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Detection of Fatigue Crack Anomaly:
A Symbolic Dynamics Approach†

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Abstract

This paper presents a novel method for early detection of fatigue crack anomaly in complex mechanical structures, which is built upon the concepts of *Symbolic Dynamics* and *Finite State Machines.* The experimental apparatus, on which the crack detection method is tested, is a multi-degree of freedom mass-beam structure excited by oscillatory motion of two vibrators. The evolution of fatigue crack at one or more of the three failure sites causes slow variations in the natural frequencies of the mechanical structure, which are detected at an early stage from the time series data of displacement sensor signals. The proposed anomaly detection method has been validated by comparison with existing pattern recognition techniques.

1 Introduction

An anomaly is defined as a deviation from the nominal behavior of a dynamical system and is often associated with parametric and non-parametric changes that may gradually evolve in time. Anomalies may manifest themselves with self excitation, or under excitation of certain exogenous stimuli. These anomalies may be benign or malignant depending on their impact on the mission objectives and operating conditions.

Major catastrophic failures in complex engineering systems could often be averted if the malignant anomalies are detected at an early stage. Anomaly detection is based on observed time series data under known external stimuli (i.e., input excitation) or under self excitation of the dynamical system. The goal is to make inferences on occurrence of *slow-time-scale* anomalies based on observed changes in behavior pattern of the *fast-time-scale* process dynamics.

From the above perspectives, anomaly detection in dynamical systems is formulated as a two-time-scale problem, in which the phase trajectories evolve in the fast time scale and anomalies, if any, progress in the slow time scale. Progression of anomalies takes place in the form of parametric or non-parametric variations in the system response and the objective is to capture this information from the observed time series data as early as possible. Thus, early detection of malignant anomalies allows the decision and control system to avert catastrophic failures and possibly satisfy the mission requirements albeit at a degraded level of performance.

A laboratory test apparatus has been constructed to experimentally validate the concepts of anomaly detection and life extending decision and control policies [ZRP00] in complex mechanical systems. The test apparatus is designed to be complex in itself due to partially correlated interactions amongst its individual components and functional modules [ZR99]. This paper focuses on experimental validation of anomaly detection due to fatigue crack in mechanical structures that exhibit self oscillations or can be excited by external stimuli.

The paper is organized in seven sections including the present one. Section 2 briefly describes the test apparatus for anomaly detection. Section 3 presents generation of fatigue crack anomalies on the test apparatus. Section 4 introduces the general concept of anomaly detection in complex systems. Section 5 presents the novel concept of anomaly detection based on symbolic dynamics, and describes the construction of state machines. Section 6 presents and discusses the experimental results. Finally, the paper is summarized and concluded in Section 7 with recommendations for future research.

2 Description of the Test Apparatus

The test apparatus is designed and fabricated as a multi-degree of freedom (DOF) mass-beam structure excited by oscillatory motion of two vibrators as shown in Fig. 1. Physical dimensions of the pertinent components are listed in Table I. Two of the three major DOF's are directly controlled by the two actuators,

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Shaker #1 and Shaker #2, and the remaining DOF is observable via displacement measurements of the three vibrating masses: Mass#1, Mass#2 and Mass#3. The inputs to the multivariable mechanical structure are the forces exerted by the two actuators; and the outputs to be controlled are the displacements of Mass #2 and Mass #3. The failure site in each specimen, attached to the respective mass is a circular hole (of radius 3.81*mm*) as shown in Fig. 1.

