Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Adaptive Robust Output Feedback Controllers Guaranteeing Uniform Ultimate Boundedness for Uncertain Nonlinear Systems

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Abstract—**The problem of robust output feedback stabilization for a class of nonlinear systems with partially known uncertainties is considered. A class of continuous adaptive robust output feedback controllers that can guarantee uniform ultimate boundedness of the resulting closed-loop systems in the presence of uncertainties is proposed. In contrast with some results presented in the control literature, the adaptive law for updating the estimate values of the unknown parameters is continuous, and the existence of the solutions to the resulting closed-loop systems in the usual sense can be guaranteed. Moreover, due to the continuity of the output feedback controller and adaptive law, the proposed adaptive robust output feedback controller is easily implemented in practical robust control, and no chattering will appear in practical systems. Finally, an illustrative example is given to demonstrate the utilization of the results.**

I. Introduction

THE problem of stabilization for uncertain systems has been widely researched over the last few decades. There are two major approaches to deal with the problem. One is the state feedback stabilization, through which many results have been achieved when all state variables of systems are available (see [1-3]). However, it is often impossible or difficult to measure all state variables in practical systems.

Another approach is the output feedback stabilization (see [4-9]). Sabrei and Khalil [9] studied a class of systems with a nominal linear part and matched uncertainties, and derived a sufficient condition for the existence of stabilizing static output feedback control. Steinberg and Corless [6] showed that if the nominal system is strictly positive real, the robust stabilization in the presence of matched uncertainties is achievable via linear static output feedback. Later, Zeheb [8] and Steinberg [7] demonstrated that for single-input/ single-output systems, the robust stabilization problem could be solved by a linear static output feedback if the nominal systems are minimum phase and have relative degree one. These results were extended by Guo [4] to multivariable systems with an equal number of inputs and outputs. And the nominal systems are minimum phase and have a non-singular high-frequency gain.

In all of the robust control schemes mentioned above, the bounds of uncertainties are assumed to be known and the designed controllers are based on the assumed bounds of uncertainties. However, it is often difficult to estimate the bounds of uncertainties for practical systems. If the actual bounds of the uncertainties exceed the assumed values used in controller design, the stability of the system could not be guaranteed. To ensure the stability, one has to use large bounds of uncertainties in the controller design, which definitely leads to large conservativeness.

When the bounds of uncertainties are unknown, there are robust control schemes that are applicable to systems with uncertainties satisfying the matching condition [10-12]. Wu and Shigemaru [11] and Wu [12] studied the robust stabilization of linear time-varying systems and nonlinear systems with unknown bounds of uncertainties and presented a class of continuous adaptive robust state feedback controllers that can guarantee the resulting closed-loop systems uniform ultimate bounded. Liu and Zhang [10] investigated the problem of decentralized output feedback stabilization for a class of interconnected systems with unknown bounds of uncertainties and suggested a class of nonlinear output feedback controllers that can guarantee the resulting closed-loop systems uniform ultimate bounded. Other works in robust control of uncertain systems have been reported in [13-15].

Based on the framework described in [10-12], this paper discusses the adaptive robust feedback stabilization for a class of nonlinear systems with uncertain parameters. The uncertainties are bounded, but the bounds of the uncertainties are unknown in controller design. The proposed controller can guarantee the resulting closed-loop systems uniform ultimate boundedness. Moreover, due to the continuity of the output feedback controller and adaptive

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law, the proposed adaptive robust output feedback controller is easily implemented in practical robust control, and no chattering will appear in practical systems. Finally, an illustrative example is given to demonstrate the utilization of the results.

II. Procedure for Paper Submission

Consider a class of nonlinear systems which are described by

$$\dot{x} = F(x,t) + G(x,t)(u + \xi(x,t)), \tag{1a}$$

$$y = H(x,t), \tag{1b}$$

where $x(\bullet) \in R^n$, $u \in R^m$ and $y \in R^m$ are the state, control input and measurable output respectively. $F(\bullet): R^n \times R \to R^n$, $G(\bullet): R^n \times R \to R^{n\times m}$, $H(\bullet): R^n \times R \to R^m$ are known. The uncertain $\xi(\bullet)$ is assumed to be bounded.

