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Resilient Design of Thau's Observer Using LMIs

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Abstract—**Much of the recent work on robust observer design has focused on the convergence of the observer in the presence of parameter perturbations in the plant equations. The present work addresses the important problem of resilience or non-fragility, which is maintenance of convergence when the observer is erroneously implemented due possibly to computational errors. A class of nonlinear system and measurement equations is considered and a linear matrix inequality approach is presented that guarantees convergence based on the knowledge of an upper bound on the observer gain perturbations. This result can be viewed as the resilient version of Thau's method of nonlinear observer design.**

I. INTRODUCTION

An observer that is destabilized by a small perturbation in the observer gain is referred to as a "fragile" or "non-resilient" observer. In fact, the fragility problem in the control systems area is not new. After the publication of [1], the subject of fragility has gained more attention [2]-[5]. In applications, the observer gains are calculated offline using available software, hence there is a need to address the consequences, in practice, of the computation error. Also, in some implementations, it is necessary to resort to manual tuning to improve performance. Yet in some other instances, the gains may very slowly drift. Therefore, the observer must be able to tolerate some perturbations in the observer coefficients.

The recent research in nonlinear systems theory has resulted in the emergence of many new nonlinear state observation techniques: feedback linearization [6]-[8], variable structure techniques [9]-[13], extended linearization [14], and high gain observers [15], and Lyapunov-based observer design [16]-[19], which also includes Thau's observer design [16].

In this paper, we introduce a novel design of resilient Thau-type observers for continuous-time nonlinear systems with incrementally conic nonlinearities. Linear matrix inequalities (LMIs) [20] are used as the main mathematical tool.

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II. MAIN RESULTS

Consider a MIMO non-linear system of the general form:

$$\dot{x} = f(x,u,t) + g(y,u,t) \qquad (1)$$
$$y = h(x,u,t)$$

where $x \in R^n$ is the state to be estimated from knowledge of the system control input $u \in R^m$ and the measurement $y \in R^p$. The nonlinear functions f, g and h are assumed to be measurable functions of their arguments.

Assume the following incrementally conic condition on the nonlinearites:

$$\left\| \begin{bmatrix} f(x,u,t) - f(\hat{x},u,t) - A(x-\hat{x}) \\ h(x,u,t) - h(\hat{x},u,t) - C(x-\hat{x}) \end{bmatrix} \right\| \le \alpha \| x - \hat{x} \| \qquad (2)$$

for some $\alpha > 0$, where (A,C) is a detectable pair.

Let $\hat{x}$, the estimate of the true state, obey the observer equation in Thau's form [16],[17]:

$$\dot{\hat{x}} = f(\hat{x},u,t) + g(y,u,t) + (K+\Delta)(y - h(\hat{x},u,t)) \qquad (3)$$

where Δ is the additive perturbation on the gain which is bounded as follows:

$$\Delta^T \Delta \le \delta I, \; \delta > 0 \qquad (4)$$

The following theorem summarizes the main result of this paper:

<u>Theorem</u> : Let the observer (3) having gain K with a perturbation satisfying (4) be used for estimating the state of the system (1) where the system and measurement nonlinearities satisfy (2). If the following LMI:

$$\begin{bmatrix} \begin{bmatrix} -A^T P + C^T Y^T - PA \\ + YC - I - C^T C \end{bmatrix} & P & Y - C^T & P \\ P & \beta I & 0 & 0 \\ (Y - C^T)^T & 0 & (\beta - 1) I & 0 \\ P & 0 & 0 & \in I \end{bmatrix} > 0 \qquad (5)$$

holds for $P > 0$,Y and $\beta, \in > 0$, then $K = P^{-1} Y$ will guarantee a convergent observer

$$\|e(t)\| < \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \right)^{\frac{1}{2}} \|e(0)\| \quad \text{with} \quad e(t) = x(t) - \hat{x}(t)$$

for any gain perturbation satisfying (4) .

Note that $\beta = \alpha^{-2}$ and $\in = \delta^{-1}$, therefore minimizing β and $\in$ will result in the maximum bounds on the nonlinearities in (2) and the resilience in (4), respectively. Due to space restrictions, the proof of this theorem and the simulation results will be omitted.

III. CONCLUSIONS

A simple solution is presented to the problem of non-fragile observer design for a class of nonlinear systems. This design is to account for possible erroneous computation or gain perturbations in implementation to maintain the convergence of the observer. The design procedure is based on linear matrix inequalities.

IV. REFERENCES

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