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Robust Stabilization of Markovian Jump Linear Singular Systems with Wiener Process

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Abstract: This paper deals with the robust stability and the robust stabilizability problems for the class of uncertain Markovian jump continuous-time singular systems with Wiener process. Our attention is focused on developing sufficient conditions on robust stability and the design of a state feedback controller such that the robust stochastic stability is assured, even if the singular system incorporates both Wiener process disturbance and norm-bounded uncertainties. The obtained sufficient conditions are based on linear matrix inequality technique. Numerical examples are given to show the usefulness of the proposed results.

Keywords: Jumping parameters, Singular system, Linear matrix inequality, Stochastic stability, Stabilization, Wiener process, norm-bounded uncertainties.

I. Introduction

During the past decades, singular systems has received considerable interest, because this class is more suitable than the conventional ones in modelling practical systems in different areas such as power systems, chemical systems, economics systems, robotics, etc., (see for instance, [6], [7]). When the system incorporates also abrupt changes in its structure, the Markovian jump linear system is very appropriate to describe its dynamics. For more details regarding this class of systems, we refer the reader to [8], [3] and the references therein. Many contributions related to problems like stability, stabilizability and their robustness, $\mathcal{H}_\infty$ control, output-feedback, filtering, have been reported in the literature, see for instance ([2], [4], [5]). For the class of stochastic systems with Markovian jumping parameters, a lot of works was done. In [2] sufficient conditions guaranteeing the exponential stabilizability for the uncertain system with Brownian motion perturbation are provided. The robust $\mathcal{H}_\infty$ control problem for uncertain continuous-time linear time delay systems with Markovian jumping parameters was studied in [11]. Recently, the robust stability and robust stabilizability of jumps linear systems with delays are developed in [4]. For the class of uncertain singular system, stability conditions for singular system with parameter uncertainties have been derived by applying Lyapunov stability theory in [5]. To the best of our knowledge, only few works have been done on the resulting class of systems that we call the class of linear singular systems with Markovian jumps. However, the uncertain Markovian jump linear singular system with Wiener process disturbance has never been tackled.

The aim of this paper is to derive sufficient condition for robust stability of the uncertain Markovian jump linear singular system with Wiener process disturbance, by using LMI technique, furthermore, a state feedback controller design method that robustly stochastically stabilizes the system under study is designed. The results obtained in this paper extend those developed on the Markovian jump linear singular systems with Wiener process disturbance in [10], to Markovian jump linear singular systems with both parameter uncertainties and Wiener process disturbance.

The rest of this paper is organized as follows. Section II states the problem to be studied and provides some assumptions. In section III, sufficient conditions are established to check the robust stability and stabilizability of the system under consideration. While section IV presents numerical examples.

Throughout this paper, the following notations will be used. The superscript "^{T}" denotes matrix transposition and for symmetric matrices X and Y the notation $X > Y$ (respectively $X < Y$) means that $(X - Y)$ is positive-definite (resp. negative-definite). $\mathbb{I}$ denotes the identity matrix with the appropriate dimension that may be understood from the context. $\mathrm{diag}\,[.]$ denotes a block diagonal matrix.

II. **Problem statement**

Let us consider the class of uncertain Markovian jump continuous-time singular linear system defined on the probability space $(\Omega, \digamma, P)$, with the following dynamics:

$$\begin{cases} Edx_t = A(r_t,t)x_t dt + B(r_t,t)u_t dt \\ +\mathbb{W}(r_t,t)x_t dw(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state, $u_t \in \mathbb{R}^p$ is the control at time t, $w(t) \in \mathbb{R}^m$ is a standard Wiener process on the given probability space, which is supposed to be independent of the Markov process $\{r_t, t \geq 0\}$, $\mathbb{W}(r_t)$ is the Wiener process matrix that is supposed to be known for each $r_t \in \mathbf{S}$, for more details related to Wiener process, see ([1], [3]). The matrix $E \in \mathbb{R}^{n_E * n_E}$ may be singular, with $rank(E) = n_E \leq n$, $A(r_t,t)$ is the state matrix, $B(r_t,t)$ is the control matrix, supposed to have the following forms:

$$\begin{cases} A(r_t,t) = A(r_t) + D_A(r_t)F_A(r_t,t)E_A(r_t) \\ B(r_t,t) = B(r_t) + D_B(r_t)F_B(r_t,t)E_B(r_t) \end{cases} \tag{2}$$

with $A(r_t), D_A(r_t), E_A(r_t), B(r_t), D_B(r_t)$ and $E_B(r_t)$ are real known matrices with appropriate dimensions, and $F_A(r_t,t)$ and $F_B(r_t,t)$ are unknown matrices that satisfy the following:

$$\begin{cases} F_A(r_t,t)F_A^T(r_t,t) < \mathbb{I} \\ F_B(r_t,t)F_B^T(r_t,t) < \mathbb{I} \end{cases} \tag{3}$$

The continuous-time Markov process $\{r_t, t \geq 0\}$ takes its values in a finite set $\mathcal{S} = \{1, 2, \ldots, N\}$ with the transition probability given by:

$$P\left[r_{t+\Delta t} = j | r_t = i\right] = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t) & i \neq j \\ 1 + \lambda_{ii}\Delta t + o(\Delta t) & i = j \end{cases} \tag{4}$$

where $\Delta t > 0, \lim_{\Delta t \to 0} \frac{o(\Delta t}{\Delta t} = 0$, and λ_{ij} is the transition probability rate from the mode i to the mode j at time t, which satisfies $\lambda_{ij} \geq 0, \forall i,j, i \neq j$, and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$.

In this paper, our goal is to address the robust stochastic stability for the class of system with $u_t = 0$, furthermore, we will determine a state feedback controller and sufficient conditions that guarantee the robust stochastic stabilizability. Before establishing these results, we begin by defining some concepts and introducing some useful lemmas that will be used in this paper.

For system (1), when $u_t \equiv 0$, we have the following definitions:

Definition 2.1: [6], [7]

a. The pair $(E, A(r_t))$ is said to be regular if $det(sE - A(r_t))$ is not identically zero

b. The pair $(E, A(r_t))$ is said to be causal if it is regular and $deg(det(sE - A(r_t))) = rank(E)$

Definition 2.2: System (1), with $u_t = 0$, for all $t \geq 0$, is said to be stochastically stable if there exists a finite positive constant $T(x_0, r_0)$ such that the following holds for any initial conditions (x_0, r_0):

$$\mathbb{E}\left[\int_0^\infty \|x(t)\|^2 dt | x_0, r_0\right] \leq T(r_0, x_0); \tag{5}$$

Definition 2.3: System (1) is termed robust stabilizable if there exists a linear state feedback:

$$u(t) = K(r_t)x_t \tag{6}$$

with $K(r_t)$ is a gain controller for each $r_t \in \mathcal{S}$, such that the closed-loop system is robustly stochastically stable for every initial condition (x_0, r_0).

Lemma 2.1: The infinitesimal generator of the Markov process $(x(t), r_t)$ under an admissible control law is given by:

$$\begin{aligned}\mathbb{L}V(x(t), r_t) &= V_x(x(t), r_t)\left[A(r_t, t)x(t) + B(r_t, t)u(t)\right] \\ &+ \frac{1}{2}\text{tr}\left[x^\top(t)\mathbb{W}^\top(r_t)E^T V_x(x(t), r_t)E\mathbb{W}(r_t)x(t)\right] \\ &+ \sum_{j=1}^{N} \lambda_{r_t j} V(x(t), j)\end{aligned}$$

For more details we refer the readers to $It\hat{o}'s$ Theorem quoted in reference [1].

Lemma 2.2: [9] Let H, F and G be real matrices of appropriate dimensions then, for any scalar $\varepsilon > 0$ for all matrices F satisfying $F^T F \leq I$, we have $HFG + G^T F^T H^T \leq \varepsilon H H^T + \varepsilon^{-1} G^T G$

III. Main results

The objective of this section is to derive sufficient conditions for robust stability and stabilizability for the class of systems we are dealing with. Our attention is also to synthesize a state feedback controller of the form (6), that will robustly stochastically stabilize the system under study.