The question is how to design an output feedback controller that can guarantee the stability of the non-linear system (1). Before investigating the robust control of the system (1), the following assumptions are proposed:

Assumption 1: for all $x \in R^n$ and $t \geq 0$, there exists known function $\rho(\bullet): R^m \times R \to R^p$ and an unknown const vector $\theta^* \in R^p$ such that

$$\|\xi(x,t)\| \leq \rho^T(y,t)\theta^*, \tag{2}$$

where $\rho(\bullet) = [\rho_1(\bullet), \rho_2(\bullet), \cdots, \rho_p(\bullet)]^T$, $\theta^* = [\theta_1^*, \theta_2^*, \cdots, \theta_p^*]^T$, and where $\rho_i(y,t) > 0, i = 1,2,\cdots,p$ for all y such that $\|y\| > 0$. And the function $\rho_i(\bullet) > 0, i = 1,2,\cdots,p$ is also assumed to be continuous uniformly bounded with respect to time and locally uniformly bounded with respect to the output y.

Assumption 2: The known functions $F(\bullet)$, $G(\bullet)$, $\rho(\bullet)$ and $H(\bullet)$, as well as the unknown function $\xi(\bullet)$, verify the Carathèodory conditions, i.e. for all t and x in a bounded domain D of the (t, x)-space.

1) they are continuous in x for almost all t;
2) they are Lebesgue measurable in t for each x;
3) there exist Lebesgue summable functions $m_i(t), i = 1, 2,3,4,5$ such that

$$\|F(x,t)\| \leq m_1(t), \quad \|G(x,t)\| \leq m_2(t), \quad \|H(x)\| \leq m_3(t),$$
$$\|\xi(x,t)\| \leq m_4(t), \quad \|\rho(y,t)\| \leq m_5(t).$$

Therefore, equation (1) can be considered as a Carathèodory equation provided that u is defined as a Carathèodory function of y and t. Furthermore, we assume that each one of these functions is locally Lipschitz continuous in its first argument x (y). Thus, under these basic assumptions, the system described by equation (1) is well posed in the sense that the local existence and uniqueness of the solutions can be proved.

Assumption 3: There exists a C^1 function $V_0(\bullet): R^n \times R \to R^+$ and an output feedback function $u_0 = \psi(y,t)$, such that for all $(x,t) \in R^n \times R$

$$c_1(\|x\|) \leq V_0(x,t) \leq c_2(\|x\|), \tag{3}$$

$$\frac{\partial V_0(x,t)}{\partial t} + \frac{\partial V_0(x,t)}{\partial x}[F(x,t) + G(x,t)\psi(y,t)] \leq -c_3(\|x\|), \tag{4}$$

where $\psi(y,t)$ is continuous for y and t, and the scalar functions $c_i(\bullet), i = 1,2$ are of K_∞-class and $c_3(\bullet)$ of K-class.

Assumption 4: There exists an nonsingular $m \times m$ matrix function $D(y,t)$ such that

$$\frac{\partial V_0(x,t)}{\partial x} G(x,t) = [D(y,t)y]^T, \tag{5}$$

where $V_0(x,t)$ is given by Assumption 3.

Remark 1: Assumption 1 defines the uncertainty bands for $\xi(x,t)$ (similar to the constraint in [12]), which are partially known (for output y). i.e. they are linear in some unknown constant vectors. Assumption 2 is a technical assumption for mathematical completeness, which can guarantee the existence and uniqueness of the solutions of state equation (1). Assumption 3 shows that the nominal system can be stabilized via output feedback in the sense that a Lyapunov function exists. Indeed, in order to guarantee the robust stability of uncertain systems, their nominal systems must be stabilizable via output feedback. Assumption 4 is the generalization of the fundamental conditions for discussing the output feedback stabilization. It is closely related to the passiveness of the systems[4-10, 16, 17].

