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A Model Reference Robust Control with Unknown High Frequency Gain Sign [1]

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Abstract - In this paper, we discuss the model reference robust control (MRRC) for plants with relative degree one without the knowledge of the sign of the high frequency gain. A switching scheme is proposed so that, after a finite number of switchings, the tracking error converges to zero exponentially. Furthermore, if some initial states of the closed-loop system are zero, we show that only one switching is needed.

1 INTRODUCTION

Model reference robust control (MRRC) was introduced by [1],[2] as a new means of I/O based controller design for linear time invariant plants with nonlinear input disturbance and has been extended to MIMO and non-minimum phase systems [3],[4]. To overcome the influence of the nonlinear input disturbance, the conventional parameter adaptive law in model reference adaptive control (MRAC) was abandoned in the MRRC. Instead, the concept of *bounding function* was introduced in the control law design.

Like most of the model following methods, one of the basic assumptions of the MRRC is that the sign of the high frequency gain is known *a priori*. The objective of this paper is to generalize the MRRC scheme to the case of unknown sign of the high frequency gain. It is worth mentioning that this kind of problem is important in fault-tolerant control, where an autonomous supervisor must be designed to treat the mode changes of the controlled process and any operator commands [5].

The relaxation of the assumption of the high frequency gain sign has long been an attractive topic in control community. Several approaches have been proposed so far and most of them, however, are based on Nussbaum gain [6],[7]. Nussbaum gain seems particularly suitable to parameter adaptive control systems, where the gain is closely related to the adaptive law and changes continuously. The main disadvantage of the Nussbausm-type gain methods lies in the fact that it lacks robustness to the measurement noise. Furthermore, the transient behavior may be unacceptable.

An alternative way is switching. In adaptive control, switching was first introduced by Martennson [8] and then was extended to more general cases by Fu, Barmish, Miller and Davison [9]-[11] with the objective to achieve Lyapunov stability or transient and steady-state performance specifications of tracking error with minimum prior information. The main idea of this kind of control is to design a switching law which may determine among a set of controller candidates when to switch from the current one to the next. It should be pointed out that the robustness to disturbance is still a problem in [9], [10]. In fact, as shown in [9], [10], if a bounded input or output disturbance exists, the Lyapunov stability may not be retained again and the system states can only tend to some neighborhood of origin that is proportional to the size of the disturbance. In [11], a switching method was proposed so that the tracking error may have an arbitrarily good transient and steady-state performance specifications given by designer in advance even when the plant high frequency gain sign is unknown. However, the price of this solution is that the control signal may be very large.

In this paper, a switching scheme is proposed to the MRRC scheme for plants with relative degree one and unknown high frequency gain sign. Based on the Comparison Lemma [12], we first construct a monitoring function to supervise the behavior of the tracking error, and then a switching scheme for the control signal is proposed. We show that under the supervision of the monitoring function, only a finite number of switchings is needed and the tracking error will converge to zero exponentially. Interestingly enough, if some initial states of the closed-loop system are zero, we show that only one switching is needed.

2 PROBLEM FORMULATION

Consider the following single input/single output linear time invariant plant

$$y = G_p(s)[u+d] = k_p(n_p(s)/d_p(s))[u+d] \quad (2.1)$$

where y and u are the system output and input, respectively, $G_p(s)$ is the plant transfer function with $d_p(s)$ and $n_p(s)$ being polynomials of degree n and m,

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respectively, and d is an input disturbance. The reference model is given by

$$y_M = M(s)[r] = k_M \frac{1}{d_M(s)}[r], k_M > 0 \qquad (2.2)$$

where $d_M(s)$ is a monic Hurwitz polynomial satisfying $\deg(d_M(s)) = n - m := n^*$ and r is any piecewise continuous, uniformly bounded reference signal.

We make the following assumptions:

(A1) *$G_p(s)$ is minimum phase. The parameters of $G_p(s)$ are unknown but belong to a known compact set;*

(A2) *The degree n of $d_p(s)$ is a known constant;*

(A3) *The relative degree $n^* = 1$;*

(A4) *The sign of the high frequency gain $k_p (\neq 0)$ is unknown;*

(A5) *The lumped disturbance and uncertainty term $d(y,t)$ is bounded by a known continuous function $\rho(y,t)$ as, for all $(y,t) \in \mathbf{R} \times \mathbf{R}^+$,*

$$|d(y,t)| \le \rho(y,t), \ \forall t \ge 0 \qquad (2.3)$$

where the bounding function $\rho(y,t)$ is assumed to be continuous, uniformly bounded with respect to t and, locally uniformly bounded with respect to the system output y. The uncertainty $d(y,t)$ is not necessarily continuous but, if it may be discontinuous, existence of the solution of y is assumed.