A. Robust stability

Before establishing our results on the robust stochastic stability of the system (1) with $u_t = 0$ for $t \geq 0$, we need the following assumption:

Now, we give the first main result in this section.

Theorem 3.1: If there exist a set of symmetric and positive-definite matrices $P = (P(1), \ldots, P(N))$ and a set of positive scalars $\varepsilon_A = (\varepsilon_A(1), \ldots, \varepsilon_A(N))$ such that the following LMI holds for every $r_t \in \mathcal{S}$:

$$\begin{bmatrix} J_w(r_t) & E^T P(r_t) D_A(r_t) \\ D_A(r_t)^T P(r_t) E & -\varepsilon_A(r_t)\mathbb{I} \end{bmatrix} < 0 \tag{7}$$

with $J_w(r_t) = E^T P(r_t) A(r_t) + A^\top(r_t) P(r_t) E$
$+ \mathbb{W}^\top(r_t) E^T P(r_t) E \mathbb{W}(r_t) + E^T \left[\sum_{j=1}^{N} \lambda_{r_t j} P(j)\right] E$
$+ \varepsilon_A(r_t) E_A(r_t)^T E_A(r_t) < 0$

then, the system (1) is robustly stochastically stable. □

Proof : Let us consider the generalized Lyapunov function as follows:

$$V(x_t, r_t) = x^\top(t) E^T P(r_t) E x(t) \tag{8}$$

Where $P(r_t)$, $r_t \in \mathcal{S}$ is a symmetric and positive-definite matrix.

In this case, the infinitesimal generator of the Markov process $(x(t), r_t)$ becomes:

$$\begin{aligned}\mathbb{L}V(x(t), r_t) &= x^\top(t) E^T P(r_t) A(r_t, t) x(t) \\ &+ x^\top(t) A^\top(r_t, t) P(r_t) E x(t) \\ &+ x^\top(t) \mathbb{W}^\top(r_t) E^T P(r_t) E \mathbb{W}(r_t) x(t) \\ &+ x^\top(t) E^T \left[\sum_{j=1}^{N} \lambda_{r_t j} P(j)\right] E x(t)\end{aligned} \tag{9}$$

By applying Lemma (2.2), and using the fact that: $A(r_t, t) = A(r_t) + E_A(r_t) F(r_t, t) D_A(r_t)$, we obtain the following relation:

$$\mathbb{L}V(x(t),r_t) \leq x^\top(t)\Big[\ E^TP(r_t)A(r_t)+A^\top(r_t)P(r_t)E + \mathbb{W}^\top(r_t)E^TP(r_t)E\mathbb{W}(r_t)+\textstyle\sum_{j=1}^{N}\lambda_{r_tj}E^TP(j)E + \varepsilon_A^{-1}(r_t)E^TP(r_t)D_A(r_t)D_A^T(r_t)P(r_t)E + \varepsilon_A(r_t)E_A^T(r_t)E_A(r_t)\ \Big]x(t) = x^\top(t)\Theta(r_t)x(t)$$

with $\Theta(r_t) = A^\top(r_t)P(r_t)E + E^TP(r_t)A(r_t)$ $+\ \mathbb{W}^\top(r_t)E^TP(r_t)E\mathbb{W}(r_t) + \varepsilon_A(r_t)E_A^T(r_t)E_A(r_t)$ $+\ \varepsilon_A^{-1}(r_t)E^TP(r_t)D_A(r_t)D_A^T(r_t)P(r_t)E$ $+\ E^T\left[\sum_{j=1}^{N}\lambda_{r_tj}P(j)\right]E$

Using (3.1), we conclude that, for each mode r_t, $\Theta(r_t) < 0$. Therefore, if this inequality holds, it results that:

$$\mathbb{L}V(x(t),r_t) \quad \leq \quad -min_{i\in\mathcal{S}}\left[\lambda_{min}\left(-\Theta(i)\right)\right]x^\top(t)x(t)$$