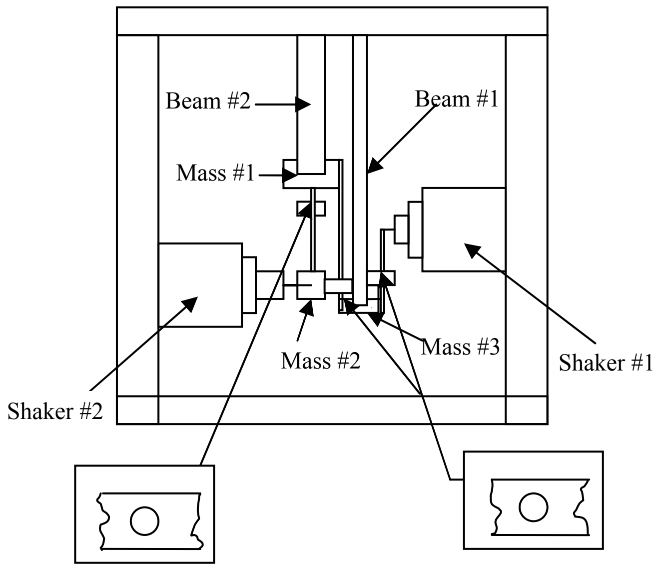

Fig. 1 Schematic Diagram for the Test Apparatus

Table I Structural dimensions of the test apparatus

Component	**Material**	**Length (mm) & Mass (kg)** **Length x width x thickness**
Mass # 1	Mild Steel	2.82
Mass # 2	Aluminium 6063-T6	0.615
Mass # 3	Mild Steel	3.87
Beam # 1	Mild Steel	800 x 22 x 11
Beam # 2	Aluminium 6063-T6	711.2 x 22.2 x 11.1
Specimens	Aluminium 6063-T6	203.2 x 22.2 x 11.1

The test apparatus system is logically partitioned into two subsystems: (i) the plant subsystem consisting of the mechanical structure including the test specimens to undergo fatigue crack damage), actuators and sensors; and (ii) the instrumentation & control subsystem consisting of computers, data acquisition and processing, and communications hardware and software. Frequency of the reference signal is 10.39 Hz that is the resonating frequency associated with Mass#3 in the mechanical structure. The test specimens are thus excited by different levels of cyclic stress as two of them are directly affected by the vibratory inputs while the remaining one is subjected to resulting stresses, thus functioning as a coupling between the two vibrating systems. In the present configuration, three test specimens are identically manufactured and their material is 6063-T6 aluminum alloy; different materials can be selected for individual specimens that may also undergo different manufacturing procedures.

The real-time instrumentation & control subsystem of the test apparatus is implemented on a Pentium PC platform. The software runs on the Real-Time Linux Operating System and is provided with A/D and D/A interfaces to the amplifiers serving the sensors and actuators of the test apparatus. The excitation signal is fed at the resonant frequency so as to facilitate the development of sufficient stress to break the specimens.

3 Generation of Fatigue Crack Anomaly

The mechanical system of the test apparatus in Fig. 1 is persistently excited near resonance so as to induce a stress level that causes fatigue failure to yield an average life of ~20,000 cycles having a total duration of approximately 40 minutes. There exists considerable scatter in fatigue data, and variations have been seen in the actual observed life of the specimens tested at same stress level. The scatter results as a consequence of fatigue sensitivity to a number of test and material parameters including specimen fabrication and surface preparation, metallurgical variables, specimen alignment in the apparatus, mean stress, and test frequency [SS92] [R99]. These uncertainty factors were taken into consideration during design of the three failure sites, shown in Fig. 1. In the present configuration of the test apparatus, the three specimens are made identical in terms of the material and manufacturing method to reduce uncertainties. Future research will allow different materials and manufacturing methods for individual specimens.

The dynamical system attains stationary behavior (in the fast time scale) under persistent excitation in the vicinity of the resonant frequency. The applied stress is dominantly flexural (bending) in nature and the amplitude of oscillations is symmetrical about the zero mean level, i.e., it is a reversed stress cycle [KL92]. Under such loading conditions, the specimens undergo high cycle fatigue where the gross stress is elastic and plasticity is only localized, which eventually leads to a catastrophic failure. Close observation indicates that fatigue failure develops in the following pattern: (i) repeated cyclic stressing causes incremental crystallographic slip and formation of persistent slip bands (PSB's); (ii) gradual reduction of ductility in the strain hardened areas results in the formation of submicroscopic cracks; and (iii) the notch effect of the submicroscopic cracks concentrates stresses until complete fracture occurs. Crack initiation may occur at a microscopic inclusion or at a site of stress concentration that is localized by creating a hole in the specimens.

Since the mechanical structure of the test apparatus consists of beams and masses, it can be approximated as a set of ordinary differential equations with parameters of damping and stiffness. The damping coefficient is essentially very small and the stiffness slowly changes due to the evolving fatigue crack. The objectives of the work reported in this paper are: (i) to detect the slowly evolving anomaly (i.e., decrease in stiffness) at an early stage by observing time series data of available sensors; and (ii) to formulate a control policy to mitigate fatigue failure in the specimen structures and thereby extend service life of the apparatus without any significant loss in performance. To achieve this goal, time series data need to be generated in real time by remote sensing if the failure site is not directly accessible to the measuring instruments.

