

Design of Robustly Stable Disturbance Observers Based on Closed Loop Consideration Using H_∞ Optimization and its Applications to Motion Control Systems

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Abstract—In disturbance-observer-based control, the closed loop system consists of a main feedback control loop and an inner disturbance observer loop. This paper presents a design method for a disturbance observer to satisfy closed loop performance specifications, provided that the main feedback controller is known. Taking advantage of certain fixed-structure disturbance observers, the disturbance observer design problem can be transformed into the synthesis problem of H_∞ (sub-)optimal static output feedback gain for an extended plant. The static output feedback gain, and therefore the disturbance observer, can be obtained by solving a series of convex optimization problems. Since the disturbance observer is designed based on closed loop consideration, the robust stability of the closed loop system is guaranteed.

I. INTRODUCTION

In motion control design, disturbance rejection and robustness to parametric uncertainties are important issues besides tracking performance. Disturbance-observer-based control is an effective method often used to deal with these issues. Experimental results in [1]–[3] have shown the effectiveness of disturbance-observer-based control. In disturbance-observer-based control design, an inner disturbance observer loop is added into a main feedback control loop. The key of the disturbance observer design is to select a proper low-pass filter. The inner disturbance observer loop has been studied by several researchers [3], [4]. Various guidelines were suggested for the selection of the low-pass filter. These suggestions make the disturbance observer loop behave more desirable. However, since the disturbance observer loop is just a part of the overall closed loop system, the robust stability of the closed loop system is not guaranteed. In this paper, we present a design method for the disturbance observer based on closed loop consideration. The resulting closed loop system satisfies given specifications, which ensure desired disturbance attenuation and robust stability.

The remainder of this paper is organized as follows. Section II provides a brief overview of disturbance observers and certain fixed-structure disturbance observers. Section III gives the problem statement of the disturbance observer design based on closed loop consideration. The problem is solved by the algorithm presented in Section IV.

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Section V gives an example to illustrate the design method. Conclusions are given in Section VI.

II. DISTURBANCE OBSERVERS

A. Overview

Fig. 1 shows the general structure of a disturbance observer for a SISO plant, where v , u , d , y and ξ are the command input, control input, external disturbance, output and sensor noise, respectively. $P(s)$ represents the physical plant to be controlled. $P_n(s)$ is a nominal plant model. In practice, $P_n(s)$ is chosen as a low order approximation of the physical plant. The disturbance observer considers the mismatch between the plant and nominal model as an equivalent disturbance acting on the nominal model. It estimates the equivalent disturbance combined with the external disturbance, and feeds back the estimate as a cancellation signal, as shown in Fig. 1. The behavior of the disturbance observer loop can be analyzed by looking at the transfer functions from v , d , and ξ to the output y .

$$y = G_{yv}(s)v + G_{yd}(s)d + G_{y\xi}(s)\xi \quad (1)$$

where

$$G_{yv}(s) = \frac{P(s)P_n(s)}{P_n(s) + Q(s)(P(s) - P_n(s))} \quad (2)$$

$$G_{yd}(s) = \frac{P(s)P_n(s)(1 - Q(s))}{P_n(s) + Q(s)(P(s) - P_n(s))} \quad (3)$$

$$G_{y\xi}(s) = \frac{P(s)Q(s)}{P_n(s) + Q(s)(P(s) - P_n(s))} \quad (4)$$

When $Q(s) \approx 1$, (1) becomes $y \approx P_n(s)v + \xi$. This indicates that the disturbance observer rejects the disturbance and compensates for the model mismatch. The disturbance observer loop behaves as the nominal model. On the other hand, when $Q(s) \approx 0$, (1) becomes $y \approx P(s)v + P(s)d$. The disturbance observer loop is essentially cut, and the output is not affected by the sensor noise. In control applications, disturbances normally dominate at low frequencies, whereas sensor noise dominates at high frequencies. This suggests that $Q(s)$ should be a low-pass filter with a steady state gain of one. Besides the low-pass characteristic of $Q(s)$, for the disturbance observer in Fig. 1 to be realizable, the relative degree of $Q(s)$ must be greater than or equal to that of $P_n(s)$.