Now this together with Dynkin's formula yield:

$$\begin{aligned}&\mathbb{E}\left[V(x(t),r_t)\right]-\mathbb{E}\left[V(x(0),r_0)\right]\\ &=\mathbb{E}\left[\int_0^t \mathbb{L}V(x(s),r_s)ds|(x_0,r_0)\right]\\ &\leq -\min_{i\in\mathbf{S}}\{\lambda_{min}(-\Theta(i))\}\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0,r_0)\right],\end{aligned}$$

implying, in turn,

$$\begin{aligned}&\min_{i\in\mathbf{S}}\{\lambda_{min}(-\Theta(i))\}\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0,r_0)\right]\\ &\leq \mathbb{E}\left[V(x(0),r_0)\right]-\mathbb{E}\left[V(x(t),r_t)\right]\\ &\leq \mathbb{E}\left[V(x(0),r_0)\right].\end{aligned}$$

This yields that:

$$\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0,r_0)\right] \leq \frac{\mathbb{E}\left[V(x(0),r_0)\right]}{\min_{i\in\mathbf{S}}\{\lambda_{min}(-\Theta(i))\}} \tag{10}$$

holds for any $t > 0$. □

B. Stabilization

This section is devoted to the design of a suitable state feedback controller that robustly stochastically stabilizes the uncertain Markovian jump singular system with Wiener process disturbance (1). Using the robust stability condition of the theorem (3.1), we will establish an LMI approach to compute the controller's gain. By setting $u(t) = K(r_t)x(t)$ in the system dynamics yields the following closed loop system:

$$\begin{cases} Ex_tdt = A(r_t,t)x_tdt + B(r_t,t)K(r_t)x_tdt \\ +\mathbb{W}(r_t)x_tdw(t), \\ = A_c(r_t,t)x_tdt + \mathbb{W}(r_t)x_tdw(t)\end{cases} \tag{11}$$

with $A_c(r_t,t) = A(r_t,t) + B(r_t,t)K(r_t)$.

The following theorem gives a robust stochastic stabilizability condition and stabilizing feedback gain:

Theorem 3.2: Let M be given symmetric matrices. The system (1) is robustly stochastically stable if there exist a set of symmetric and positive definite matrices $P = (P(1),\ldots,P(N))$ and $L = (L(1),\ldots,L(N))$, a matrix $Y = (Y(1),\ldots,Y(N))$ and a set of positive scalars $\varepsilon_A = (\varepsilon_A(1),\ldots,\varepsilon_A(N))$ and $\varepsilon_B = (\varepsilon_B(1),\ldots,\varepsilon_B(N))$ such that the following holds for each $r_t \in S$:

$$P(r_t)B(r_t) = B(r_t)L(r_t) \tag{12}$$

$$\begin{bmatrix} \Upsilon_w(r_t) & E^TP(r_t)D_A(r_t) & E^TP(r_t)D_B(r_t) & \mathbb{W}^\top(r_t)E^TP(r_t) \\ D_A^T(r_t)P(r_t)E & -\varepsilon_A(r_t)\mathbb{I} & \mathbf{0} & \mathbf{0} \\ D_B^T(r_t)P(r_t)E & \mathbf{0} & -\varepsilon_B(r_t)\mathbb{I} & \mathbf{0} \\ P(r_t)EW(r_t) & \mathbf{0} & \mathbf{0} & -P(r_t)\end{bmatrix} < 0 \tag{13}$$

where $\Upsilon_w(r_t) = E^T\left[P(r_t)A(r_t) + B(r_t)Y(r_t)\right]$ $+\left[P(r_t)A(r_t) + B(r_t)Y(r_t)\right]^TE$ $+\ \varepsilon_A(r_t)E_A^T(r_t)E_A(r_t) + E^T\left[\sum_{j=1}^{N}\lambda_{r_tj}P(j)\right]E < 0$

The controller gain is given by $K(r_t) = L^{-1}(r_t)Y(r_t)$.