The control signal of the MRRC system is of the following form:

$$u = \hat{\theta}^T \omega + u_R \qquad (2.4)$$

where u_R is the nonlinear control to be designed to ensure that the tracking error

$$e := y - y_M \qquad (2.5)$$

tends to zero, the constant vector $\hat{\theta} \in \mathbf{R}^{2n}$ will be defined below and ω, the regressor vector, is defined as

$$\omega := [v_1^T, y, v_2^T, r]^T \qquad (2.6)$$

where v_1 and v_2 are generated by input/output filters according to

$$\dot{v}_1 = \Lambda v_1 + b_\lambda u, \quad \dot{v}_2 = \Lambda v_2 + b_\lambda y, \ v_1(0) = 0,$$
$$v_2(0) = 0, \Lambda \in \mathbf{R}^{(n-1)\times(n-1)}, b_\lambda \in \mathbf{R}^{n-1}, \qquad (2.7)$$

where Λ is a matrix such that $\det(sI - \Lambda)$ is a Hurwitz polynomial and (Λ, b_λ) is controllable pair. It is well known [13] that under the above assumptions, there exits a unique constant vector

$$\theta^* = [(\theta_1^*)^T, \theta_0^*, \ (\theta_2^*)^T, k^*]^T \in \mathbf{R}^{2n}, \qquad (2.8)$$

such that, modulo exponentially decaying terms due to initial conditions,

$$y = G_p(s)[(\theta^*)^T \omega] = M(s)[r] = y_M. \qquad (2.9)$$

Since the plant parameters are assumed to be uncertain, the constant vector $\hat{\theta}$ in (2.4) is then defined as

$$\hat{\theta} := [\ (\hat{\theta}_1)^T, \ \hat{\theta}_0, (\hat{\theta}_2)^T, \hat{k}]^T \in \mathbf{R}^{2n}, \qquad (2.10)$$

which can be obtained from nominal plant and is a rough estimate of θ^*. The tracking error e can therefore be readily expressed from (2.1)-(2.10) as

$$e = M(s)\kappa^*[\tilde{\theta}^T \omega + d_f + u_R] + \bar{\varepsilon} \qquad (2.11)$$

where $\bar{\varepsilon}$ denotes a bounded, differentiable and exponentially decaying real function that represents non-zero initial conditions for all internal states of the MRRC system,

$$\tilde{\theta} := \hat{\theta} - \theta^*, \ \kappa^* := k_p / k_M = 1/k^*,$$
$$d_f := (1 - d_1(s))[d], d_1(s) := \tilde{\theta}_1^T adj(Is - \Lambda) b_\lambda . (2.12)$$

3 CONTROL LAW DESIGN

In this section, we consider the control law design for plants with $n^* = 1$. From (2.2), $n^* = 1$ implies that we can write the reference model as

$$M(s) = \frac{k_M}{s + \lambda}, \ \lambda > 0. \qquad (3.1)$$

Hence, from (2.11) and (3.1),

$$\dot{e} = -\lambda e + k_p(\tilde{\theta}^T \omega + d_f + u_R) + \varepsilon \qquad (3.2)$$

where $\bar{\varepsilon}$ in (2.11) is in fact divided into two parts: one is those related to $e(0)$, and the other one is ε, which is also a bounded, differentiable and exponentially decaying real function.

