and bounded away from zero. $N_p^{-1}(s,t)$ is an ES *PIO*. $D_m(s) and N_m(s)$ are monic and Hurwitz with $deg[N_m(s)] \leq deg[D_m(s)] - 1$. $deg[D_m(s)] \leq deg[D_p(s,t)]$, $k_m > 0$ and the plant have the same relative degree as the reference model. Consider the controller structure shown in Fig(2) with a cascade compensator $c_0(t)N_2^{-1}(s,t)D(s)$ and a feedback compensator $D^{-1}(s)N_1(s,t)$.Without loss of generality, $N_2^{-1}(s,t)$ is chosen as monic. Also $D(s)$ is chosen as a monic and Hurwitz *PDO* to ensure that the cancellation of $D(s)$ and $D^{-1}(s)$ is permitted.The results about MRC design is summarized in the following theorem:

Theorem 4 (Tsakalis,Ioannou,[17]): Consider the plant (Eq.3) or (Eq.4) and the control law:

$$u = c_0(t)N_2^{-1}(s,t)D(s)[r + D^{-1}(s)N_1(s,t)y] \qquad (6)$$

where $D(s)$ is monic,Hurwitz *PDO* of degree $n-1$ and such that $N_m(s)$ is right factor of $D(s)$,i.e.,$D(s) = D_z(s)N_m(s)$;$N_1(s,t) and N_2(s,t)$ are *PDO*'s of degree n-1 with $N_2(s,t)$ monic and $c_0(t)$ is scalar function of time. Then the controller can be designed so that the closed-loop I/O operator $S_{ry} : r \rightarrow y$ is *BIBO* stable and equal to the reference model(Eq.5).The controller parameters are smooth,*UB* functions of time and can be calculated by solving the algebraic design equations

$$N_2(s,t)c_0^{-1}(t)D_p(s,t) - N_1(s,t)k_p(t)N_p(s,t) = D_z(s)D_m(s)k_m^{-1}k_p(t)N_p(s,t) \qquad (7)$$

for a plant with I/O operator in the P_R form(Eq.3),or

$$\tilde{N}_2(s,t)D_p(s,t) - k_m N_1(s,t) = k_m D_z(s) D_m(s) k_m^{-1} \qquad (8)$$

$$N_2(s,t) = k_m^{-1}\tilde{N}_2(s,t)N_p(s,t)k_m \qquad (9)$$

for a plant with I/O operator in the P_L form(Eq.4).

Note the change to minus sign in the Eq.(7) and Eq.(8) results from rearranging the controller structure with positive feedback.

State space realization of I/O operators:

Till now, the MRC design problem is formulated in the I/O operator perspective for plants also described by an I/O operator. The controller I/O operator has been solved from a Diophantine equation. Next we are going to find the state-space realization of the controller given by I/O operator and address the internal stability of it. Recall that the controller I/O operators to be realized are $D^{-1}(s)\bar{N}_2(s,t)$ and $D^{-1}(s)N_1(s,t)$.For the purpose of state-space realization, we need to use right form *PDO*'s now,which will be clearly seen later.Note $D^{-1}(s)\bar{N}_2(s,t)$ is strictly proper and $D^{-1}(s)N_2(s,t)$ is proper, since $deg[\bar{N}_2(s,t)] = n-2$, $deg[D(s)] = n-1$, and $deg[N_1(s,t)] = n-1$. Assuming the leading coefficient of $N_1(s,t)$ is $\theta_3(t)$ and let $\bar{N}_1(s,t) = N_1(s,t) - D(s)\theta_3(t)$, we can write $D^{-1}(s)N_1(s,t) = D^{-1}(s)\bar{N}_1(s,t) + \theta_3$,where $D^{-1}(s)\bar{N}_1(s,t)$ is strictly proper. Now let

$$\mathbf{F} = \begin{bmatrix} -d_1 & 1 & 0 & \dots & 0 \\ -d_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & 1 \\ -d_{n-1} & 0 & 0 & \dots & 0 \end{bmatrix}$$