4 Anomaly Detection in Complex Systems

Anomaly detection in complex dynamical systems is formulated as a two-time-scale problem, in which the phase trajectories evolve in the fast time scale and anomalies may evolve in the slow time scale. Anomalies take place as parametric or non-parametric variations in the system response. The goal is to capture this information by recognizing the patterns of the time series data as early as possible. The existing approaches for pattern recognition [DHS01] that are potentially applicable to the above anomaly detection problem include: (i) Feature Extraction; (ii) Artificial Neural Networking.

Linear or nonlinear feature extraction methods determine an appropriate subspace of dimension m (using either linear or nonlinear methods) in the original feature space of dimension n $(m \le n)$. The best known linear feature extractor is the Principal Component Analysis (PCA) that computes the m largest eigenvectors of the $(n \times n)$ covariance matrix of the patterns.

The most commonly used family of neural networks for pattern classification is the feed-forward network, including the Multilayer Perceptron (MLP) that is a collection of connected processing elements called nodes or neurons, arranged together in layers. Different layers in the MLP may contain different numbers of neurons. Signals pass into the input layer nodes, progress forward through the network hidden layers and finally emerge from the output layer.

This paper has adopted a novel approach to anomaly detection resulting from small cracks, which relies on the time series data of vibration signals in the test apparatus. The anomaly detection method that is built upon the concepts of *Symbolic Dynamics* and *Finite State Machines* is described in the next section.

5 Symbolic Dynamics for Anomaly Detection

The idea of behavior identification of complex dynamical systems stems from using *formal languages* [M97] and conversion from continuous to discrete representation using *symbolic dynamics* [LM99]. The concept of using *automata theory* for measure of *complexity* was suggested by Kolmogorov in terms of algorithmic complexity [CF01]. This idea was primarily based on a deterministic automaton. Crutchfield and Young [CY89] applied this concept to stochastic automata to construct the so-called ε-machine that was updated by Shalizi et al. [SSC02]. A data sequence (e.g., time series data) can be converted to a symbol sequence by partitioning the space Ω (over which the data evolves) into finitely many discrete blocks [A96] [BP97]. If $\Phi \equiv \{\varphi_1, \varphi_2, \cdots, \varphi_m\}$ is a partition of Ω (i.e., $\bigcup_{j=1}^{m} \varphi_j = \Omega$ and $\varphi_j \cap \varphi_k = \varnothing \ \ \forall j \neq k$), then each block $\varphi_j \in \Phi$ is labeled as the symbol $\sigma_j \in \Sigma$, where the symbol set Σ is called the *alphabet* consisting of m different symbols. (Note that a block $\varphi_j \in \Phi$ is not necessarily a connected subset of the space Ω .) In this way, a data sequence, obtained from a trajectory of the dynamical system, is converted to a symbol sequence $\{\sigma_i, \sigma_j, \sigma_k, \cdots\}$ that characterizes the system dynamics represented by the data sequence. The graphical display in Fig. 2 depicts a partitioning of a finite region of the phase space and a mapping from the partitioning into the symbol alphabet, which becomes a representation of the system dynamics defined by the trajectories.

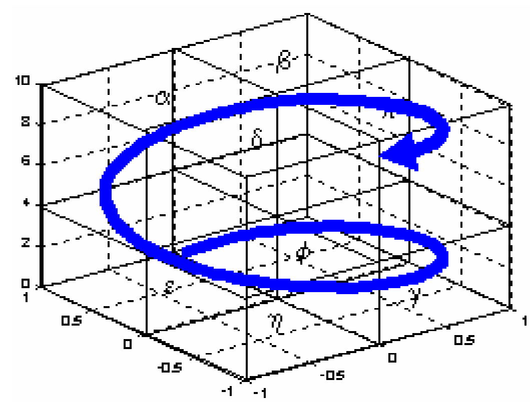

Fig. 2 Continuous Dynamics to Symbolic Dynamics

Finding dimensionality of the phase space of mechanical system dynamics can be difficult especially if the time series data are noise-corrupted [A96] [KB03]. Since the phase space dimension could often be very large, finding a generating partition, in which the set of time series data is bijectively mapped onto the set of symbols, can be difficult if not impossible. There are potentially a number of ways to (non-bijectively) partition the phase-space. Two such methods to perform the partitioning are described below.

