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ADAPTIVE INTERVAL MODEL CONTROL AND APPLICATION

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Abstract: In many manufacturing processes, an interval model is a useful description of the processes for control. Conditions for the same process can vary widely. Interval models take into account these variations and consequently can have wide applications. However, interval models present problems when their intervals are too wide or not accurate. An adaptive interval model control which alters the interval has been proposed in this paper. Simulations verify its effectiveness. It has been used to successfully control a novel arc welding process.

I. Introduction

In many manufacturing processes, an interval model is a useful description of the processes for control. Conditions for the same process can vary widely. Interval models take into account these variations and consequently can have wide applications. Abdallah et al. [1] and Olbrot and Nikodem [2] addressed a class of interval plants with one interval parameter. In another study [3], a prediction based algorithm with guaranteed robust steady-state performance in tracking a given set-point is proposed to control interval plants described using linear impulse response models. Despite the advantages interval model control algorithms offer over traditional control algorithms, an interval model has problems. If there is too much variation, the intervals of the model parameters become wide and system speed is affected. If the system parameters lie altogether outside the given intervals, the system is not guaranteed stability. Adaptive interval model control algorithm offers solutions to these problems. An adaptive interval model control algorithm, while starting with a set of intervals, can narrow, widen, and otherwise change the intervals based on the performance of an individual system. Thus, an adaptive interval model control algorithm should have wide applications in manufacturing processes.

II. Problem Formulation

The first problem encountered using the adaptive interval model control algorithm is that the original interval model control algorithm [3, 4] uses an impulse response model of the form:

$$y_k = \sum_{j=1}^{n} h(j) u_{j-j} \tag{1}$$

where k is the current instant, y_k is the output at k, u_{k-j} is the input at $(k-j)$ $(j>0)$, while n and $h(j)'s$ are the order and the real parameters of the impulse response function. Assume $h(j)'s$ $(1 \le j \le n)$ are time-invariant. They are unknown but bounded by the following intervals:

$$h_{\min}(j) \le h(j) \le h_{\max}(j) \quad (j = 1, ..., n) \tag{2}$$

where $h_{\min}(j) \le h_{\max}(j)$ are the minimum and maximum value of $h(j)$ and known. Assume y_0 is the given set-point. It was proved that the prediction based control algorithm guarantees

$$\lim_{k \to +\infty} y_k = y_0 \tag{3}$$

if the parameters are bounded as given in (2). In order for the control algorithm to predict and thus determine the input, the model needs to be given in the form (1) and (2). For identification purposes, this model becomes cumbersome due to the large number of parameters. To minimize the number of parameters needing identification, a second order model with auto-regression of the form

$$y_k = a_1 y_{k-1} + a_2 y_{k-2} + b_1 u_{k-1} \tag{4}$$

may be used. This type of model is referred to as an autoregressive model [5]. Thus the system parameters in the autoregressive model (4) must be converted to those in (1).

III. Model Conversion

The z-transform function of the auto-regressive model (1) can be written as:

$$H(z) = b_1 z^{-1} / (1 - a_1 z^{-1} - a_2 z^{-2}) \tag{5}$$

If the system has two real poles $z = \alpha_1$ and $z = \alpha_2$, the parameters can be converted into poles and the function can be rewritten. Partial fraction expansion can be used to manipulate the equation (5). Hence,

$$H(z) = h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + ... \tag{6}$$

That is, the auto-regressive model is equivalent to the moving average model (1) where:

$$h(j) = b_1 (\alpha_1^j - \alpha_2^j) / (\alpha_1 - \alpha_2) \quad (j = 1,2,3,...) \tag{7}$$

A similar method of manipulation may be used for repeated real roots and complex conjugate roots. If the two real poles are the same, i.e., $z = \alpha$, then

$$h(j) = j b_1 \alpha^{j-1} \quad (j = 1,2,3...,) \tag{8}$$

If the two poles are complex conjugates, $z = u \pm iv$ where $u \in R$, $v \in R$, and $v > 0$,

$$H(z) = \frac{b_1 z^{-1}}{1 - 2\sqrt{u^2+v^2} \dfrac{u}{\sqrt{u^2+v^2}} z^{-1} + (u^2+v^2) z^{-2}} \tag{9}$$