where d_i is the coefficient of $D(s) = s^{n-1} + s^{n-2}d_1 + \ldots + d_n$,

$$\boldsymbol{\theta_1(t)} = \begin{bmatrix} n_{2,0} \\ n_{2,1} \\ \vdots \\ n_{2,n-2} \end{bmatrix}, \boldsymbol{\theta_2(t)} = \begin{bmatrix} n_{1,0} \\ n_{1,1} \\ \vdots \\ n_{1,n-2} \end{bmatrix}$$

where $n_{2,i}$ is the coefficient of $\bar{N}_2(s,t) = s^{n-2}n_{2,0} + \ldots + n_{2,n-2}$, $n_{1,i}$ is the coefficient of $\bar{N}_1(s,t) = s^{n-2}n_{1,0} + \ldots + n_{1,n-2}$, and

$$\mathbf{q^T} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}$$

Then it is trivial to verify that

$$D^{-1}(s)\bar{N}_2(s,t) = q^T(sI - F)^{-1}\theta_1(t)$$
$$D^{-1}(s)\bar{N}_1(s,t) = q^T(sI - F)^{-1}\theta_2(t) + \theta_3(t)$$

It is clear from the above that Fig.(3) is the state space realization of the controller I/O operators solved from the Diophantine equation. The above discussion leads to the following two theorems about the state-space realization of MRC law and it internal stability issue.

Theorem 5 (State-space realization,[17]): To realize in state-space the TV MRC scheme of Theorem (4) the plant output y_p and input u_p are used to generate a $(2n-1)$-

dimensional auxiliary vector ω as follows:

$$\begin{aligned}\dot{\omega}_1 &= F\omega_1 + \theta_1 u_p \qquad (10)\\ \dot{\omega}_2 &= F\omega_2 + \theta_2 y_p\\ \omega_3 &= \theta_3 y_p\end{aligned}$$

$\omega = [\omega_1^T, \omega_2^T, \omega_3]^T, \theta = [\theta_1^T, \theta_2^T, \theta_3]^T$ and $F \in R^{(n-1)\times(n-1)}$ is stable matrix with $det(sI - F) = D(s)$. The input to the plant is taken as

$$u_p = c_0(t)[g^T\omega + r] \qquad (11)$$

where $g = [q^T, q^T, 1]^T$ is constant vector such that (q^T, F) is an observable pair. Then, there exists $[\theta_*, c_{0*}(t)]^T$such that the control law Eq.(11) satisfies the TV MRC objective. Further, $[\theta_*, c_{0*}(t)]^T$ is UB and at least once differentiable with UB derivative,provided that the plant parameters are UB and possess a sufficiently large but finite number of UB derivatives.

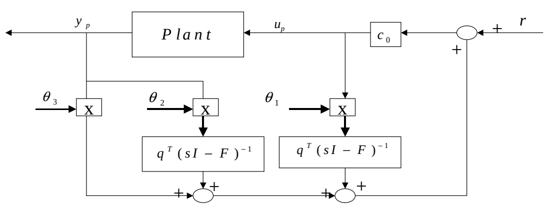

Fig. 3. State-space realization of the MRC scheme

Theorem 6 (Stability,[17]): Under the conditions given in Theorem (4) and Theorem (5),the closed-loop plant is ES and therefore BIBS stable for any external UB input.

Angle domain MRC design analysis:

Let's see how the above design techniques can be applied to the angle domain motor plant described in Eq.(2).First we need to show that the angle domain plant given in state-space form of Eq.(2) admits a PDO factorization form such that MRC design techniques can be used. The following assumptions are made on the plant: 1. A,B,C are constant finite dimension matrices and $\omega(t)$ is smooth,UB and bounded away from zero. 2. The order of the plant is constant and finite. 3. The original system in the time domain described by Eq(1) is completely controllable and observable, which means [A,B,C] is completely controllable and observable. And we can see from the following theorem that the converted angle domain system will be strongly controllable under these assumptions,which ensures that the angle domain system admits a left or right PDO factorization.

Theorem 7: The triple$[\frac{A}{\omega(t)}, \frac{B}{\omega(t)}, C]$ for the angle domain plant is strongly controllable and observable under the above three assumptions.