Kennel and Buhl [KB03] have formulated a phase-space partitioning method that is built upon the concept of symbolic false nearest neighbors (SFNN), where a statistical algorithm is introduced to define empirical partitions for symbolic state reconstruction. This method avoids topological degeneracy; this is an essential feature of a generating partition [BP97]. A salient feature of the SFNN method is that the partitioning is entirely based on the time series data. The partitions are defined with respect to a set of radial-basis influence functions: $f_k(x) = \dfrac{\alpha_k}{\|x - z_k\|^2}$, each associated with a symbol s_k with the center z_k and weight α_k. For each element x of the time series data set, one $f_m(x)$ is generally expected to be greater than other $f_k(x)$ with $k \neq m$. Then, the data point x in the phase space is transformed to a symbol s in the symbol space. The parameters of z_k and α_k are the free optimization variables, with the constraint $\alpha_k \geq 0 \ \forall k$. There may be one or more influence functions assigned to each of the symbols in the alphabet. The partitions remain invariant at all epochs of the slow time scale.

An alternative scheme for obtaining partitions is based on wavelet transform of time series data, which yields a graph of coefficients versus scale at each time shift. After the wavelet transform is applied to the data, we partition the space of wavelet coefficients that is a function of scale and time. These graphs are stacked from end to end starting with the smallest value of scale and ending with the largest value. For example, the wavelet coefficients versus scale at time shift t_k are stacked after the ones at time shift t_{k-1} to obtain the so-called *scale series data* in the wavelet space, which is analogous to the time series data in the phase space. The wavelet space is partitioned into horizontal slabs. The number of blocks in a partition is equal to the size of the alphabet and each block of the partition is associated with a symbol in the alphabet. For a given stimulus, the partitioning of wavelet space must remain invariant at all epochs of the slow time scale.

5.1 Finite State Machine Construction

Finite state machines, generated from the symbol sequences of a dynamical system, identify its behavioral pattern. This section presents the concept of finite state machine construction from the wavelet transformation of time series data.

A probabilistic finite state machine is constructed from each symbol sequence, where the states of the machine are defined corresponding to the given alphabet of size $\mathcal{A}$ and window length $\mathcal{D}$. The states are joined by edges labeled by a symbol in the alphabet. The state machine moves from one state to another upon occurrence of an event as a new symbol in the symbol sequence is received. The machine language is complete in the sense that there are $\mathcal{A}$ different outgoing edges marked by different symbols $\sigma \in \Sigma$; however, it is possible that the some of these arcs may have zero probability. The effects of an anomaly are reflected in the respective state transition matrices. Thus, the structure of the finite state machine is fixed for a given alphabet size $\mathcal{A}$ and window length $\mathcal{D}$. Furthermore, the number of edges is also finite because of the finite alphabet size. The elements of the state transition matrix that is a stochastic matrix are identified from the symbol sequence.

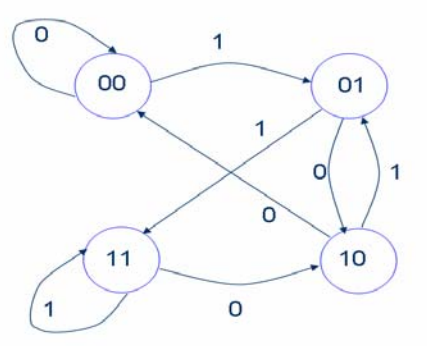

Fig. 3 State Machine with $\mathcal{D}$ =2, and Σ={0,1}

The states are chosen as words of length $\mathcal{D}$ from the symbol sequence, thereby making the total number of states to be equal to the total permutations of the alphabet symbols within word of length $\mathcal{D}$. Thus, the number of states is $\mathcal{A}^{\mathcal{D}}$ because each symbol takes on one of the $\mathcal{A}$ possible values. For machine construction, the window of length $\mathcal{D}$ is shifted to the right by one symbol upon receiving a new symbol $\sigma \in \Sigma$, such that it retains the last ($\mathcal{D}$-1) symbols of the previous state and appends it with the new symbol σ in the end. The symbolic permutation in the current window gives rise to a new state that might be a different one or the same as the previous one (i.e., forming a self loop on that state). The entire state machine is constructed in this way. As an example, let us choose $\mathcal{D} = 2$ and $\Sigma = \{0,1\}$, i.e., $\mathcal{A} = 2$. Consequently, the number of states are: $\mathcal{A}^{\mathcal{D}} = 4$; and the states are 00, 01, 10 and 11. Figure 3 elucidates the state machine construction for this specific example.