The following lemma summarizes the main result when the sign of k_p is known:

Lemma 1: *Let the relative degree one MRRC system satisfy the assumptions* (A1), (A2), (A3) *and* (A5). *Suppose the sign of k_p is known. If the control signal is defined as*

$$u_R := \begin{cases} -\dfrac{\mu|\mu|^\tau}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]} g, & \text{if} \quad k_p > 0 \\ \dfrac{\mu|\mu|^\tau}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]} g, & \text{if} \quad k_p < 0 \end{cases}$$
$$(3.3)$$

where $\beta \ge 0$, $\sigma > 0$ and $\tau \ge 0$ are design parameters, the functions g and μ are chosen such that

$$g = \mathrm{BND}(|\tilde{\theta}^T \omega + d_f|), \ \mu = eg \qquad (3.4)$$

where $\mathrm{BND}(\cdot)$ *is a bounding function* [1]. *Then, e converges exponentially to either zero (if $\beta > 0$) or a residual set (if $\beta = 0$) whose radius becomes zero in the limit as σ approaches zero. Furthermore, the control u_R is continuous and uniformly bounded for $\sigma > 0$, $\beta \ge 0$.*

Proof. See [1].

The proof of the following corollary can be found in [1].

Corollary 1: *The MRRC system is stable if and only if the tracking error e is uniformly bounded.*

Remark 3.1: *The bounding function of a signal f, say, $\mathrm{BND}(|f|)$ is a known, continuous, nonnegative function that bounds the magnitude (or Euclidean norm) of f. Readers may see* [1] *for detail about the definition.*

Since, however, the sign of k_p is unknown, we have to redefine the control as

$$u_R := \begin{cases} u_R^+ = -\dfrac{\mu|\mu|^\tau}{|\mu|^{\tau+1}+\sigma^{\tau+1}\exp[-\beta(\tau+1)t]}g, & \text{if} \quad t\in\mathbf{T}^+ \\ u_R^- = \dfrac{\mu|\mu|^\tau}{|\mu|^{\tau+1}+\sigma^{\tau+1}\exp[-\beta(\tau+1)t]}g, & \text{if} \quad t\in\mathbf{T}^- \end{cases} \tag{3.6}$$

and design a monitoring function to decide when u_R would be switched from u_R^+ to u_R^- and vice versa, where the sets $\mathbf{T}^+$ and $\mathbf{T}^-$ satisfy $\mathbf{T}^+\bigcup\mathbf{T}^-=[0,\infty)$ and $\mathbf{T}^+\bigcap\mathbf{T}^-=\phi$, and both $\mathbf{T}^+$ and $\mathbf{T}^-$ have the form $[t_k,t_{k+1})\bigcup\cdots\bigcup[t_j,t_{j+1})$. Here, t_k or t_j denotes the switching time for u_R and will be defined later. The difference between (3.3) and (3.6) is that if the sign of k_p is known, we only need one control signal while if the sign of k_p is unknown, two control signals, say, u_R^+ and u_R^- are needed, where u_R^+ and u_R^- correspond to $\mathrm{sgn}(k_p)>0$ and $\mathrm{sgn}(k_p)<0$, respectively.

To this end, we consider the *Lyapunov* function

$$V=\frac{1}{2}e^2. \tag{3.7}$$

The time derivative of V along the trajectory of (3.2) yields

$$\dot V=-\lambda e^2+k_p[(\tilde\theta^T\omega+d_f)e+eu_R]+e\varepsilon. \tag{3.8}$$

Suppose we have correctly estimated the sign of k_p for some $t\ge\bar t_0\ge 0$ where $\bar t_0$ is any finite initial time, then, by replacing (3.3) in (3.8) it follows that

$$\begin{aligned}
\dot V &\le -\bar\lambda e^2+\frac{1}{2c_\varepsilon}\varepsilon^2+ \\
&\quad |k_p|\{|(\tilde\theta^T\omega+d_f)e|-\frac{\mu|\mu|^\tau eg}{|\mu|^{\tau+1}+\sigma^{\tau+1}\exp[-\beta(\tau+1)t]}\} \\
&\le -\bar\lambda e^2+\frac{1}{2c_\varepsilon}\varepsilon^2+|k_p|\{|\mu|-\frac{|\mu|^{\tau+2}}{|\mu|^{\tau+1}+\sigma^{\tau+1}\exp[-\beta(\tau+1)t]}\} \\
&\le -\bar\lambda e^2+\frac{1}{2c_\varepsilon}\varepsilon^2 \\
&\quad +|k_p|\{\frac{|\mu|\sigma^\tau\exp(-\beta\tau t)}{|\mu|^{\tau+1}+\sigma^{\tau+1}\exp[-\beta(\tau+1)t]}\}\sigma\exp(-\beta t) \\
&\le -2\bar\lambda V+|k_p|\sigma\exp(-\beta t)+\frac{1}{2c_\varepsilon}\varepsilon^2
\end{aligned} \tag{3.9}$$