III. Adaptive Robust Control Scheme

For equation (1), we propose an output feedback controller described by

$$u(t) = u_0(t) + u_1(t), \tag{6}$$

where

$$u_0 = \psi(y,t), \tag{7}$$

$$u_1 = -\frac{(\rho^T(y,t)\hat{\theta}(t))^2 D(y,t)y}{\|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) + \varepsilon}, \tag{8}$$

where the scalar $\varepsilon > 0$, and the vector $\hat{\theta}(t) \in R^p$ is the estimate of the unknown parameter vector $\theta^* \in R^p$, hich is updated by the following adaptive law

$$\dot{\hat{\theta}}(t) = -\delta\Gamma\hat{\theta}(t) + \frac{1}{2}\|D(y,t)y\|\Gamma\rho(y,t), \tag{9}$$

where the parameter δ is any positive constant, $\Gamma \in R^{p\times p}$ is any constant symmetric positive definite matrix, and $\hat{\theta}(t_0)$ is finite.

Applying (7) and (8) to equation (1) yields a closed-loop system of the form

$$\dot{x} = F(x,t) + G(x,t)(\psi(y,t) + u_1 + \xi(x,t)). \tag{10}$$

Moreover, we define

$$\tilde{\theta} = \hat{\theta}(t) - \theta^* . \quad (11)$$

Then, equation (9) can be rewritten as the following error equation

$$\dot{\tilde{\theta}}(t) = -\delta\Gamma\tilde{\theta}(t) + \tfrac{1}{2}\|D(y,t)y\|\Gamma\rho(y,t) - \delta\Gamma\theta^* . \quad (12)$$

In the following, we denote $(x,\tilde{\theta})$ as the solution for the closed-loop system (10) and the error equation (12).

Remark 2: Under the assumptions stated in §2, it is obvious that both the closed-loop system (10) and the error system (12) are continuous, and the existence and continuity of the solution to equations (10) and (12) in the usual sense can be guaranteed. Moreover, the controller (6) consisting of (7), (8) and adaptive law (9) can be easily implemented in practical control design.

Remark3: From Assumption 1 and 2, it is obvious that the output feedback controller (6) is locally uniformly continuous. Moreover, from equation (6) we can easily prove that

$$\|u(t)\| \le \|\psi(y,t)\| + \rho^T(y(t),t)\hat{\theta}(t) , \quad (13)$$

which shows that the control $u(t)$ is locally uniformly bounded when the solution to the adaptive law (9) exists.

The following theorem shows that the solution $(x,\tilde{\theta})$ to the close-loop system (10), the error system (12) is uniformly ultimately bounded.

Theorem 1: Consider the closed-loop system (10) and the error system (12) satisfying Assumptions 1-4. Then the solution $(x,\tilde{\theta})(t;t_0,x(t_0),\tilde{\theta}(t_0))$ to the closed-loop system (10) and the error system (12) is uniformly ultimately bounded in the presence of the uncertain $\xi(x(t),t)$.

Proof: For the close-loop system (10), the error system (12), we define a Lyapunov function candidate as

$$V(x,\tilde{\theta}) = V_0(x,t) + \tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t) , \quad (14)$$

where $V_0(x,t)$ is a Lyapunove function for the nominal system of the system (1), and Γ^{-1} is any symmetric positive definite matrix. Then, taking the derivative of $V(x,\tilde{\theta})$ along the trajectories of the closed-loop system (10) and the error equation (12) leads to