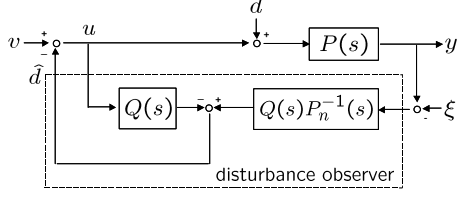


Fig. 1. Disturbance observer loop

B. Fixed-structure Disturbance Observers

In the state estimator design, assuming that the disturbance model is available, the model can be included in the estimator equations, and the disturbance can be estimated along with the plant state [5]. Although the disturbance is estimated under the assumption of known disturbance modes, it has been shown that the augmented state estimator is indeed a disturbance observer if the assumed disturbance model includes modes $1/s^i$, where $i \geq 1$ [6].

It is assumed that the plant model $P_n(s)$ is strictly proper and does not have a zero at the origin. The state space representation of $P_n(s)$ is given by

$$\begin{aligned}\dot{x} &= Ax + B(u + d) \\ y &= Cx\end{aligned}\quad (5)$$

where $x \in \mathbf{R}^n$, (A, B) controllable and (A, C) observable. It is also assumed that the disturbance model is represented by

$$\begin{aligned}\dot{x}_d &= A_d x_d \\ d &= C_d x_d\end{aligned}\quad (6)$$

where $x_d \in \mathbf{R}^{n_d}$, (A_d, C_d) observable, and A_d has at least one zero eigenvalue. In order to estimate the disturbance d , the plant model (5) is augmented with the disturbance model (6), resulting in an augmented plant P_z . Let $z = [x^T \ x_d^T]^T$ be the augmented state, then the augmented plant P_z is given by

$$\begin{aligned}\dot{z} &= \bar{A}z + \bar{B}u \\ y &= \bar{C}z\end{aligned}\quad (7)$$

where

$$\bar{A} = \begin{bmatrix} A & BC_d \\ 0 & A_d \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = [C \ 0] \quad (8)$$

It can be shown that the augmented plant P_z is observable if none of the eigenvalues of A_d are at the same locations as the zeros of the plant model $P_n(s)$, that is, the disturbance state can be "seen" at the plant output y .

Suppose that the above assumptions are all satisfied, then the state estimator of P_z can be constructed as follows.

$$\begin{aligned}\dot{\hat{z}} &= \bar{A}\hat{z} + \bar{B}u - L(y - \bar{C}\hat{z}) \\ &= (\bar{A} + L\bar{C})\hat{z} + \begin{bmatrix} \bar{B} & -L \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}\end{aligned}\quad (9)$$

The poles of the estimator can be placed anywhere in the left-half plane. The disturbance estimate \hat{d} can be obtained by

$$\hat{d} = \begin{bmatrix} 0 & C_d \end{bmatrix} \hat{z} := \bar{C}_d \hat{z} \quad (10)$$

Following (9) and (10), \hat{d} can be written as

$$\hat{d} = -G_1(s)u + G_2(s)y \quad (11)$$

where

$$G_1(s) = \begin{bmatrix} \bar{A} + L\bar{C} & -\bar{B} \\ \bar{C}_d & 0 \end{bmatrix}; G_2(s) = \begin{bmatrix} \bar{A} + L\bar{C} & -L \\ \bar{C}_d & 0 \end{bmatrix} \quad (12)$$

Notice that the disturbance estimate \hat{d} in Fig. 1 is given by (let $\xi = 0$)

$$\hat{d} = -Q(s)u + Q(s)P_n^{-1}(s)y \quad (13)$$

Hence, for (11) to be directly comparable to (13), $G_1(s)$ should be a low-pass filter with a steady state gain of one and $G_2(s)$ should be equal to $G_1(s)P_n^{-1}(s)$. These are true if the assumptions imposed on the plant and disturbance models hold [6].