The relationships $u = a_1/2$ and $(u^2+v^2) = -a_2$, given in the above equation hold because

$$\begin{aligned} 1 - a_1 z^{-1} - a_2 z^{-2} &= (1 - (u+iv) z^{-1})(1 - (u-iv) z^{-1}) \\ &= 1 - 2uz^{-1} + (u^2+v^2) z^{-2} \end{aligned} \tag{10}$$

For a stable system, $\sqrt{u^2+v^2} < 1$. It can be shown

$$h(j) = \frac{b_1}{v} (u^2+v^2)^{\frac{j}{2}} \sin(j\{\sin^{-1} \frac{v}{\sqrt{u^2+v^2}}\}) \quad (j = 1,2,3,...) \tag{11}$$

IV. Interval Conversion

Assume the intervals of the parameters in model (5) can be obtained from on-line identification:

$$a_{j\min} \le a_j \le a_{j\max} (j=1,2), b_{1\min} \le b_1 \le b_{1\max} \quad (12)$$

These intervals need to be used to find the intervals in (2). To this end, letís use the case of two distinctive real poles as an example. In this case, (7) can be written as

$$h(j) = b_1(\alpha_1^{j-1} + \alpha_1^{j-2}\alpha_2 + \alpha_1^{j-3}\alpha_2^2 + ... + \alpha_2^{j-1}) \quad (j = 1,2,3,...) \quad (7A)$$

Because b_1 is independent from α_1 and α_2, $h_{\max}(j)$ and $h_{\min}(j)$ will occur at four possible locations $(b_{1\min}, \min f(\alpha_1, \alpha_2))$, $(b_{1\min}, \max f(\alpha_1, \alpha_2))$, $(b_{1\max}, \min f(\alpha_1, \alpha_2))$, and $(b_{1\max}, \max f(\alpha_1, \alpha_2))$ where $f(\alpha_1, \alpha_2) = \alpha_1^{j-1} + \alpha_1^{j-2}\alpha_2 + \alpha_1^{j-3}\alpha_2^2 + ... + \alpha_2^{j-1}$. While $b_{1\min}$ and $b_{1\max}$ are given, one must find $\min f(\alpha_1, \alpha_2)$ and $\max f(\alpha_1, \alpha_2)$. To achieve this, an analytic method will be used that divides the possible distinct real pole interval combinations into three categories: 1) both positive 2) both negative and 3) differing polarities. Overlapping intervals, because of the difficulty of analysis will use a quasiñanalytic method in Section E. Systems with complex poles, because of the complexity of the equation, will also use a numerical method whereby *u* and *v* intervals will be searched for instead of α_1 and α_2 intervals.

The control algorithm will determine which case the particular α_1, α_2 interval pair satisfies by first testing the sign of the products of $\alpha_{1\min}$, $\alpha_{1\max}$ and $\alpha_{2\min}$, $\alpha_{2\max}$. A positive product indicates consistency of polarity within each α interval. Consistency of polarity between the two α intervals can be determined if a positive product results from multiplying any one element of the set $[\alpha_{1\min}, \alpha_{1\max}]$ by an element of the set $[\alpha_{2\min}, \alpha_{2\max}]$. Satisfaction of both conditions narrows the α_1, α_2 pair to either case 1.) both positive or case 2.) both negative. Differentiation between case 1 and case 2 is easily accomplished. All α_1, α_2 interval pairs that do not fit all of the above requirements are assigned to 3.) differing polarities.

<u>IV.A Two Positive α Intervals</u>

$$f(\alpha_1, \alpha_2) = \alpha_1^{j-1} + \alpha_1^{j-2}\alpha_2 + \alpha_1^{j-3}\alpha_2^2 + ... + \alpha_2^{j-1} \quad (13)$$

Finding $f(\alpha_1, \alpha_2)_{\max}$ and $f(\alpha_1, \alpha_2)_{\min}$ with two positive α intervals is relatively straightforward. Positive α interval values guarantee positive values when exponentiated. To find $f(\alpha_1, \alpha_2)_{\max}$, $\alpha_{1\max}$ and $\alpha_{2\max}$ will be used. Similarly, using $\alpha_{1\min}$ and $\alpha_{2\min}$ will yield $f(\alpha_1, \alpha_2)_{\min}$.

<u>IV.B Two Negative α Interval</u>

Unlike positive α intervals, negative α intervals have different polarities when raised to different powers. Even exponents yield positive values whereas odd exponents yield negative values. When finding $f(\alpha_1, \alpha_2)_{\max}$ and $f(\alpha_1, \alpha_2)_{\min}$ with negative α intervals, analysis is aided by further dividing α interval pairs into two subclasses.