Proof:First,we prove the strong controllability. Without loss of generality, we can write the original time domain system in the controllability canonical form,i.e.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \dots & 1\\ a_1 & a_2 & a_3 & \dots & a_n \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ 1 \end{bmatrix}$$

then we have:

$$\frac{\mathbf{A}}{\omega(\mathbf{t})} = \begin{bmatrix} 0 & \frac{1}{\omega(t)} & 0 & \dots & 0\\ 0 & 0 & \frac{1}{\omega(t)} & \dots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \dots & \frac{1}{\omega(t)}\\ \frac{a_1}{\omega(t)} & \frac{a_2}{\omega(t)} & \frac{a_3}{\omega(t)} & \dots & \frac{a_n}{\omega(t)} \end{bmatrix}, \frac{\mathbf{B}}{\omega(\mathbf{t})} = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ \frac{1}{\omega(t)} \end{bmatrix}$$

Recalling the definition of the controllability matrix of time-variable systems,we have the controllability matrix as follows:

$$Q_c = [p_0 p_1 \dots p_{n-1}]$$

where

$$p_{k+1} = -\frac{A}{\omega(t)} p_k + \frac{d}{dt} p_k, p_0 = \frac{B}{\omega(t)}.$$

Plugging the expressions of the $\frac{A}{\omega(t)}, \frac{B}{\omega(t)}$ we can get the controllability matrix:

$$\mathbf{Q_c} = \begin{bmatrix} 0 & 0 & 0 & 0 & (-1)^{n+1}\frac{1}{\omega(t)^n}\\ 0 & 0 & 0 & (-1)^n\frac{1}{\omega(t)^{n-1}} & \diamond\\ \vdots & \vdots & & \diamond & \diamond\\ 0 & -\frac{1}{\omega(t)^2} & \diamond & \diamond & \diamond\\ \frac{1}{\omega(t)} & \diamond & \diamond & \diamond & \diamond \end{bmatrix}$$

It is clear that if $\omega(t)$ is smooth,UB, and bounded away from zero, then $|det(Q_c)| \geq c > 0$. So the angle domain plant is strongly controllable. Similarly the strong observability can be proved.

Following the theorem, the angle domain plant can be written as the I/O operator form. The MRC control design discussed before can be applied to match the plant to a LTI reference model in the I/O operator perspective.

2) Repetitive Controller Design: According to the internal model principle proposed by Francis and Wonham[1], asymptotic tracking of periodic signals of period L may be achieved by including a periodic signal generator in the control loop.The discrete prototype repetitive controller is applied to the nominal linear time-invariant plant resulting from the linearized plant model by using the model reference control design techniques.

Prototype Discrete Repetitive Controller:

Let the discrete nominal linear time-invariant model be described as:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \qquad (12)$$

where d is the plant delay and b_0 is nonzero.
The prototype repetitive controller $M(z^{-1})$ is given by:

$$M(z^{-1}) = \frac{R(z^{-1})}{S(z^{-1})(1-z^{-N})} \quad (13)$$

$$\frac{R(z^{-1})}{S(z^{-1})} = k_r \frac{z^{-N+d+nu}A(z^{-1})(z^{-nu}B^-(z^{-1}))}{B^+(z^{-1})b} \quad (14)$$

where k_r is the repetitive control gain.nu is the order of $B^-(z^{-1})$,and $b \geq max|B^-(e^{-j\omega})|^2, w \in [0,\pi]$. The asymptotic stability of the closed loop system is guaranteed.[4]
To ensure robust stability, a low pass filter $Q(z,z^{-1})$can be incorporated into the control law.$Q(z^1,z^{-1})$ can be chose as:

$$Q(z,z^{-1}) = \left[\frac{z+2+z^{-1}}{4}\right]^n$$

And it should be noted that robust stability sacrifices tracking performance.