$$\Rightarrow V\to\begin{cases}0, & \text{if} \quad \beta>0 \\ \dfrac{|k_p|}{2\bar\lambda}\sigma, & \text{if} \quad \beta=0\end{cases}, \quad t\ge\bar t_0$$

where the triangle inequality

$$\varepsilon e\le(c_\varepsilon e^2+\varepsilon^2/c_\varepsilon)/2 \tag{3.10}$$

has been used with c_ε being any positive constant, the constant $\bar\lambda$ is defined as

$$\bar\lambda:=\lambda-c_\varepsilon/2>0 \tag{3.11}$$

and is given by designer in advance, and the following inequalities

$$\begin{gathered}|(\tilde\theta^T\omega+d_f)e|\le|\mu|, \\ \frac{|\mu|\sigma\exp(-\beta\tau t)}{|\mu|^{\tau+1}+\sigma^{\tau+1}\exp[-\beta(\tau+1)t]}\le 1\end{gathered} \tag{3.12}$$

have been used also, where the first inequality is from (3.4), and the proof of the second one can be found in [1].

To construct the monitoring function, we consider the following differential equation:

$$\begin{gathered}\dot\xi=-2\bar\lambda\xi+|k_p|\sigma\exp(-\beta t)+\frac{1}{2c_\varepsilon}\varepsilon^2, \\ \xi(\bar t_0)=V(\bar t_0),\ t\ge\bar t_0.\end{gathered} \tag{3.13}$$

Comparing (3.13) to (3.9), it follows that

$$\dot V(t)\le\dot\xi(t),\ \forall t\ge\bar t_0. \tag{3.14}$$

Note that $\xi(\bar t_0)=V(\bar t_0)$, then by using the *Comparison Lemma* [12, Th.7, p.214], we have

$$V(t)\le\xi(t),\ \forall t\ge\bar t_0, \tag{3.15}$$

provided a correct sign of k_p has been estimated for all $t\ge\bar t_0$.

We therefore consider the solution of (3.13). Since ε decays exponentially, there exist constants $\delta>0$ and $c>0$, such that

$$|\varepsilon(t)|\le c\exp(-\delta t),\ t\ge 0. \tag{3.16}$$

Hence, the solution of (3.13) satisfies

$$\begin{aligned}\xi(t)&\le\exp[-2\bar\lambda(t-\bar t_0)]V(\bar t_0) \\ &+c_\beta\exp[-\beta_0(t-\bar t_0)]\exp(-\beta\bar t_0) \\ &+c_\delta\exp[-2\delta_0(t-\bar t_0)]\exp(-2\delta\bar t_0)\end{aligned} \tag{3.17}$$

where

$$\beta_0=\min\{2\bar\lambda,\beta\},\ \delta_0=\min\{\bar\lambda,\delta\}, \tag{3.18}$$

$$c_\beta = 2\frac{|\bar{k}_p|\sigma}{|2\bar{\lambda}-\beta|},\ c_\delta = \frac{c^2}{|\bar{\lambda}-\delta|} \tag{3.19}$$

where $|\bar{k}_p|$ is an upper bound of $|k_p|$ which, from the assumption (A1), can be obtained *a priori*. Since β is a design parameter, we can choose β such that $\beta < 2\bar{\lambda}$; also, we can let $\delta < \bar{\lambda}$ due to the fact that a less δ can only make (3.16) more conservative. As a result,

$$\beta_0 = \beta,\ \delta_0 = \delta. \tag{3.20}$$

By taking into account (3.20), the inequality (3.17) can be rewritten as

$$\begin{aligned}\xi(t) \le \exp[-2\bar{\lambda}(t-\bar{t}_0)]V(\bar{t}_0) + c_\beta \exp(-\beta t)\\ + c_\delta \exp(-2\delta t), \qquad t \ge \bar{t}_0.\end{aligned} \tag{3.21}$$

Since ε as well as c_δ and δ are unknown, based on the above inequality, we define the monitoring function $\varphi_k(t)$ as follows:

$$\begin{aligned}\varphi_k(t) = \exp[-2\bar{\lambda}(t-t_k)]V(t_k) + c_\beta \exp(-\beta t)\\ +(k+1)\exp(-2\delta_k t),\\ t\in[t_k, t_{k+1}), t_0 := 0, k = 0,1,\cdots\end{aligned} \tag{3.22}$$

where t_k is the switching time to be defined and, $\{\delta_k\}$ is any monotonically decreasing sequence satisfying

$$\delta_k \to 0 \text{ as } k \to \infty. \tag{3.23}$$

It is clear that we obtain $\varphi_k(t)$ from (3.21) mainly by replacing both c_δ and δ by integers $k+1$ and δ_k , respectively. Note that the value of k increases as the switching proceeds while δ_k satisfies (3.23).