As the system trajectory evolves, different states are visited with different frequencies. The number of times a state is visited as well as the number of times a particular symbol $\sigma \in \Sigma$ is received, while sliding the window from a state leading to another state, is counted. The state probabilities as well as the state to state transition probabilities are calculated for each state in this way.

The transition probabilities associated with state to state transitions are dependent on the dynamics of the complex system as reflected in the symbol sequence from which the state transition probabilities are generated. This is the key factor in detecting an anomaly because perturbations in the system dynamics may cause significant changes in the state probabilities that, of course, are also dependent on the space partitioning.

Having obtained the state probability vector at (slow-time) epochs, the next step is to calculate the anomaly measure that signifies the change in the stationary behavior under the specific stimulus. First, the state probability vector under the nominal condition is determined as a benchmark. At different slow-time epochs (when an anomaly might have occurred), the state probability vector is determined again from the time series data collected on the fast-time scale at that particular slow-time epoch. The *anomaly measure* $\mathcal{M}$ at a given (slow-time) epoch is obtained as the distance (e.g., a norm of the vector difference, or the angle between two directions) between the state probability vector at that epoch and the state probability vector under the nominal condition. Obviously, the anomaly measure at the nominal condition is zero. In general, the anomaly measure at an epoch is different under different stimuli.

From the above perspective, the problem of anomaly detection is categorized into two parts:

a) *Forward problem*: The primary objective of the forward problem is to identify how the system performance is affected by gradually evolving anomalies and to classify the parametric and non-parametric conditions that affect the system behavior. The problem of anomaly detection focuses on identification of the patterns followed by the dynamical system as the anomaly develops slowly. Solution of the forward problem requires the following steps:

- Generation of time series data from an experimental apparatus under a number of exogenous stimuli.
- Partitioning of the phase space (or wavelet space) for generation of symbolic sequences (on the fast time scale) at different epochs of the slow time scale.
- Finite state machine construction from the symbol sequences and computation of the respective state probabilities.
- Computation of anomaly measures the respective state probability vectors with reference to the state probability vector under the nominal condition.

b) *Inverse Problem*: The inverse problem focuses on inferring the anomalies from the anomaly measures based on the observed time series data. Since this problem may be ill-posed, selection of appropriate stimuli is critical for prediction of the anomaly range.

6 Experimental Results and Discussion

The proposed anomaly detection methodology has been evaluated with time-series data generated from the test apparatus in Fig. 1. Both vibrators were excited by a sinusoidal input of amplitude 0.85 V and frequency 10.39 Hz throughout the run of each experiment. The time series data of Mass#3 displacement sensor, which serve as an indicator of the system performance, were collected from the beginning of the experiments until breakage of specimens. The ensemble of data were saved in a total of 80 files, with each file containing half a minute of sensor time-series data. Following the procedure outlined in Sections 4 and 5, the anomaly measure was obtained from the data at each half minute interval from the sensor data contained in each file. The time-series data sets were collected after the dynamic response attained the stationary behavior. The first data set was taken as the reference point representing the nominal behavior of the dynamical system. These data sets were used to compare the anomaly detection capability of the symbolic dynamics approach relative to that of two existing pattern recognition techniques: Principal Component Analysis (PCA) and Multilayer Perceptron Neural Network (MLP NN). Since symbol generation from time series data is the crucial step in symbolic-dynamics-based anomaly detection, we have investigated two alternative approaches – Symbolic False Nearest Neighbor (SFNN) partitioning and Wavelet Space (WS) partitioning.