III. Design Example and Simulation

Consider a second order linear motor model in the time domain:

$$G_p(s) = \frac{b_0 s + b_1}{s^2 + a_0 s + a_1} \quad (15)$$

The model in the angle domain is is derived using the following two equations:

$$\frac{dy}{dt} = \frac{dy}{d\theta}\tilde{\omega}$$
$$\frac{d^2y}{dt^2} = \tilde{\omega}^2\frac{d^2y}{d\theta^2} + \tilde{\omega}\tilde{\omega}'\frac{dy}{d\theta}$$

And we get that the model plant in the angle domain is described as:

$$\left(s^2 + \left(\frac{\tilde{\omega}\tilde{\omega}' + a_0\tilde{\omega}}{\tilde{\omega}^2}\right)s + \frac{a_1}{\tilde{\omega}^2}\right)y = \left(\frac{b_0\tilde{\omega}'}{\tilde{\omega}}s + \frac{b_1}{\tilde{\omega}^2}\right)u \quad (16)$$

Now the system can be written in the *PDO* factorization form as follows:

$$y_p = D_p^{-1}(s,t)N_p(s,t)k_p(t)[u_p] \quad (17)$$
$$D_p(s,t) = s^2 + \left(\frac{\tilde{\omega}\tilde{\omega}' + a_0\tilde{\omega}}{\tilde{\omega}^2}\right)s + \frac{a_1}{\tilde{\omega}^2} \quad (18)$$
$$N_p(s,t) = s + \frac{b_1}{\tilde{\omega} * b0} + \frac{\tilde{\omega}'}{\tilde{\omega}} \quad (19)$$
$$k_p(s,t) = \frac{b_0}{\tilde{\omega}} \quad (20)$$

Suppose the reference model is:

$$(s^2 + p_0 s + p_1)y_m = k_m(s+q)r$$

In order to get the MRC controller parameters, we need to solve the corresponding Diaphantine equation:

$$\tilde{N}_2(s,t)D_p(s,t) - k_m N_1(s,t) = k_m D_z(s)D_m(s)k_m^{-1}$$

Here,$D(s) = D_z(s)N_m(s) = N_m(s) = s+q, D_z(s) = 1$,
Plugging $D_p(s,t), N_p(s,t)$ into the above equation ,we get:

$$\tilde{N}_2(s,t) = 1 \quad (21)$$
$$N_1(s,t) = \frac{\frac{\tilde{\omega}\tilde{\omega}' + a_0\tilde{\omega}}{\tilde{\omega}^2} - p_0}{k_m} \quad (22)$$
$$N_2(s,t) = s + \frac{b_1}{\tilde{\omega} * b0} + \frac{\tilde{\omega}'}{\tilde{\omega}} \quad (23)$$

And the corresponding state-space realization parameters are:

$$\theta_1 = \left(q - \frac{\tilde{\omega}\tilde{\omega}' + a_0\tilde{\omega}}{\tilde{\omega}^2}\right)k_p(t)/k_m$$
$$\theta_2 = \frac{\frac{a_1}{\tilde{\omega}^2} - p_1 - q(\frac{\tilde{\omega}'}{\tilde{\omega}} + \frac{a_0}{\tilde{\omega}} - p_0) - \frac{\tilde{\omega}''}{\tilde{\omega}} + \frac{\tilde{\omega}^2}{\tilde{\omega}^2} + a_0\frac{\tilde{\omega}'}{\omega^2}}{k_m}$$
$$\theta_3 = \frac{\frac{\tilde{\omega}''}{\tilde{\omega}} + \frac{a_0}{\tilde{\omega}} - p_0}{k_m}$$

From the above expressions, we can see that in order to design MRC controller for the linear time-varying plant in the angle domain, we need to know about the information of how the rotation speed changes,*i.e.*, $\omega, \dot{\omega}, \ddot{\omega}, \ldots$. The actual coefficients are as follows:
$b_0 = 105.3065, b_1 = 5.7926e5, a_0 = 1.3785e2, a_1 = 7.9076e5$.
Now let's assume the nominal rotational speed is $\omega_n = 600rpm = 62.8319rad/s$, a reasonable nominal reference model in the angle domain would be replacing s in the above equation with $\omega_n\sigma$, and we get:

$$G_P(\sigma) = \frac{1.676\sigma + 146.73}{\sigma^2 + 2.194\sigma + 200.3} = \frac{1.676(\sigma + 87.5477)}{\sigma^2 + 2.194\sigma + 200.3} \quad (24)$$

so the reference model parameters will be: $p_0 = 21.94, p_1 = 200.3, k_m = 1.676, q = 87.5477$
Plugging into Eq. 22-24, we can get the MRC controller parameters.
And for 1.40625 degree (256 interrupts per revolution) sampling interval time, the discrete transfer function with zero order hold is

$$G_P(z) = \frac{0.0822z + 0.0030}{z^2 - 1.8313z + 0.9476} = \frac{z^{-1}(0.0822 + 0.0030z^{-1})}{1 - 1.8313z^{-1} + 0.9476z^{-2}} \quad (25)$$