Remark 3.2: *The sequence* $\{(k+1)\}$ *in* (3.21) *may be replaced by any monotonically increasing sequence* $\{z_k\}$ *that tends to infinity.*

Recalling that the inequality (3.17) holds if the sign of k_p is correctly estimated, it seems natural to use ξ as a benchmark to decide whether a switching of u_R is needed. However, since ε is not available for measurement, we have to use φ_k to replace ξ and invoke the switching of φ_k . Note that from (3.7) and (3.22), we always have $V(t_k) < \varphi_k(t_k)$ at the switching point $t = t_k$. Hence, we define the switching time for u_R from u_R^- to u_R^+ (or u_R^+ to u_R^-) as follows:

$$t_{k+1} = \begin{cases}\min\{t : t > t_k,\quad V(t) = \varphi_k(t)\},\\ \qquad\qquad \text{if the minimum exists}.\\ +\infty, \text{otherwise}\end{cases} \tag{3.24}$$

We have the following main result of this section.

Theorem 1: *Suppose the MRRC system given by equations* (2.1), (2.2) *and* (3.1) *satisfies the assumptions* (A1)-(A5). *Let the control signal* u_R *be defined by* (3.6) *and the switching time of* u_R (*from* u_R^+ *to* u_R^- *and vice versa*) *be defined by* (3.24). *Then, the switching will stop after at most finite number of switchings, and the tracking error will converge to zero exponentially.*

Proof. By contradiction, suppose u_R switches between u_R^+ and u_R^- without stopping. Since c_δ and δ (see (3.19) and (3.16), respectively) are constants, and from (3.6), u_R only has two choices, $u_R = u_R^+$ or $u_R = u_R^-$, then after a finite number of k-th switchings, from (3.21) and (3.22), the following inequalities must be satisfied:

$$c_\delta < (k+1),\ \exp(-2\delta t) < \exp(-2\delta_k t),\ \forall t > t_k, \tag{3.25}$$

and, at the same time, u_R has a correct sign, *i.e.*, $u_R = u_R^+$ if $k_p > 0$ or $u_R = u_R^-$ if $k_p < 0$.

Note that we can design the control signals u_R^+ and u_R^- to be continuous (or piece-wise continuous)[1], hence, for any finite number of switchings, u_R is piece-wise continuous. That is, the solution of (3.2) exists and is continuous for any finite number of switchings [13]. The continuity of e implies that $V(t_k)$ as well as $\varphi_k(t_k)$ is bounded. Now, from (3.25) and (3.21), we have

$$\xi(t) < \varphi_k(t),\ \forall t > t_k \tag{3.26}$$

where we have replaced $\bar{t}_0$ by t_k in (3.21). However, since for a correct estimate of the sign of k_p, V satisfies (3.15), the above inequality implies that

$$V(t) < \varphi_k(t),\ \forall t > t_k. \tag{3.27}$$

Hence, from (3.24), no switching will occur again, a contradiction. Since $\varphi_k(t_k)$ is bounded and no switching is needed for all $t > t_k$, $\varphi_k(t)$ is uniformly bounded and converges to zero exponentially. Thus, (3.27) shows that V as well as e will converge to zero exponentially. Finally, by invoking the Corollary 1, the system is stable. This completes the proof. Q.E.D.

The following corollary shows a more interesting (probably surprising) fact for the relative degree one MRRC system.