1. Odd *j* value: For every odd *j* value, *j-1* is even. The sum of the exponents of both α_1 and α_2 in any one term of $f(\alpha_1, \alpha_2)$ is *j-1* and therefore, even. Because both α interval values are negative and negative values raised to an even power become positive, all terms of $f(\alpha_1, \alpha_2)$ will be positive. To derive $f(\alpha_1, \alpha_2)_{\max}$, α ís with the greatest absolute value, $\alpha_{1\min}$ and $\alpha_{2\min}$ are required. Similarly, to derive $f(\alpha_1, \alpha_2)_{\min}$, α ís with the least absolute value, $\alpha_{1\max}$ and $\alpha_{2\max}$ should be used.
2. Even *j* value: For every even *j* value, *j-1* is odd. The sum of the exponents of both α_1 and α_2 in any one term of $f(\alpha_1, \alpha_2)$ is *j-1* and therefore, odd. Because both α interval values are negative and negative values raised to an odd power remain negative, all terms of $f(\alpha_1, \alpha_2)$ will be negative. To derive $f(\alpha_1, \alpha_2)_{\max}$, α ís with the greatest value, $\alpha_{1\max}$ and $\alpha_{2\max}$, should be used. Similarly, to derive $f(\alpha_1, \alpha_2)_{\min}$, α ís with the least value, $\alpha_{1\min}$ and $\alpha_{2\min}$, are used.

<u>IV. C Differing Polarities</u>

1. Even *j* values

For all even *j* values, $f(\alpha_1, \alpha_2)$ will have *j*, or an even number of terms. Consecutive terms can be grouped into *(j-1)/2* pairs. Factoring an $(\alpha_1 + \alpha_2)$ from each pair yields the following result:

$$f(\alpha_1, \alpha_2) = (\alpha_1 + \alpha_2)(\alpha_1^{j-2} + \alpha_1^{j-4}\alpha_2^2 + ... + \alpha_2^{j-2}) \quad (13A)$$

Analysis reveals that the polarity of the first factor, $(\alpha_1 + \alpha_2)$, depends on the relative size of the positive and negative α values. Because every term of the second factor, $(\alpha_1^{j-2} + \alpha_1^{j-4}\alpha_2^2 + ... + \alpha_2^{j-2})$, contains only α ís raised to even powers, the polarity of the second factor is guaranteed to be positive. The possibility of the first factor being either negative or positive complicates analysis, and thus it is advantageous to divide α_1, α_2 intervals further into two subclasses: a.) $|\alpha_+| > |\alpha_-|$ and b.) $|\alpha_-| > |\alpha_+|$ where $\alpha_+ > 0; \alpha_- < 0$

1a. $|\alpha_+| > |\alpha_-|$: Let it be assumed that $\alpha_1 = \alpha_-$ and that $\alpha_2 = \alpha_+$. From (13A), it can be shown that under the conditions imposed ($|\alpha_+| > |\alpha_-|$), the first factor, $(\alpha_1 + \alpha_2)$, will be positive. It has already been proved that the second factor, $(\alpha_1^{j-2} + \alpha_1^{j-4}\alpha_2^2 + ... + \alpha_2^{j-2})$ will be positive. To find $f(\alpha_1, \alpha_2)_{\max}$, both factors should be maximized. To maximize the first factor, α's with the greatest value, $\alpha_{1\max}$ and $\alpha_{2\max}$, should be used. To maximize the second factor, α ís with the greatest absolute value, $\alpha_{2\max}$ and $\alpha_{1\min}$ should be used. Both factors indicate that $\alpha_{2\max}$ should be used; however, there is inconsistency on α_1 value to be used. A numerical method explained in III.D will be used to obtain the α_1 value. To find $f(\alpha_1, \alpha_2)_{\min}$, both factors should be minimized. To minimize the first factor, α's with the least value, $\alpha_{1\min}$ and

$\alpha_{2\min}$, should be used. To minimize the second factor, α ís with the least absolute value, $\alpha_{2\min}$ and $\alpha_{1\max}$ should be used. Similar to finding $f(\alpha_1, \alpha_2)_{\max}$, both factors indicate that $\alpha_{2\min}$ should be used; but there is inconsistency on α_1 value to be used. A numerical method explained in III.D will be used to obtain the α_1 value.