We can consider (9) and (10) as an alternative way to design and implement the disturbance observer. $Q(s)$ filter in Fig. 1 is designed via the selection of the disturbance model and estimator gain L . Once the disturbance model (i.e. the pair (A_d, C_d)) is selected, the order of the disturbance observer is fixed. The remaining design of the disturbance observer is to select a proper estimator gain L , such that the poles of $Q(s)$ are at desired locations.

III. DESIGN OF DISTURBANCE OBSERVERS

A. Design Based on Disturbance Observer Loop

As mentioned in the previous section, the key issue of the disturbance observer design is to select $Q(s)$. Assuming that $P = P_n$ in Fig. 1, $1 - Q(s)$ and $Q(s)$ represent the sensitivity and complementary sensitivity functions of the disturbance observer loop, respectively. The selection of $Q(s)$ is a design trade-off between disturbance rejection versus noise rejection and robust stability. The following structure of $Q(s)$ is commonly utilized by several researchers [1]–[3], [7]:

$$Q(s) = \frac{1 + \sum_{k=1}^{N-r} a_k (\tau s)^k}{1 + \sum_{k=1}^N a_k (\tau s)^k} \quad (14)$$

where N is the order of $Q(s)$, r is the relative degree of $Q(s)$ and the a_k 's are normally chosen to be a binomial model or Butterworth low-pass filter. The design trade-off of $Q(s)$ is achieved through the selection of the cut-off frequency, $\omega_c = 1/\tau$, of $Q(s)$. The allowable cut-off frequency ω_c is limited by the model uncertainty when we consider the robust stability of the disturbance observer loop. Suppose that the model uncertainty can be treated as a multiplicative perturbation, that is,

$$P(s) = P_n(s)(1 + \Delta(s)) \quad (15)$$

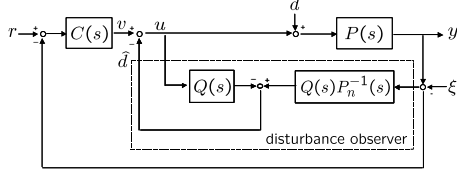


Fig. 2. Closed loop system with the disturbance observer and $C(s)$

where $\Delta(s)$ is stable. The disturbance observer loop is robustly stable [3] if,

$$|Q(s)\Delta(s)| < 1 \quad \forall s = j\omega \quad (16)$$

Therefore, the cut-off frequency ω_c is selected as high as reasonable, while (16) is satisfied.

Although the selection of $Q(s)$ based on the disturbance observer loop is straightforward with clear physical interpretation, it is important to notice that the disturbance observer is only a part of the overall controller. There is also a feedback controller $C(s)$ in the closed loop system as shown in Fig. 2. Assuming $P = P_n$, the output y of the closed loop system can be written as

$$y = \frac{P_n(s)C(s)}{1 + P_n(s)C(s)}r + \frac{P_n(s)(1 - Q(s))}{1 + P_n(s)C(s)}d + \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)}\xi \quad (17)$$

The sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ of the closed loop system are respectively defined as

$$S(s) = \frac{1 - Q(s)}{1 + P_n(s)C(s)}; T(s) = \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)} \quad (18)$$

Notice that $S(s)$ and $T(s)$ depend on $Q(s)$ and $C(s)$. As a result, even though $Q(s)$ is selected such that the disturbance observer loop is robustly stable, there is no guarantee of the robust stability of the closed loop system. This suggests that the selection of $Q(s)$ should take into consideration the feedback controller $C(s)$ as well.

B. Design Based on Closed Loop Consideration

Since the disturbance observer loop behaves as $P_n(s)$ in the low frequency region, the feedback controller $C(s)$ is normally designed based on the nominal model $P_n(s)$. Hence, it is assumed that the feedback controller $C(s)$ has been designed before the selection of $Q(s)$. It is suggested that the design of $C(s)$ should focus on the tracking performance $\frac{P_n(s)C(s)}{1 + P_n(s)C(s)}$, while the disturbance observer will improve the attenuation of disturbance.