$$\begin{aligned}\frac{dV(x,\tilde{\theta})}{dt} &= \frac{\partial V_0(x,t)}{\partial t} + \frac{\partial V_0(x,t)}{\partial x}[F(x,t) \\ &\quad + G(x,t)\psi(y,t)] + \frac{\partial V_0(x,t)}{\partial x}G(x,t)(u_1(t) \\ &\quad + \xi(x,t)) + 2\tilde{\theta}^T(t)\Gamma^{-1}\dot{\tilde{\theta}}(t) \quad (15)\end{aligned}$$

and with equation (4), (5) and (12) , we get

$$\begin{aligned}\frac{dV(x,\tilde{\theta})}{dt} &\le -c_3(\|x\|) + (D(y,t)y)^T u_1(t) \\ &\quad + (D(y,t)y)^T\xi(x,t) - 2\delta\tilde{\theta}^T(t)\tilde{\theta}(t) \\ &\quad + \tilde{\theta}^T(t)\|D(y,t)y\|\rho(y,t) - 2\delta\tilde{\theta}^T\theta^* . \quad (16)\end{aligned}$$

Substituting the inequality (2) into (16), we obtain

$$\begin{aligned}\frac{dV(x,\tilde{\theta})}{dt} &\le -c_3(\|x\|) + (D(y,t)y)^T u_1(t) \\ &\quad + \|D(y,t)y\|\|\xi(x,t)\| - 2\delta\tilde{\theta}^T(t)\tilde{\theta}(t) \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\tilde{\theta}(t) + 2\delta\|\tilde{\theta}\|\|\theta^*\| \\ &\le -c_3(\|x\|) + (D(y,t)y)^T u_1(t) \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\theta^* - 2\delta\tilde{\theta}^T(t)\tilde{\theta}(t) \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\tilde{\theta}(t) + 2\delta\|\tilde{\theta}\|\|\theta^*\| \\ &= -c_3(\|x\|) + (D(y,t)y)^T u_1(t) \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) - 2\delta\tilde{\theta}^T(t)\tilde{\theta}(t) \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\tilde{\theta}(t) + 2\delta\|\tilde{\theta}\|\|\theta^*\| \\ &\quad -\|D(y,t)y\|\rho^T(y,t)\tilde{\theta}(t) \\ &= -c_3(\|x\|) + (D(y,t)y)^T u_1(t) \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) \\ &\quad -2\delta\tilde{\theta}^T(t)\tilde{\theta}(t) + 2\delta\|\tilde{\theta}\|\|\theta^*\| . \quad (17)\end{aligned}$$

It can be obtained from the equation (8) that

$$\begin{aligned}&(D(y,t)y)^T u_1(t) + \|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) \\ &= -\frac{(D(y,t)y)^T(D(y,t)y)(\rho^T(y,t)\hat{\theta}(t))^2}{\|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) + \varepsilon} \\ &\quad + \|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) \\ &= \frac{\|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t)\varepsilon}{\|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) + \varepsilon} . \quad (18)\end{aligned}$$

Therefore, it follows from the equation (18) and the inequality

$$0 \le \frac{ab}{a+b} \le b , \quad \forall a \ge 0, b > 0 , \quad (19)$$

that

$$(D(y,t)y)^T u_1(t) + \|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) \le \varepsilon . \quad (20)$$

On the other hand

$$\begin{aligned}&-2\delta\tilde{\theta}^T(t)\tilde{\theta}(t) + 2\delta\|\tilde{\theta}\|\|\theta^*\| = -\delta\|\tilde{\theta}(t)\|^2 - \delta(\|\tilde{\theta}(t)\|^2 \\ &\quad - 2\|\tilde{\theta}(t)\|\|\theta^*\|) \le -\delta\|\tilde{\theta}(t)\|^2 + \delta\|\theta^*\|^2 . \quad (21)\end{aligned}$$

Then, with (20) and (21), the inequality (17) can be rewritten as

$$\begin{aligned}\frac{dV(x,\tilde{\theta})}{dt} &\le -[c_3(\|x\|) + \delta\|\tilde{\theta}(t)\|^2] + \varepsilon + \delta\|\theta^*\|^2 \\ &= -\tilde{c}_3(\|\tilde{x}\|) + \tilde{\varepsilon} , \quad (22)\end{aligned}$$

where

$$\tilde{x} := [x^T, \tilde{\theta}^T]^T , \ \tilde{c}_3(\|\tilde{x}\|) := c_3(\|x\|) + \delta\|\tilde{\theta}(t)\|^2 ,$$

$$\tilde{\varepsilon} = \varepsilon + \delta\|\theta^*\|^2 , \quad (23)$$

where is of *K*-class.