Proof: Under the condition of the theorem (3.1), the closed-loop system is robustly stochastically stable if the following condition holds for every $r_t \in S$:

$$A_c^\top(r_t,t)P(r_t)E + E^T P(r_t)A_c(r_t,t) + \mathbb{W}^\top(r_t)E^T P(r_t)E\mathbb{W}(r_t) + E^T\left[\sum_{j=1}^{N}\lambda_{r_t j}P(j)\right]E < 0 \quad (14)$$

By applying Lemma (2.2), replacing $A_c(r_t,t)$ by its expression, and taking into consideration that $\varepsilon_B(r_t)K^T(r_t)E_B^T(r_t)E_B(r_t)K(r_t) > 0$, the inequality (14) will be satisfied if the following one holds:

$$\begin{aligned} &A^\top(r_t)P(r_t)E + E^T P(r_t)A(r_t) \\ &+K^T(r_t)B^T(r_t)P(r_t)E + E^T P(r_t)B(r_t)K(r_t) \\ &+\varepsilon_A(r_t)E_A^T(r_t)E_A(r_t) \\ &+\varepsilon_A^{-1}(r_t)E^T P(r_t)D_A(r_t)D_A^T(r_t)P(r_t)E \\ &+\varepsilon_B^{-1}(r_t)E^T P(r_t)D_B(r_t)D_B^T(r_t)P(r_t)E \\ &+\mathbb{W}^\top(r_t)E^T P(r_t)E\mathbb{W}(r_t) + E^T\left[\sum_{j=1}^{N}\lambda_{r_t j}P(j)\right]E < 0 \quad (15)\end{aligned}$$

Let us now transform the condition (15) in the LMI form. For this purpose, we assume that there exists a symmetric and definite positive matrix $L(r_t)$ such that:

$$P(r_t)B(r_t) = B(r_t)L(r_t) \quad (16)$$

this requires that $B(r_t)$ is full column rank for each mode. Substituting (16) in (15) and letting $L(r_t)K(r_t) = Y(r_t)$, then by and using Schur complement, the previous inequality becomes:

$$\begin{bmatrix} \Upsilon_w(r_t) & E^T P(r_t)D_A(r_t) & E^T P(r_t)D_B(r_t) & \mathbb{W}^\top(r_t)E^T \\ D_A^T(r_t)P(r_t)E & -\varepsilon_A(r_t)\mathbb{I} & \mathbf{0} & \mathbf{0} \\ D_B^T(r_t)P(r_t)E & \mathbf{0} & -\varepsilon_B(r_t)\mathbb{I} & \mathbf{0} \\ EW(r_t) & \mathbf{0} & \mathbf{0} & -P^{-1}(r_t) \end{bmatrix} < 0 \quad (17)$$

where $\Upsilon_w(r_t) = E^T\left[P(r_t)A(r_t) + B(r_t)Y(r_t)\right] + \left[P(r_t)A(r_t) + B(r_t)Y(r_t)\right]^T E + \varepsilon_A(r_t)E_A^T(r_t)E_A(r_t) + E^T\left[\sum_{j=1}^{N}\lambda_{r_t j}P(j)\right]E < 0.$

Let us pre- and post-multiplying (17) by $diag[\mathbb{I},\mathbb{I},\mathbb{I},P(r_t)]$, this yields to LMI (13). □

In the following section, we give a numerical example to demonstrate the validity of the proposed results, which shows how we can compute a state feedback controller that robustly stochastically stabilizes the system under study.

IV. Examples

let us suppose that the transition probability rate matrix and the matrix E are given by:

$$\Lambda = \begin{bmatrix} -2.00 & 2.00 \\ 1.00 & -1.00 \end{bmatrix}, E = \begin{bmatrix} 1.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

Example 4.1: Let us consider a system with two modes with the dynamics described by (1), and suppose that the system data is as follows:

- mode 1:

$$\begin{aligned} A(1) &= \begin{bmatrix} -1.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & -1.00 & -1.00 \end{bmatrix}, \\ B(1) &= \begin{bmatrix} 0.30 & 0.00 & 1.00 \\ 0.00 & 0.30 & 0.00 \\ 0.10 & 0.00 & 0.30 \end{bmatrix}, \\ W(1) &= \begin{bmatrix} 0.36 & 0.00 & 1.20 \\ 0.00 & 0.36 & 0.0 \\ 0.12 & 0.0 & 0.36 \end{bmatrix}, \\ E_A(1) &= \begin{bmatrix} 0.20 & 0.10 & 0.01 \end{bmatrix}, \\ D_A(1) &= \begin{bmatrix} 0.0001 \\ 0.0020 \\ 0.0012 \end{bmatrix}, D_B(1) = \begin{bmatrix} 0.0100 \\ 0.2000 \\ 0.1000 \end{bmatrix}, \\ \varepsilon_B(1) &= 0.01, \varepsilon_A(1) = 0.50 \end{aligned}$$

- mode 2:

$$
\begin{aligned}
A(2) &= \begin{bmatrix} -1.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & -1.00 \\ 0.00 & 1.00 & -1.00 \end{bmatrix}, \\
B(2) &= \begin{bmatrix} 0.10 & 0.00 & 1.00 \\ 1.00 & 0.00 & 0.40 \\ 0.00 & 0.00 & 0.30 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
W(2) &= \begin{bmatrix} 0.12 & 0.00 & 1.20 \\ 1.20 & 0.00 & 0.48 \\ 0.00 & 0.00 & 0.36 \end{bmatrix}, \\
E_A(2) &= \begin{bmatrix} 0.03 & 0.01 & 0.02 \end{bmatrix}, \\
D_A(2) &= \begin{bmatrix} 0.0013 \\ 0.0010 \\ 0.0010 \end{bmatrix}, D_B(2) = \begin{bmatrix} 0.20 \\ 0.30 \\ 0.10 \end{bmatrix}, \\
\varepsilon_A(2) &= 0.30 \varepsilon_B(2) = 0.01,
\end{aligned}
$$

Using theorem 3.2 yields the following matrices:

$$
\begin{aligned}
P(1) &= \begin{bmatrix} 570.4145 & -0.0000 & 2.7212 \\ -0.0000 & 553.3931 & 0.0002 \\ 2.7212 & 0.0002 & 562.2511 \end{bmatrix}, \\
P(2) &= \begin{bmatrix} 561.7627 & -16.2165 & -27.3174 \\ -16.2165 & 551.5207 & -1.9652 \; , \\ -27.3174 & -1.9652 & 642.3248 \end{bmatrix}, \\
L(1) &= \begin{bmatrix} 562.2484 & 0.0000 & 2.7220 \\ 0.0000 & 553.3931 & 0.0000 \\ 2.7220 & 0.0000 & 570.4143 \end{bmatrix}, \\
L(2) &= \begin{bmatrix} 556.1616 & 0.0000 & -15.6564 \\ 0.0000 & 449.9976 & 0.0000 \\ -15.6564 & 0.0000 & 548.6464 \end{bmatrix},
\end{aligned}
$$

the corresponding gain matrices are given by:

$$
\begin{aligned}
K(1) &= 10^4 \begin{bmatrix} -501.30 & -0.00 & -508.80 \\ -334.60 & -0.00 & -1390.20 \\ -417.40 & -0.00 & -476.70 \end{bmatrix}, \\
K(2) &= 10^4 \begin{bmatrix} -179.80 & -0.00 & -854.36 \\ -683.00 & -683.00 & -683.00 \\ -386.16 & 0.00.00 & -234.23 \end{bmatrix}
\end{aligned}
$$

Based on the results of Theorem 3.2, we conclude that we can design a robust stabilizing state feedback controller of the form (6) by using the developed method.

V. **Conclusion**

In this paper, we studied the problem of robust stability and stabilizability for stochastic uncertain singular systems with both Markovian jumping parameters and Wiener process disturbance. Sufficient LMI based condition that assures the robust stability of the class of systems under study has been presented. LMI approach has also been developed to design a robust stabilizing state feedback controller which guarantees the robust stabilizability of the systems.

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