Suppose that the closed loop performance specifications are selected as follows.

$$|W_s(s)S(s)| < 1 \quad \forall s = j\omega \quad (19)$$

$$|W_u(s)T(s)| < 1 \quad \forall s = j\omega \quad (20)$$

where $S(s)$ and $T(s)$ are as defined in (18), and $W_s(s)$ and $W_u(s)$ are two stable weighting functions. Equation

(19) specifies desired disturbance attenuation, while (20) guarantees the robust stability of the closed loop system, if $W_u(s)$ is chosen such that the multiplicative perturbation $\Delta(s)$ in (15) satisfies

$$|\Delta(s)| \leq |W_u(s)| \quad \forall s = j\omega. \quad (21)$$

Notice that (19) can be rewritten as

$$\left| \left(W_s(s) \frac{1}{1 + P_n(s)C(s)} \right) \cdot (1 - Q(s)) \right| < 1 \quad \forall s = j\omega \quad (22)$$

which can be considered as a constraint on $1 - Q(s)$. If a stable weighting function $W_p(s)$ is selected so that

$$\left| W_p(s) \frac{1}{1 + P_n(s)C(s)} \right| \leq |W_p(s)| \quad \forall s = j\omega, \quad (23)$$

then (24) is a sufficient condition for (22).

$$|W_p(s)(1 - Q(s))| < 1 \quad \forall s = j\omega \quad (24)$$

It is preferable to use (24) over (22) as a design criterion, since $W_p(s)$ can also impose a lower bound on the cut-off frequency of the high-pass filter $1 - Q(s)$. Hence, the selection of $W_p(s)$ should also consider the desired bandwidth of $Q(s)$, which is implicitly assumed to be high enough for the design of $C(s)$ based on $P_n(s)$.

The disturbance observer design based on closed loop consideration is to select $Q(s)$ so that (20) and (24) are simultaneously satisfied. It is straightforward to verify that, in Fig. 2, the transfer function from d to u , $G_{ud}(s)$, is $-T(s)$, while the transfer function from d to \hat{d} is $Q(s)$. Let $w = d - \hat{d}$, then the transfer function from d to w , $G_{wd}(s)$, is $1 - Q(s)$. The disturbance observer design can be stated as the following H_∞ optimization problem:

$$\text{Find } Q(s) \text{ such that} \quad (25)$$

- (i) the closed loop system is stable and
- (ii) $\left\| \begin{array}{l} W_p(s)G_{wd}(s) \\ W_u(s)G_{ud}(s) \end{array} \right\|_\infty < 1$

Notice that $Q(s)$ in the problem statement (25) is searched in the proper rational transfer function space, the dimension of which is infinite, subject to the constraints: 1) $Q(s)$ is a low-pass filter with a steady state gain of one, and 2) the relative degree of $Q(s)$ is greater than or equal to that of the plant $P_n(s)$. This problem may not be easy to solve.

The problem (25) can be simplified, if the fixed-structure disturbance observer is utilized. As mentioned in Section II-B, the design of the fixed-structure disturbance observer is to select estimator gain L , once the disturbance model (6) is chosen. Thus, the problem (25) becomes:

$$\text{Find } L \in \mathbf{R}^{n+n_d} \text{ such that} \quad (26)$$

- (i) the closed loop system is stable and
- (ii) $\left\| \begin{array}{l} W_p(s)G_{wd}(s) \\ W_u(s)G_{ud}(s) \end{array} \right\|_\infty < 1$

This is a problem to search the static gain L in the finite dimensional vector space. By extracting the gain L

from the closed loop system, the problem (26) can be transformed into the synthesis problem of H_∞ (sub-)optimal static output feedback gain L for an extended plant $P_L(s)$, as shown in Figure 3(a), such that the transfer function matrix $F_L(P_L(s), L) = G_{ed}(s)$ satisfies

$$\|F_L(P_L(s), L)\|_\infty < 1 \quad (27)$$

The details of the extended plant $P_L(s)$ are shown in Fig. 3(b). The combination of $P_{obs}(s)$ and L is the disturbance observer defined in (11). Following (9) and (10), the state space representation of $P_{obs}(s)$ in Fig. 3(b) is given by

$$P_{obs}(s) = \begin{bmatrix} \bar{A} & \bar{B} & 0 & I \\ \bar{C}_d & 0 & 0 & 0 \\ \bar{C} & 0 & -I & 0 \end{bmatrix} \quad (28)$$

The H_∞ (sub-)optimal static output feedback gain L for $P_L(s)$ can be found by the algorithm given in the next section.