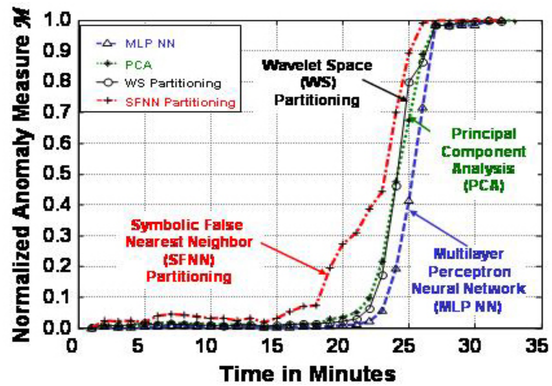

Fig. 4 Anomaly Measure under Persistent Stimulus

The four plots in Fig. 4 compare the anomaly measures obtained by using the afore-described four anomaly detection approaches, SFNN, WS, PCA and MLP NN, for the first 70 files (i.e., up to 35 minutes) when the service life of the test specimen is largely expired, i.e., the specimen is about to break. (Note: The estimated service life of the specimen under this load excitation is about 40 minutes.) The symbolic dynamics-based anomaly detection with SFNN partitioning (shown by red dash-dot line) yields the best performance and the MLP neural network (shown by blue dash line) yields the worst performance in terms of early detection of anomalies. SFNN detects the anomaly within 18 minutes when the remaining life is about 50 percent of the total service life of 35 minutes. In contrast, MLP neural network takes about 24 minutes to detect the anomaly at a similar level, which is equivalent to having the remaining life about 30 percent of the total service life.

The remaining two plots in Fig. 4 show that the methods of symbolic dynamics with WS-partitioning (shown by black solid line) and PCA (shown by green dotted line) are comparable. However, experience shows that WS-partitioning is significantly more robust than PCA. The rationale is that the PCA method is dependent on eigenvalues and eigenvectors of the covariance matrix that is sensitive to measurement noise in the data acquisition process. In contrast, the symbolic dynamic approach, with both SFNN and WS partitioning, are much less sensitive to (zero-mean) measurement noise because of the inherent averaging due to repeated path traversing in the finite-state machine.

7 Summary, Conclusions and Future Research

The anomaly detection technique, presented in this paper, is built upon the principles of Symbolic Dynamics and Finite State Machines, where anomalies are assumed to evolve slowly relative to the process dynamics. The goal is to detect fatigue crack anomalies well ahead of reaching a critical condition such as the onset of wide spread fatigue damage. This information, in turn, could be used for decision and control leading to life extension of complex mechanical system [ZRP00].

Upon the partitioning of the phase space (or wavelet space), a sequence of symbols is generated from the time series data under SFNN partitioning (or scale series data under WS partitioning) at slow-time epochs. Then, a probabilistic finite state automaton is constructed from the symbol sequences at these slow-time epochs. The anomaly measure at a given epoch is obtained as the distance, based on a chosen metric, between the state probability vector of the finite state machine at that epoch and the state probability vector of the finite state machine at the nominal condition. Thus, the above measure quantifies possible growth of fatigue crack anomaly from the nominal condition as the system progresses in the slow time scale.

A laboratory test apparatus has been constructed to experimentally validate the concepts of fatigue crack anomaly detection in complex mechanical systems. Time series data were generated from sensor signal outputs in the test apparatus to demonstrate efficacy of the anomaly detection method. The test data sets were used to compare the anomaly detection capability of the symbolic dynamics approach relative to two existing pattern recognition techniques: Principal Component Analysis (PCA) and Multilayer Perceptron Neural Network (MLP NN). Since symbol generation from time series data is a crucial step in the symbolic-dynamics-based anomaly detection, two alternative approaches have been investigated for two types of partitioning – Symbolic False Nearest Neighbor (SFNN) and Wavelet Space (WS).

The symbolic dynamics-based anomaly detection with SFNN partitioning yields the best performance and the MLP neural network yields the worst performance in terms of early detection of anomalies. Although the symbolic dynamics-based anomaly detection with WS-partitioning and Principal Component Analysis (PCA) yield comparable performance, the former is significantly more robust than the latter.

The results of analysis show that the effects of fatigue crack damage are detected within about fifty percent of the total service life. This is an early prediction of incipient fatigue crack failures, which may not be easily detected by conventional fault detection techniques [B03].

Future work would involve implementation of these anomaly detection techniques in real time and synthesis of control policies to mitigate failure and extend life without any significant loss in performance. A variety of sensor data (e.g., ultrasonic, acoustic emission, optical metrology, and displacement transducers) would be used to accurately assess the fatigue crack damage and predict the onset of widespread fatigue damage.

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