1b. $|\alpha_-| > |\alpha_+|$: Let it be assumed that $\alpha_1 = \alpha_-$ and that $\alpha_2 = \alpha_+$. From (13A), it can be shown that under the conditions imposed ($|\alpha_-| > |\alpha_+|$), the first factor, $(\alpha_1 + \alpha_2)$, will be negative. It has already been proved that the second factor, $(\alpha_1^{j-2} + \alpha_1^{j-4}\alpha_2^2 + ... + \alpha_2^{j-2})$ will be positive. To find $f(\alpha_1, \alpha_2)_{\max}$, the first factor should be maximized while the second factor should be minimized. To maximize the first factor, α's with the greatest value, $\alpha_{1\max}$ and $\alpha_{2\max}$, should be used. To minimize the second factor, α ís with the least absolute value, $\alpha_{2\min}$ and $\alpha_{1\max}$ should be used. Both factors indicate that $\alpha_{1\max}$ should be used; however, there is inconsistency on the α_2 value to be used. A numerical method explained in III.D will be used to obtain the α_2 value. To find $f(\alpha_1, \alpha_2)_{\min}$, the first factor should be minimized while the second factor should be maximized. To minimize the first factor, α's with the least value, $\alpha_{1\min}$ and $\alpha_{2\min}$, should be used. To maximize the second factor, α ís with the greatest absolute value, $\alpha_{2\max}$ and $\alpha_{1\min}$ should be used. Similar to finding $f(\alpha_1, \alpha_2)_{\max}$, both factors indicate that $\alpha_{1\min}$ should be used; but there is inconsistency on the α_2 value to be used. A numerical method explained in III.D will be used to obtain the α_2 value.

2. Odd *j* values

For all odd *j* values, $f(\alpha_1, \alpha_2)$ will have *j*, or an odd number of terms. Separating the last term, α_2^{j-1} from the other terms, grouping consecutive terms into *(j-1)/2* pairs, and subsequently factoring an $(\alpha_1 + \alpha_2)$ from each pair yields the following result:

$$f(\alpha_1,\alpha_2)=(\alpha_1+\alpha_2)(\alpha_1^{j-2}+\alpha_1^{j-4}\alpha_2^2+...+\alpha_1\alpha_2^{j-3})+\alpha_2^{j-1} \quad (13B)$$

By separating the first term instead of the last, a different factorization may result of the form:

$$f(\alpha_1,\alpha_2)=(\alpha_1+\alpha_2)(\alpha_2^{j-2}+\alpha_2^{j-4}\alpha_1^2+...+\alpha_2\alpha_1^{j-3})+\alpha_1^{j-1} \quad (13C)$$

If it is assumed that $\alpha_1 = \alpha_-$ and that $\alpha_2 = \alpha_+$ and (13C) is used, analysis reveals that the polarity of the first factor, $(\alpha_1 + \alpha_2)$, depends on the relative size of the positive and negative α values. Taking into account our assumptions, the second factor, $(\alpha_2^{j-2} + \alpha_2^{j-4}\alpha_1^2 + ... + \alpha_2\alpha_1^{j-3})$, contains either negative α ís raised to even powers or positive α ís raised to odd powers; therefore, the polarity of the second factor is guaranteed to be positive. The added element will also be positive because it consists of an α value raised to an even power. The possibility of the first factor being either negative or positive complicates analysis, and it is advantageous to divide α_1, α_2 intervals further into two subclasses: a.) $|\alpha_+| > |\alpha_-|$ and b.) $|\alpha_-| > |\alpha_+|$ where $\alpha_+ > 0; \alpha_- < 0$.