We summarize below the procedure for the disturbance observer design based on the closed loop consideration.

Design Procedure:

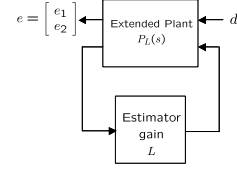
- 1) Design the feedback controller $C(s)$ based on the nominal model $P_n(s)$, focusing on the tracking performance.
- 2) Select weighting functions $W_p(s)$ and $W_u(s)$.
- 3) Determine the disturbance model (6). The simplest choice of the pair (A_d, C_d) is $(A_d = 0, C_d = 1)$, i.e. assume that the disturbance model is $1/s$. If any disturbance mode is explicitly known, it can also be included in the disturbance model.
- 4) Construct the extended plant $P_L(s)$ as shown in Fig. 3(b). Use the algorithm provided in the next section to find the gain L .
- 5) If the gain L can not be found, increase the order of the disturbance model and/or relax the specifications of the closed loop system. Go to step 4).

IV. H_∞ OPTIMIZATION ALGORITHM FOR SEARCHING GAIN L

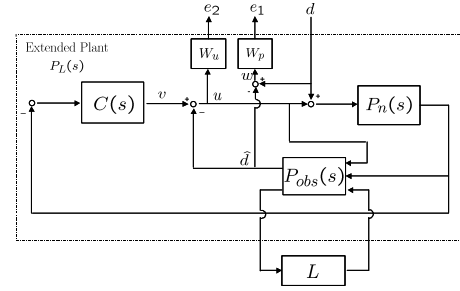
Define the state space realization of $P_L(s)$ as:

$$P_L(s) = \left[\begin{array}{c|cc} A_L & B_{L1} & B_{L2} \\ \hline C_{L1} & D_{L11} & D_{L12} \\ C_{L2} & D_{L21} & D_{L22} \end{array} \right] \quad (29)$$

where $A_L \in \mathbf{R}^{n_L \times n_L}$, $D_{L11} \in \mathbf{R}^{2 \times 1}$ and $D_{L22} \in \mathbf{R}^{1 \times (n+n_d)}$. Due to the separation property, the closed loop system is stable, if $C(s)$ stabilizes $P_n(s)$ and the gain L is chosen such that the estimator (9) is stable (see Appendix). If $W_p(s)$ and $W_u(s)$ are stable transfer functions, then there exists a stabilizing static output feedback gain L for the extended plant $P_L(s)$. As a result, (A_L, B_{L2}) is stabilizable and (A_L, C_{L2}) is detectable. It is also straightforward to verify that $D_{L22} = 0$. Under these conditions, the following theorem can be used to determine the existence of static



(a) Extraction of gain L



(b) Block diagram of $P_L(s)$

Fig. 3. Static output feedback of the extended plant $P_L(s)$

output feedback gain L such that $\|F_L(P_L(s), L)\|_\infty < \gamma$ for given $\gamma > 0$ [8].

Theorem 1: There exists a gain L such that $\|F_L(P_L(s), L)\|_\infty < \gamma$ if and only if there exist two symmetric matrices $X \in \mathbf{R}^{n_L \times n_L}$ and $Y \in \mathbf{R}^{n_L \times n_L}$ such that

$$\begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_L X + X A_L^T & X C_{L1}^T & B_{L1} \\ C_{L1} X & -\gamma I & D_{L11} \\ B_{L1}^T & D_{L11}^T & -\gamma I \end{bmatrix} \begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix} \prec 0 \quad (30)$$

$$\begin{bmatrix} N_2 & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_L^T Y + Y A_L & Y B_{L1} & C_{L1}^T \\ B_{L1}^T Y & -\gamma I & D_{L11}^T \\ C_{L1} & D_{L11} & -\gamma I \end{bmatrix} \begin{bmatrix} N_2 & 0 \\ 0 & I \end{bmatrix} \prec 0 \quad (31)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \succeq 0 \quad (32)$$

$$\text{rank}(XY - I) = 0 \quad (33)$$

where N_1 and N_2 denote bases of the null spaces of $\begin{bmatrix} B_{L2}^T & D_{L12}^T \end{bmatrix}$ and $\begin{bmatrix} C_{L2} & D_{L21} \end{bmatrix}$, respectively.