From (22), it is obvious that the Lyapunov function $V(x,\tilde{\theta})$ decreases monotonically along any solution of the equation (10) and (12) until the solution reaches the compact set

$$\Omega_f = \left\{ (x,\tilde{\theta}) \,\middle|\, c_3(\|x(t)\|) + \delta\|\tilde{\theta}(t)\|^2 \le \tilde{\varepsilon} \right\}. \quad (24)$$

Therefore, it can be concluded that the solution $(x,\tilde{\theta})(t;\, t_0, x(t_0), \tilde{\theta}(t_0))$ of the closed-loop system (10) and the error system (12) is uniformly ultimately bounded with the bound $\tilde{\varepsilon}$ given by (24).

Remark 4: It is worth pointing out that the parameters ε and δ can be selected by the system designer. Therefore, by choosing these parameters correctly, we can guarantee better stability results for adaptive systems. In fact, it can be seen from equation (23) that a smaller $\tilde{\varepsilon}$ can be guaranteed by choosing parameters ε and δ which are small enough. However, we should note that making δ small will lead to a high adaptive gain, and letting $\varepsilon \to 0$, the controller (6) will be reduced to a standard saturation-type controller, resulting in a tradeoff between the better stability results and large gains, and the loss of continuity of the controller.

Remark 5: In contrast to some results in the control literature ([16,17]), the adaptive controller (6) has good continuity. Thus, it can be easily implemented in practical engineering.

In the rest of this section, as a special case of the results obtained above, we consider the uncertain linear system described by

$$\dot{x}(t) = Ax(t) + B(u(t) + \xi(x(t),t)), \quad (25a)$$

$$y = Cx, \quad (25b)$$

where $x \in R^n$. u and $y \in R^m$ are the states, input and output, and A, B and C are constant matrices of appropriate dimensions.

Here, for the system (25), we make the following standard assumption.

Assumption 5: The matrix pair (A,B,C) defined in equation (25) is stabilizable and detectable. Transfer function $G(s)=C(sI-A)^{-1}B$ is minimum phase. The nominal system of the system (25) is nonsingular high-frequency gain, i.e. $\det(CB) \neq 0$.

Lemma 1: Consider the system satisfying Assumption 5 as described

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx, \end{cases} \quad (26)$$

Then, there exists positive definite matrix P and nonsingular matrix K such that

$$(A + \alpha I - \tfrac{1}{2}\beta BKC)^T P + P(A + \alpha I - \tfrac{1}{2}\beta BKC) + \gamma I < 0, \quad (27)$$

$$B^T P = KC, \quad (28)$$

where α, β and γ are constants. And the closed-loop system consisting of the system (26) and $u = -\frac{1}{2}\beta Ky$ is asymptotically stable.

Lemma 1 can be derived directly from the theorem 2.11 and theorem 3.3 in Guo [4].