Corollary 2: *if* $\varepsilon = 0$, *then at most one switching of* u_R *is needed.*

Proof. From (3.16), $\varepsilon = 0$ implies that the term $c_\delta \exp(-2\delta t)$ in (3.21) should be cancelled, *i.e.*,

$$\xi(t) \le \exp[-2\bar{\lambda}(t-\bar{t}_0)]V(\bar{t}_0) + c_\beta \exp(-\beta t),\ t \ge \bar{t}_0. \tag{3.28}$$

Therefore, once the correct sign of k_p is chosen, from (3.15), (3.28) and taking into account (3.22), the following inequality holds *for any finite* k :

$$\begin{aligned} V(t) \le \xi(t) \le \exp[-2\bar{\lambda}(t-t_k)]V(t_k) \\ +c_\beta \exp(-\beta t) < \varphi_k(t) \end{aligned}, \forall t > t_k \quad (3.29)$$

where we have replaced $\bar{t}_0$ by t_k. In comparison (3.29) with (3.24), it is clear that if the correct sign of k_p is chosen at $t_0 = 0$, no switching occurs; whereas, one switching is enough. Q.E.D.

4 SIMULATION RESULTS

We consider the following relative degree one plant:

$$G_p(s) = \frac{-(s+1)}{s^2 - s - 1}, \; x(0) = [0.5 \, , \; 0.5]^T \qquad (4.1)$$

where x is the state vector of the controllable canonical form of the plant. Note that $\operatorname{sgn}(k_p) < 0$. The reference model is

$$M(s) = \frac{1}{s+2}. \qquad (4.2)$$

The parameters of the input/output filters are $\Lambda = -2$ and $g = 1$. We choose the reference signal $r = \sin(2t)$ and the disturbance $d(y,t) = 0.2 \sin t + 0.5 \sin y + \; y^2 \cos t$. The design parameters defined by (3.6) are $\tau = 0$, $\sigma = 0.15$ and $\beta = 1$. To obtain the bounding function of (3.4), similar to [1], we write

$$\begin{aligned} \left|\tilde{\theta}^T \omega + d_f\right| &\le \mathrm{BND}(\tilde{k})r + \mathrm{BND}(\tilde{\theta}_0)|y| + \mathrm{BND}(\tilde{\theta}_1)|v_1| \\ &+ \mathrm{BND}(\tilde{\theta}_2)|v_2| + \rho + d_1(s)[\rho] := \mathrm{BND}(\left|\tilde{\theta}^T \omega + d_f\right|) \end{aligned} \qquad (4.3)$$

and choose $\hat{\theta} = 0$, $\mathrm{BND}(\tilde{k}) = 4$, $\mathrm{BND}(\tilde{\theta}_0) = \mathrm{BND}(\tilde{\theta}_1) = \mathrm{BND}(\tilde{\theta}_2) = 5$ and $\rho(y,t) = 1 + y^2$, where ω, $\tilde{\theta}$, $d_1(s)$ and d_f are defined by (2.6) and (2.12), respectively. We choose at $t = 0$ the control signal of (3.6) to be $u_R(0) = u_R^+(0)$, that is, an incorrect control is given at the beginning of the simulation since $\operatorname{sgn}(k_p) < 0$. The monitoring function φ_k is given by (3.22) where δ_k is chosen as $\delta_k = 1/(k+1)$ and $\left|\bar{k}_p\right|$ in (3.19) as $\left|\bar{k}_p\right| = 5$. The simulation results are shown in Fig.1 where we can see that after one switching of u_R from u_R^+ to u_R^-, the plant output y soon follows y_M and the tracking error converges to zero exponentially.

We then consider different initial conditions and control gains. In the first case, we increase the value of the initial condition and find that our scheme is quite insensitive to the change. Fig.2 shows that only one switching occurs even when the initial condition has been increased to $x(0) = [100 \, , \; 100]^T$. In the second case, we decrease the gain of (3.6) by replacing g with $0.95g$ since the conservativeness always exists in the design of g. Fig.3 shows that there are three switchings with the same initial condition, *i.e.*, $x(0) = [100 \, , \; 100]^T$.

The conclusion is that our scheme is quite insensitive to the initial condition, the nonlinear input disturbance and the choice of the sequences $\{z_k\}$ and $\{\delta_k\}$ (see Remark 3.2 and (3.23), respectively) provided we design the controller gain according to the way introduced by [1]. In fact, an appropriately chosen controller gain may guarantee that $V(t)$ converges faster than $\varphi_k(t)$ once a correct sign of k_p is chosen and therefore, the condition $V(t) < \varphi_k(t) \;\; \forall t > t_k$ is satisfied which, from (3.24), implies that no switching will occur again. This may give an explanation for the fact that in almost every simulation, only one switching is observed.