Notice that (30) ~ (32) are convex constraints on X and Y , but the rank condition (33) is not. The problem of searching the matrices X and Y that satisfy constraints (30) ~ (33) is equivalent to the following trace minimization problem [9].

$$\min_{X, Y \in \mathbf{R}^{n_L \times n_L}} \text{tr}(XY) = n_L \quad \text{subject to (30) ~ (32)} \quad (34)$$

This is not a convex optimization problem, since the objective function in (34) is not a convex function of both X and Y . As suggested in [9], we use the cone complementarity linearization algorithm [10] to solve the minimization problem. It starts with linearizing the objective function with respect to X and Y . The minimization problem (34) becomes

$$\min_{X_{k+1}, Y_{k+1}} \text{tr}(X_{k+1} Y_k + X_k Y_{k+1}) \quad \text{subject to (30) ~ (32)} \quad (35)$$

The objective function in (35) is a convex function of X_{k+1} and Y_{k+1} , when X_k and Y_k are fixed. It can be solved by convex optimization then. The algorithm is stated as follows [10].

Algorithm 1:

- 1) Find feasible $X_0 = X_0^T \in \mathbf{R}^{n_L \times n_L}$ and $Y_0 = Y_0^T \in \mathbf{R}^{n_L \times n_L}$ that satisfy (30) \sim (32). If there are none, exit. Set $k = 0$.
- 2) Solve the convex optimization problem (35) for X_{k+1} and Y_{k+1} .
- 3) If a stopping criterion is satisfied, exit. Otherwise, set $k = k + 1$ and go to Step 2).

El Ghaoui et al. [10] showed the algorithm converges and demonstrated its search performance by extensive numerical experiments. Once the optimal X and Y that satisfy (34) are found, the (sub-)optimal gain L can be found by solving the following convex feasibility problem [8].

$$\begin{bmatrix} A_L^T Y + Y A_L & Y B_{L1} & C_{L1}^T \\ B_{L1}^T Y & -\gamma I & D_{L11}^T \\ C_{L1} & D_{L11} & -\gamma I \end{bmatrix} + \begin{bmatrix} C_{L2}^T \\ D_{L21}^T \\ 0 \end{bmatrix} L^T [B_{L2}^T Y \ 0 \ D_{L12}^T] \\ + \begin{bmatrix} Y B_{L2} \\ 0 \\ D_{L12} \end{bmatrix} L [C_{L2} \ D_{L21} \ 0] \prec 0 \quad (36)$$

V. EXAMPLE

This section provides an example to illustrate the design method for the disturbance observer. We consider track-following control of a hard disk drive. The goal of track-following control is to maintain the read/write head on the track in the presence of external disturbances. The controlled plant is modelled as follows [11].

$$P(s) = \frac{K \omega_{n1}^2}{s^2 + 2\zeta_1 \omega_{n1} s + \omega_{n1}^2} \cdot \frac{\omega_{n2}^2}{s^2 + 2\zeta_2 \omega_{n2} s + \omega_{n2}^2} \cdot \frac{-0.5T_D s + 1}{0.5T_D s + 1} \quad (37)$$

The first term represents the low frequency dynamics, while the second and last term represent a structural resonant mode and Pade approximation of time delay, respectively. The bode plot of $P(s)$ is shown in Fig 4. The resonant mode appears at 3.6 kHz. The nominal model $P_n(s)$ is chosen as (38), ignoring the resonant mode and time delay.