Thus, for the system (25), the following adaptive robust feedback controller is presented

$$u(t) = p_1(y(t),t) + p_2(y(t),t), \quad (29)$$

where

$$p_1(y(t),t) = -\tfrac{1}{2}\beta Ky(t), \quad (30)$$

$$p_2(y(t),t) = -\frac{(\rho^T(y;t)\hat{\theta}(t))^2 Ky}{\|Ky\|\rho^T(y;t)\hat{\theta}(t) + \varepsilon}, \quad (31)$$

where the matrix K is defined by the equations (27) and (28), ε is any positive constant, $\hat{\theta}(\bullet) \in R^p$ the estimate of the unknown parameter $\theta^* \in R^p$ with adaptive variable satisfying

$$\dot{\hat{\theta}}(t) = -\delta\Gamma\hat{\theta}(t) + \|Ky\|\Gamma\rho(y,t), \quad (32)$$

where is δ is any positive constant, $\Gamma \in R^{p\times p}$ is any symmetric positive definite matrix, $\hat{\theta}(t_0) = \hat{\theta}_0$ is finite.

Remark 6: The controller described in equation (29) consists of two parts $p_1(\bullet)$ and $p_2(\bullet)$. Here, $p_1(\bullet)$ is a linear feedback controller which is used to stabilize the nominal system, and $p_2(\bullet)$ is a bounded, continuous adaptive output feedback controller which is used to compensate for the system uncertainties, which are partially known, to produce some type of stability result.

Applying equation (28) and (29) to (25) yields a closed-loop system of the form

$$\dot{x}(t) = [A - \tfrac{1}{2}\beta BB^T P]x(t) + B[p_2(y(t),t) + \xi(x(t),t)]. \quad (33)$$

On the other hand, the equation (32) can be rewritten as the error equation

$$\dot{\tilde{\theta}}(t) = -\delta\Gamma\tilde{\theta}(t) + \|Ky\|\Gamma\rho(y,t) - \delta\Gamma\theta^*, \quad (34)$$

where

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta^*. \quad (35)$$

Thus, the closed-loop system consisting of the system (33) and the error equation (34) is uniformly ultimately bounded, which can be verified by the following Theorem 2.

Theorem 2: Confider the closed-loop system (33) and the error equation (34) satisfying Assumption 1, 2 and 5. Then, the solution $(x,\tilde{\theta})(t;t_0,x(t_0),\tilde{\theta}(t_0))$ to (33) and (34) is uniformly ultimately bounded in the presence of the uncertain $\xi(x(t),t)$.

Proof: As in the proof of Theorem 1, we define a Lyapunov function candidate for the equations (33) and (34) as

$$V(x,\tilde{\theta}) = x^T(t)Px(t) + \tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t), \quad (36)$$

where $P \in R^{n\times n}$ is the solution to the equations (33) and (34), Γ is defined by the equation (32). Similar to the proof for Theorem 1, taking the derivative of $V(x,\tilde{\theta})$ along the trajectories of the equations (33) and (34) leads to

$$\frac{dV(x,\tilde{\theta})}{dt} \le -\bar{\delta} V(x,\tilde{\theta}) + \bar{\varepsilon}\,, \tag{37}$$

where

$$\bar{\delta} = \min\{2\alpha\,,\ \delta\lambda_{\min}(\Gamma)\},\ \bar{\varepsilon} = 2\varepsilon + \delta\|\theta^*\|^2. \tag{38}$$

From the equation (37), it is obvious that $V(x,\tilde{\theta})$ decreases monotonically along any solution of the equations (33) and (34) until the solution reaches the compact set

$$\Omega_f = \{(x,\tilde{\theta}) \mid V(x,\tilde{\theta}) \le V_f\}, \tag{39}$$

where

$$V_f = \bar{\delta}^{-1}\bar{\varepsilon}\,, \tag{40}$$

Therefore, it can be concluded that the solutions $(x,\tilde{\theta})$ $(t;t_0, x(t_0), \tilde{\theta}(t_0))$ of the equations (33) and (34) are uniformly ultimately bounded with the bound given by (40).