5 CONCLUSION

In this paper, we have introduced a switching methodology for the controller design of the MRRC system without the knowledge of the sign of the high frequency gain. The main idea of the scheme is to construct a monitoring function to supervise the behavior of the tracking error. Then, a switching scheme is proposed. We have shown that for plants with relative degree one, our scheme can guarantee the tracking error to converge to zero exponentially. Furthermore, if some of the initial states of the closed-loop system are zero, we have shown that at most one switching is needed. Generalization to plants of higher relative degree is being developed by the authors.

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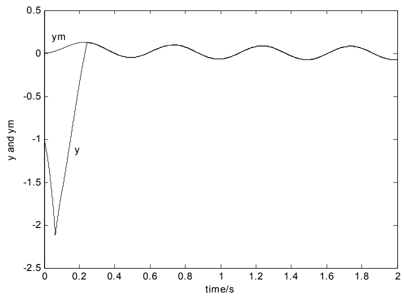

Fig. 1-1. Tracking error.

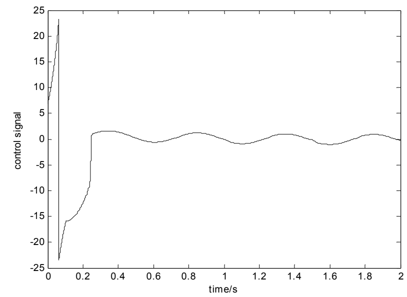

Fig.1-2. Control signal u_R.

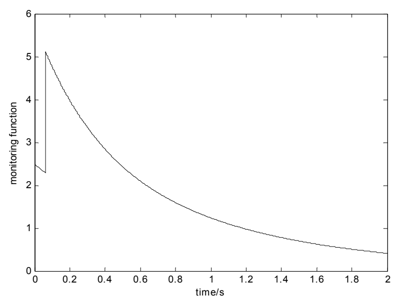

Fig.1-3. Monitoring function φ_k.

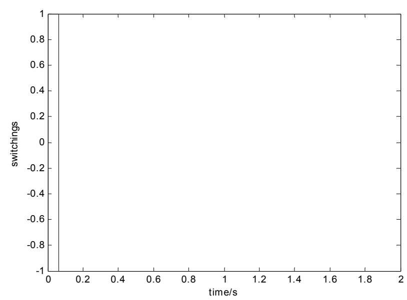

Fig.1-4. Sign switching.

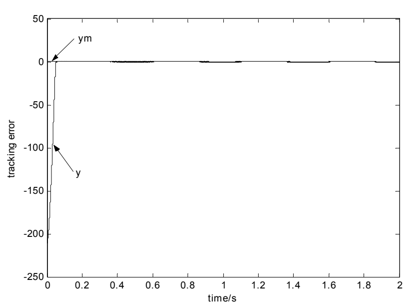

Fig. 2-1. Tracking error.

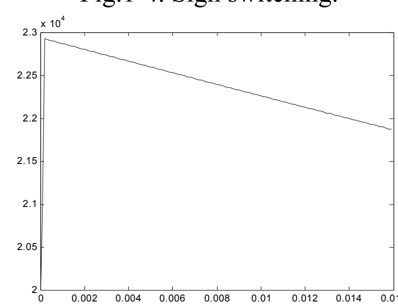

Fig.2-2. Monitoring function φ_k.

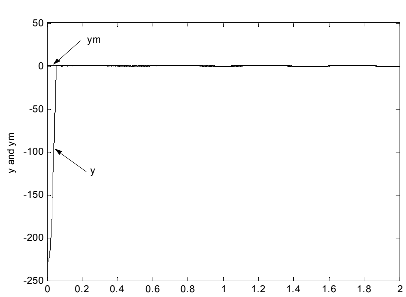

Fig.3-1. Tracking error.

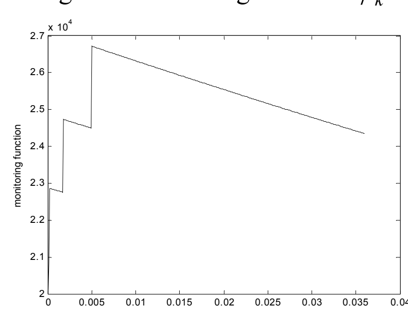

Fig.3-2. Monitoring function φ_k.