2a. $|\alpha_+| > |\alpha_-|$: From (13C), it can be shown that under the conditions imposed ($|\alpha_+| > |\alpha_-|$), the first factor, $(\alpha_1 + \alpha_2)$, will be positive. It has been proved that the second factor, $(\alpha_2^{j-2} + \alpha_2^{j-4}\alpha_1^2 + ... + \alpha_2\alpha_1^{j-3})$ and the added element, α_1^{j-1} will both be positive. To find $f(\alpha_1, \alpha_2)_{\max}$, both factors and the added element should be maximized. To maximize the first factor, α's with the greatest value, $\alpha_{1\max}$ and $\alpha_{2\max}$, should be used. To maximize the second factor, α ís with the greatest absolute value, $\alpha_{2\max}$ and $\alpha_{1\min}$ should be used. To maximize the added element, an α with the greatest absolute value, $\alpha_{1\min}$, will be used. Both factors indicate that $\alpha_{2\max}$ should be used; however, there is inconsistency on the α_1 value to be used. A numerical method explained in III.D will be used to obtain the α_1 value. To find $f(\alpha_1, \alpha_2)_{\min}$, both factors should be minimized. To minimize the first factor, α's with the least value, $\alpha_{1\min}$ and $\alpha_{2\min}$, should be used. To minimize the second factor, α ís with the least absolute value, $\alpha_{2\min}$ and $\alpha_{1\max}$ should be used. To minimize the added element, an α with the least absolute value, $\alpha_{1\min}$, will be used. Similar to finding $f(\alpha_1, \alpha_2)_{\max}$, both factors indicate that $\alpha_{2\min}$ should be used; but there is inconsistency on the α_1 value to be used. A numerical method explained in III.D will be used to obtain the α_1 value.

2b. $|\alpha_-| > |\alpha_+|$: From (13C), it can be shown that under the conditions imposed ($|\alpha_-| > |\alpha_+|$), the first factor, $(\alpha_1 + \alpha_2)$, will be negative. It has already been proved that the second factor, $(\alpha_1^{j-2} + \alpha_1^{j-4}\alpha_2^2 + ... + \alpha_2^{j-2})$ and the added element, α_1^{j-1}, will be positive. To find $f(\alpha_1, \alpha_2)_{\max}$, the first factor and the added element should be maximized while the second factor should be minimized. To maximize the first factor, α's with the greatest value, $\alpha_{1\max}$ and $\alpha_{2\max}$, should be used. To maximize the added element, an α with the greatest absolute value, $\alpha_{1\max}$, will be used. To minimize the second factor, α ís with the least absolute value, $\alpha_{2\min}$ and $\alpha_{1\max}$ should be used. Both factors and the added element indicate that $\alpha_{1\max}$ should be used; however, there is inconsistency on the α_2 value to be used. A numerical method explained in III.D will be used to obtain the α_2

value. To find $f(\alpha_1,\alpha_2)_{\min}$, the first factor and the added element should be minimized while the second factor should be maximized. To minimize the first factor, α's with the least value, $\alpha_{1\min}$ and $\alpha_{2\min}$, should be used. To minimize the added element, an α with the least absolute value, $\alpha_{1\max}$ should be used. To maximize the second factor, α ís with the greatest absolute value, $\alpha_{2\max}$ and $\alpha_{1\min}$ should be used. Because there is no agreement at all, a numerical method explained in IV.D will be used to obtain both the α_1 and α_2 value.

IV.D Numerical Method for $f(\alpha_1,\alpha_2)_{\max}$ and $f(\alpha_1,\alpha_2)_{\min}$

For instances where analysis fails to yield a clear α value, a numerical method is needed. For the α value in question, the interval between $\alpha_{\min}$ and $\alpha_{\max}$ is divided into *n* sections. The boundary points between each two consecutive sections are stored as test points. These test points are inputted into one form of equation 13 and the output stored. After all test points have been tested, a comparison will reveal the $f(\alpha_1,\alpha_2)_{\max}$ and $f(\alpha_1,\alpha_2)_{\min}$. If neither α value can be found using analytic methods, both α intervals will be divided using the method above. Every combination of α test points will be inputted into one form of equation 13 to yield $f(\alpha_1,\alpha_2)_{\max}$ and $f(\alpha_1,\alpha_2)_{\min}$. For a model with two complex conjugate roots, interval conversion follows a slightly different path. Because both roots can be expressed in the form $u \pm iv$, instead of using α intervals, the *u* and *v* intervals can be used. However, similar to the α intervals, the *u* and *v* intervals will be divided into sections and the boundary points tested with the equation 13A to find $f(\alpha_1,\alpha_2)_{\max}$ and $f(\alpha_1,\alpha_2)_{\min}$.