According to the repetitive control law in Eq. (14) we have:

$$R(z^{-1}) = z^{-255}(1 - 1.8313z^{-1} + 0.9476z^{-2}) \quad (26)$$
$$S(z^{-1}) = 0.0822 + 0.0030z^{-1} \quad (27)$$

We also add a low pass filter $q(z^1,z^{-1}) = \frac{z+2+z^{-1}}{4}$ to ensure robust stability against unmodelled dynamics.

A simulink model was constructed using the above con-

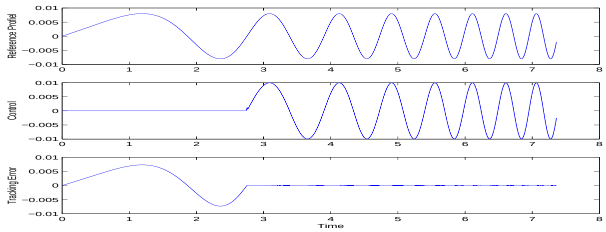

Fig. 4. Nominal Performance in the time domain

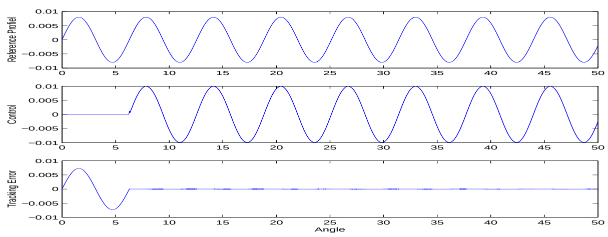

Fig. 5. Nominal Performance in the angle domain

troller design. Simulation results are presented as follows. Fig. 4 and 5 shows the nominal performance. A sine reference and disturbance of 2π are applied in the angle domain to the simulated system. We can clearly see that the reference is periodic in the angle domain, where in the time domain, its period is varying.Nonetheless, the tracking error goes to zero in both domains, which validate our approach of repetitive control plus model reference control in the angle domain. Also this approach show some extent of

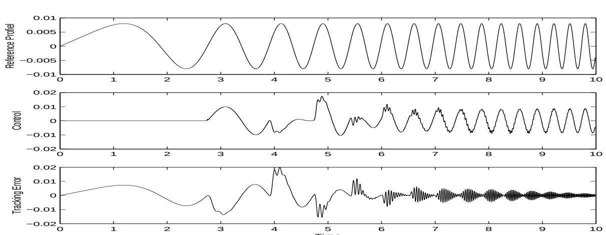

Fig. 6. Tracking Performance in the time domain

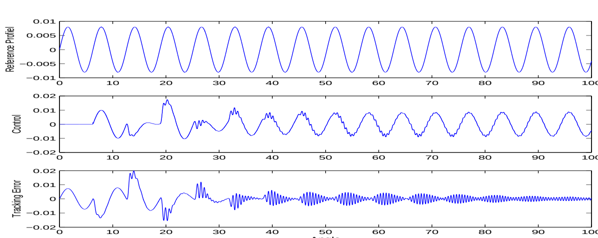

Fig. 7. Tracking Performance in the angle domain

robustness. Fig 6 and 7 shows the tracking performance of this approach when the plant was perturbed in the angle domain from its nominal model a little bit . Though the tracking error becomes larger, we can see that asymptotical stability can be still achieved.

IV. Conclusion

We have addressed asymptotic output regulation of exogenous periodic signals for linear time varying systems by employing Model Reference Control to render linear time invariant input-output model and then applying repetitive control to achieve asymptotic tracking of periodic signals. This approach advances the widely used electronic cam follower motion control from the present open loop master-slave reference generation to closed loop repetitive control with asymptotic tracking performance. Simulation results have shown the validity of this control algorithm. Experimental work is being conducted to implement this control algorithm.

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