$$P_n(s) = \frac{K \omega_{n1}^2}{s^2 + 2\zeta_1 \omega_{n1} s + \omega_{n1}^2} \quad (38)$$

The multiplicative perturbation $\Delta(s)$ in (15) can be computed by

$$\Delta(s) = P(s)/P_n(s) - 1 \quad (39)$$

The feedback controller $C(s)$ is designed to be

$$C(s) = \frac{5850(0.00084s + 1)}{0.00011s + 1} \quad (40)$$

This controller sets the gain cross-over frequency of $P_n(s)C(s)$ to about 3310 rad/s and the (nominal) phase margin to 55 degrees. The controller does not include an

integral action because disturbances are to be taken care of by the disturbance observer. The weighting functions $W_p(s)$ and $W_u(s)$ are selected as

$$W_p(s) = \frac{0.5(s+2000)}{(s+0.1)}; W_u(s) = \frac{1647.41s(s+1.3 \times 10^4)^2}{(s+10^6)(s+5 \times 10^4)^2} \quad (41)$$

$W_p(s)$ specifies the cut-off frequency of $1 - Q(s)$ to be higher than 1150 rad/s , and $W_u(s)$ satisfies $|\Delta(s)| \leq |W_u(s)|$ for all $s = j\omega$.

The disturbance model is chosen to be $1/s$, and the order of the disturbance observer is fixed to be three. The extended plant $P_L(s)$ is constructed according to Fig. 3(b). The remaining step is to find the (sub-)optimal gain L by implementing the algorithm presented in the previous section. In this paper, the algorithm has been carried out by using *LMI Control Toolbox* of *MATLAB*. The gain L is found to place the poles of the estimator (9) at -4412 and $-3801 \pm 279j$. The achieved $\|F_L(P_L(s), L)\|_\infty$ is 0.97. This choice of L corresponds to the use of the following $Q(s)$ in the fixed-structure disturbance observer:

$$Q(s) = \frac{6.409 \cdot 10^{10}}{s^3 + 1.2014 \cdot 10^4 s^2 + 4.8068 \cdot 10^7 s + 6.409 \cdot 10^{10}} \quad (42)$$

Fig. 5 shows the frequency magnitude response of the complementary sensitivity function with the fixed-structure disturbance observer (DOB1). The magnitude response is under that of $W_u^{-1}(s)$. The robust stability of the closed loop system is guaranteed. Fig. 5 also shows the magnitude response of the complementary sensitivity function with another disturbance observer (DOB2). The Q -filter of DOB2 is selected as (43).

$$Q_2(s) = \frac{3\tau s + 1}{\tau^3 s^3 + 3\tau^2 s^2 + 3\tau s + 1} \quad (43)$$

where τ is equal to $1/3900$, that is, the Q -filter bandwidth of DOB2 is close to that of DOB1. The infinity norm $\|Q_2(s)W_u(s)\|_\infty$ is computed to be 0.80. According to (16), $Q_2(s)$ seems to be an acceptable choice based on the disturbance observer loop. However, Fig. 5 shows that the closed loop system with DOB2 may not be robustly stable. Indeed, when $C(s)$ and DOB2 are applied to the plant $P(s)$, the resulting system is unstable.

VI. CONCLUSIONS

This paper presented a design method for a disturbance observer based on closed loop consideration. Given that the feedback controller of the closed loop system has been selected, the disturbance observer is designed to directly satisfy closed loop performance specifications. Taking advantage of the fixed-structure disturbance observers, the disturbance observer design problem can be transformed into the synthesis problem of H_∞ (sub-)optimal static output feedback gain for an extended plant. The static output feedback gain, and therefore the disturbance observer, can be obtained by implementing the algorithm presented in

Section IV. An example was provided to illustrate the design method. In the example, the disturbance observer designed based on closed loop consideration guaranteed robust stability, while the disturbance observer designed based on the disturbance observer loop resulted in an unstable system.