IV.E Overlapping Intervals with consistent polarity

For instances where intervals overlap and there is consistent polarity between the intervals, a quasi-analytic method can also be developed. First the overlapping section of one pole interval will be ignored, thus reducing one pole interval. The pole interval that is reduced will be called the changed interval. With one original pole interval and one changed interval, it is then possible to use analytic method to determine which α_1, α_2 values should be used. To account for the overlapping section, a numerical method is then used with the overlapping part of the changed interval. The function values from the analytic method and the numerical method can be compared, and the true find $f(\alpha_1,\alpha_2)_{\max}$ and $f(\alpha_1,\alpha_2)_{\min}$.

After $f(\alpha_1,\alpha_2)_{\max}$ and $f(\alpha_1,\alpha_2)_{\min}$ have been attained using any method, they will be multiplied by $b_{1\min}$ and $b_{1\max}$ and the $h_{\max}(j)$ and $h_{\min}(j)$ found for a particular *j*.

V. Adaptation

The control algorithm is adaptive because it utilizes online identification. A system with auto-regression can be expressed in the form $y(k) = \varphi^T(k)\theta$ where $\varphi(k) = [y(k-1) \quad y(k-2) \quad u(k-1)]$ and $\theta = [a_1 \quad a_2 \quad b_1]$. At every time instant *k*, $\varphi(k)$ changes, causing a change in θ. A recursive algorithm [5] can be used to calculate the new θ. Once system parameters are identified online, they are converted into poles and pole intervals acquired using history. The last *n* previous instantsí poles are searched and a maximum and minimum are found. The use of pole intervals rather than parameter intervals guarantees stability within the system. Because the parameters α_1, α_2 are dependent on each other, their stability is guaranteed only for that particular combination. If a parameter interval is established and α_1, α_2 from two different instances are paired with each other, the system could become unstable. Poles are inherently stable because they are within the unit circle and consequently the better interval for adaptivity. History, while lacking a theoretical basis has been shown to be practically the same as other more complex methods.

VI. Simulation

In a series of simulations, the poles are the same (α_1=.5, α_2=.5) but the value of b_1 varies. Examination of these simulations that the b_1 location with relation to the nominal b_1 interval has a great effect on the relative performance of non-adaptive interval model control algorithms. When the actual b_1 is near the maximum of an interval (Figure 2) the non-adaptive interval model control algorithm performs slightly worse than its adaptive counterpart. However, when the actual b_1 is located near the minimum of an interval (Figure 1), the non-adaptive interval model control algorithm performs drastically worse than the adaptive. Further, as b_1 increases gradually from .2 to .8, the relative advantage of the adaptive over the non-adaptive also increases. It is apparent that the performance of the adaptive algorithm is consistent while that of the non- adaptive algorithm varies with the value of b_1 in the interval. The performance of the non-adaptive algorithm becomes worse when b_1 decreases.

It is possible that pole location can also affect the relative performance of adaptive and non-adaptive interval model control algorithms. A series of simulations has been done by varying the location of the pole. As can be seen in Figs. 7 and 11, when the pole is close to the maximum of its interval, the difference between the adaptive interval model control algorithm and its non-adaptive equivalent is small. However, when the actual pole is located near the minimum of its interval, the difference is quite large. Once again, the adaptive is performs more consistently than its non-adaptive counterpart.

Oftentimes it will be the case that a known uncertainty interval will not be centered on the true system parameter value. Taking into account the results of the previous simulations, a theoretical explanation for the differences in relative performance of adaptive and non-adaptive interval model control algorithms can be developed. Because for the system:

$$H(z) = b_1 z^{-1} / (1-\alpha_1 z^{-1})(1-\alpha_2 z^{-1}) \tag{14}$$

The gain can be expressed as [6]:

$$K = b_1 / (1-\alpha_1)(1-\alpha_2) \tag{15}$$

the discrepancy in the relative performance of adaptive and non-adaptive interval model control algorithms makes sense. From Equation 22, it is seen that the larger the actual pole or actual b_1 value, the larger the gain. Keep in mind that the interval control algorithm [3] tends to use the highest gain of the gain interval to assure stability. Hence, when the gain is near the maximum of its interval, the control algorithm accurately predicts the system response and the system response speed is thus fast. However, when the systemís gain is at the low end of the interval, the control algorithm cannot accurately predict the system response. The control action determined based on the highest gain will reduce the systemís response speed. As a result, the relative location of the actual gain in the gain interval, which can be determined by the intervals of the model parameters affect the performance of non-adaptive algorithm.