IV. Simulations

Consider the uncertain nonlinear system as described

$$\dot{x} = f(x,t) + g(x,t)(u + \xi(x,t))\,,\ y = x_2\,, \tag{41}$$

where

$$f(t) = \begin{pmatrix} -x_1 - e^{-2t}x_2 \\ x_1 \\ -x_1 - (1+2e^{-2t})x_2 - x_3 \end{pmatrix},\ g(x,t) = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix},$$

$$\xi(x,t) = \theta(1 + x_2^2)t\sin t \sin x_1\,,$$

where θ is an unknown uncertain parameter. If $u_0 = y$, $V_0(x,t) = x_1^2 + (1+e^{-2t})x_2^2 + (x_3 - 4x_1 + x_2)^2$ the system (41) satisfying Assumption 1-4. From the equations (6)-(9), the following adaptive output feedback controller is obtained

$$u = y - \frac{(\rho^T(y,t)\hat{\theta}(t))^2 D(y,t)y}{\|D(y,t)y\|\rho^T(y,t)\hat{\theta}(t) + \varepsilon}\,,$$

$$\dot{\hat{\theta}}(t) = -\delta\Gamma\hat{\theta}(t) + \tfrac{1}{2}\|D(y,t)y\|\Gamma\rho(y,t)$$

where $D(y,t) = -2(1+e^{-2t})$, $\rho(y,t) = (1+y^2)t|\sin t|$. Letting $\theta = -1$, $\delta = 0.01$, $\Gamma = 0.1$, $\hat{\theta}(0) = 0$ and $\varepsilon = 0.1$, the initial condition $x(0) = (5,3,-5)$, the simulation results of the system responses and the adaptive variables are shown in Figs. 1-2.

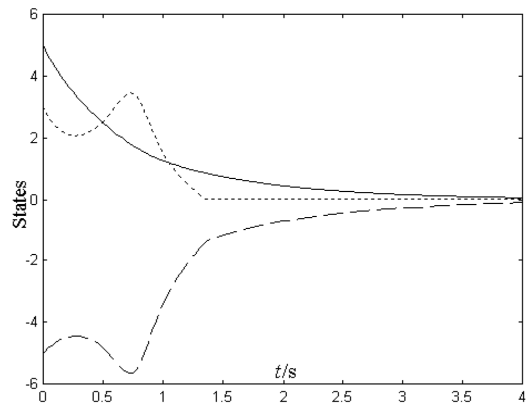

Fig 1. The system (40) responses

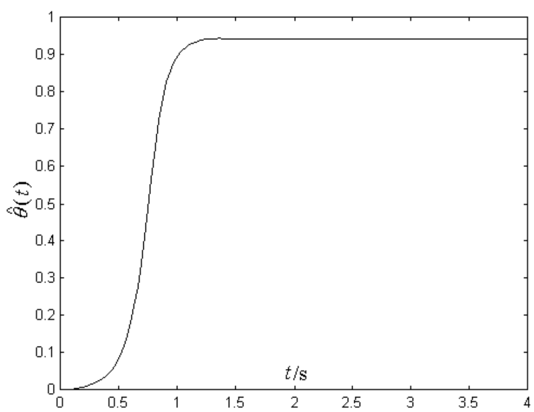

Fig 2. The adaptive variable $\hat{\theta}(t)$.

V. Conclusion

The problem of robust output feedback stabilization for a class of nonlinear systems with partially known uncertainties is considered. A class of continuous adaptive robust output feedback controllers that can guarantee uniform ultimate boundedness of the resulting closed-loop systems in the presence of uncertainties is proposed. In contrast with some results presented in the control literature, the adaptive law for updating the estimate values of the unknown parameters is continuous, and the existence of the solutions to the resulting closed-loop systems in the usual sense can be guaranteed. Moreover, due to the continuity of the output feedback controller and adaptive law, the proposed adaptive robust output feedback controller is easily implemented in practical robust control, and no chattering will appear in practical systems.

Acknowledgment

The work is supported partially by the Nation Natural Science Foundation of China (Grant no. 60374004), partially by Henan Science Fund for Distinguished Young Scholar (Grant no. 0412000200) and HAIPURT (Grant no. 2001KYCX008).

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