APPENDIX

Assume that $C(s) = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}$ stabilizes $P_n(s) = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$, (i.e. $\begin{bmatrix} A - BD_cC & BC_c \\ -B_cC & A_c \end{bmatrix}$ is Hurwitz), and $L = \begin{bmatrix} L_1^T & L_2^T \end{bmatrix}^T$ is chosen so that the estimator (9) is stable (i.e. $\bar{A} + L\bar{C} = \begin{bmatrix} A + L_1C & BC_d \\ L_2C & A_d \end{bmatrix}$ is Hurwitz). The "A" matrix of the closed loop system with $C(s)$ and the disturbance observer is given by

$$A_{closed} = \begin{bmatrix} A - BD_cC & BC_c & 0 & -BC_d \\ -B_cC & A_c & 0 & 0 \\ -BD_cC - L_1C & BC_c & A + L_1C & 0 \\ -L_2C & 0 & L_2C & A_d \end{bmatrix} \quad (44)$$

Equation (45) shows the separation property.

$$\begin{aligned} \det \begin{bmatrix} \lambda I - A + BD_cC & -BC_c & 0 & BC_d \\ B_cC & \lambda I - A_c & 0 & 0 \\ BD_cC + L_1C & -BC_c & \lambda I - A - L_1C & 0 \\ L_2C & 0 & -L_2C & \lambda I - A_d \end{bmatrix} \\ = \det \begin{bmatrix} \lambda I - A + BD_cC & -BC_c & 0 & BC_d \\ B_cC & \lambda I - A_c & 0 & 0 \\ 0 & 0 & \lambda I - A - L_1C & -BC_d \\ 0 & 0 & -L_2C & \lambda I - A_d \end{bmatrix} \\ = \det \begin{bmatrix} \lambda I - A + BD_cC & -BC_c \\ B_cC & \lambda I - A_c \end{bmatrix} \det \begin{bmatrix} \lambda I - A - L_1C & -BC_d \\ -L_2C & \lambda I - A_d \end{bmatrix} \end{aligned} \quad (45)$$

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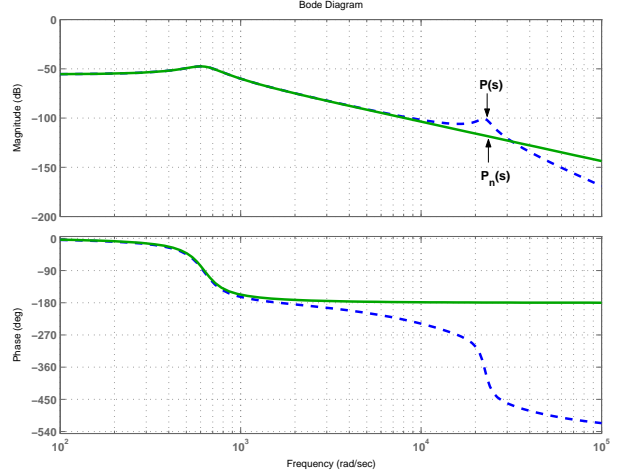


Fig. 4. Frequency responses of $P(s)$ and $P_n(s)$

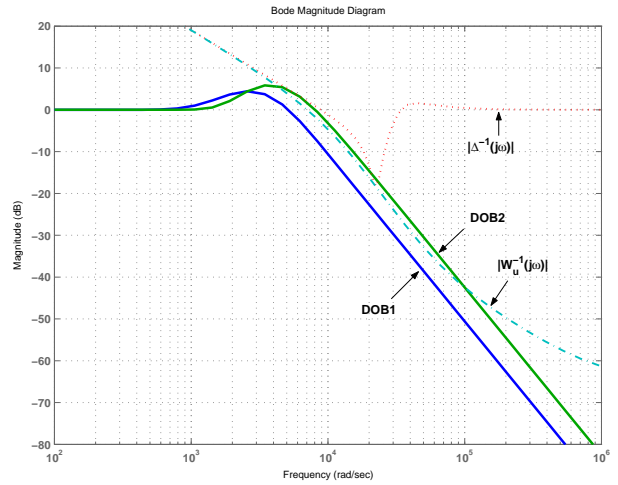


Fig. 5. Frequency magnitude responses of complementary sensitivity functions

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