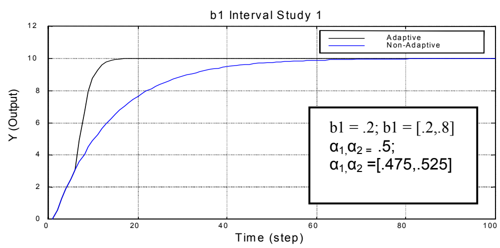

Figure 1: Simulation with b_1 = .2

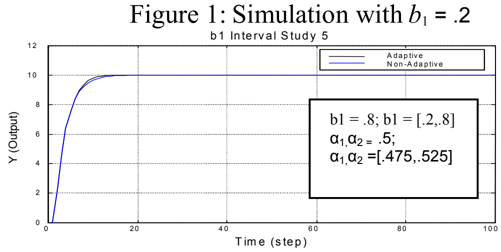

Figure 2: Simulation with b_1=. 8

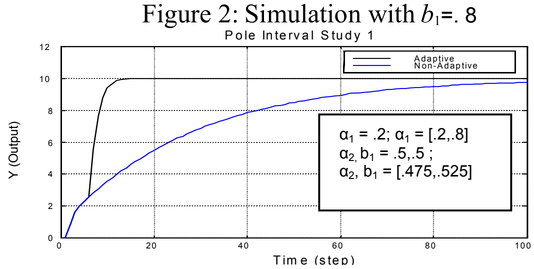

Figure 3: Simulation with α_1 = .2

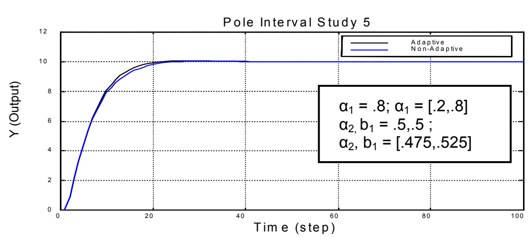

Figure 4: Simulation with α_2 = .8

VII. Application

To verify the effectiveness of the proposed adaptive interval model control algorithm, experiments were conducted with a quasi-keyhole plasma arc welding process [7]. The quasi-keyhole plasma arc welding process is a novel arc welding process which switches the current from a peak value to a base value after a keyhole is established. If the peak current is maintained, excessive metal will separate from the work piece [7]. For this process, an appropriate peak current is needed to establish the keyhole in an appropriate period, which is controlled through the amplitude of peak current. This can be measured as the time between the start of peak amperage and the time a current is detected between the work piece and detection plate. In order to detect the establishment of the keyhole for measuring the keyhole establishment time, various methods have been utilizes in the past. Some of these include monitoring the light of the plasma efflux from the keyhole [8] and the spectral lines of hydrogen and argon [9]. The utilized plasma efflux detection method for this study is the EPCS, a sensor based on the backside efflux plasma charge [10]. Once detection using this method has been proven valid [11], a control algorithm can be developed using the current as input and the keyhole establishment time as output. Previously this system was modeled by a non-linear auto-regressive model. However, the on-line identified model used in this study may be considered as a locally linearized model. Hence, using experimental data and least squares method, the system is fit to the linear auto-regressive model and controlled by the adaptive interval model control algorithm.

Using experimental data, an interval model was derived with α_1: [0.6145, 0.9555]; α_2: [-0.2244,-0.0853] and b_1: [1.2487,1.9345]. In order to examine the effectiveness of the adaptive interval model control for the plasma arc welding process, simulation has been done for a model with α_1=0.7, α_2=-0.2, b_1=1.5. As can be seen in the simulation of the plasma quasi-keyhole arc welding process shown in Figure 5, the adaptive algorithm reaches the set point (250 ms) very quickly. However, the closed-loop response speed of the non-adaptive system is very slow. In another simulation shown in Figure 6, the model parameters are α_1=0.9, α_2=-0.1, b_1=1.9. In this particular simulation, the non-adaptive performs acceptably in relation to the adaptive control algorithm. This is consistent with the findings from the previous simulations showing that system parameters affect the performance of the non-adaptive interval model control algorithm. In summary, the effectiveness of the adaptive algorithm is consistent but the non-adaptive is not assured. Hence, the adaptive algorithm offers a large advantage over its analogous non-adaptive control algorithm and can be considered for the control of quasi-keyhole arc welding process.

To test the effectiveness of the designed system, control experiments have been done. In experiment 1, the desired peak current duration is set to 250ms. The orifice diameter of the plasma arc welding nozzle is 2mm. The welding speed is kept to be 2 mm/s. The plasma gas and shielding gas flow rates are 6.5fph and 25fph, respectively. Welding experiments are fulfilled on stainless steel 304 plate with the thickness of 3.66mm. The output signal and the control signal are shown in Figure 7 and Figure 8, respectively. As can be seen, the first 10 control signals are predetermined with the average at 150A and biased by some white noises. At the 11th step, Least Square estimation is implemented according to the input-output pairs. Then, recursive Least Square estimation is implemented online to obtain the current parameters. For the first few steps right after the online estimation, the control process is oscillating a little bit. However, after a few steps, it converges to the set point and maintains it, which verifies the validity of the control algorithm. In experiment 2, the plasma gas and shielding gas flow rates are changed to be 4.5fph and 15fph, respectively.

The welding speed is 4mm/s. In order to prove the proposed algorithm can track different set points, 500ms peak current duration is selected to be the desired value. Bead-on-plate PAW is fulfilled on 4.40mm thick stainless steel plate. The output and input are shown in Figure 9 and Figure 10, respectively. The parameters variations are shown in Figure 9 and Figure 10. Same result can be concluded as the first experiment.

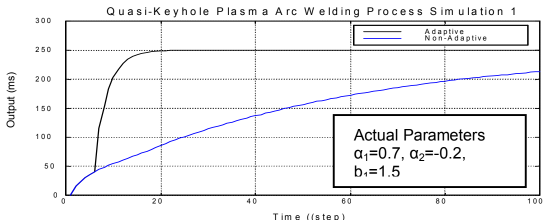

Figure 5: Plasma Arc Welding Process Simulation 1

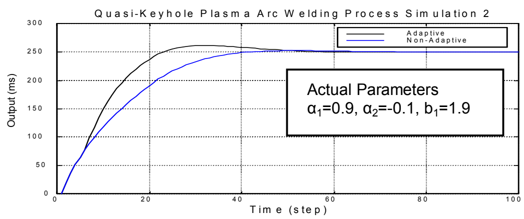

Figure 6: Plasma Arc Welding Process Simulation 2

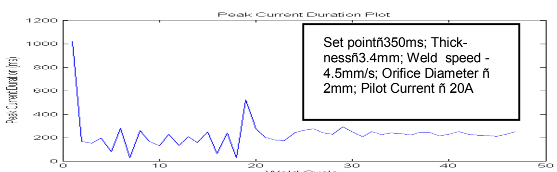

Figure 7: Control Experiment 1-Peak Current Duration

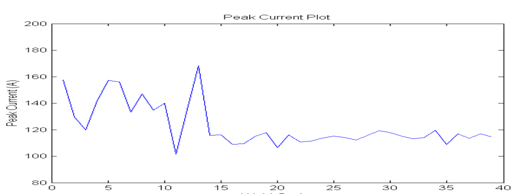

Figure 8: Control Experiment 1-Peak Current

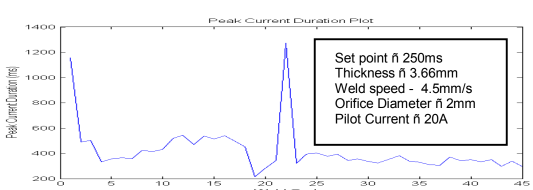

Figure 9: Control Experiment 2 - Peak Current Duration

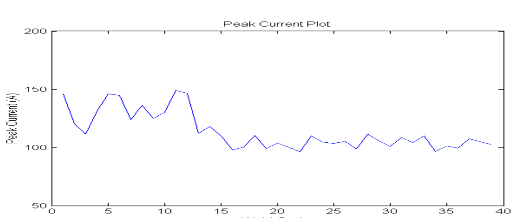

Figure 10: Control Experiment 2 - Peak Current

VIII. Conclusion

The interval model has been proved an acceptable model of control for many manufacturing processes. However, it still has its problems when the intervals are either inaccurate or too wide. The adaptive interval model from simulation can be concluded as superior to the interval model. The alteration of parameters based on system response that characterizes the adaptive interval model guarantees attaining the set point faster. The main problem with the adaptive interval model, model conversion from an impulse response model to a model with auto-regression, can be overcome through a combination of analytic and numerical methods. Application of the adaptive interval model to a real world plasma arc welding process shows its practical viability.

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