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STATE AND OUTPUT FEEDBACK STABILIZATION OF A CLASS OF DISCRETE-TIME NONLINEAR SYSTEMS

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***Abstract*— For multi-input multi-output (MIMO) discrete-time nonlinear systems whose free dynamics can be unstable, we show how the problem of global stabilization via state and dynamic output feedback can be solved. Sufficient conditions for stability are deduced, from the Lyapunov approach, and expressed in terms of matrix inequality that depend on arbitrary matrices fixed by the designer. An example is presented to illustrate the high performances of the proposed approach.**

Keywords: : nonlinear discrete-time systems, bilinear systems, stabilization, state feedback, dynamic output feedback, unstable free dynamics.

I.. Introduction

Over the past four decades, stabilization of nonlinear dynamical systems has received a great deal of attention in the literature as can be shown through basic works in this field [1], [2], [3] and [4]. Several design methodologies have been developed for local and global stabilization problems of continuous and discrete-time nonlinear systems, see for instance [5], [6], [7], [8], [9], [10] and the references inside.

When the control laws are designed, the state variables are assumed to be available. But in general, this is not true in practice and the current state must be estimated by another dynamical system, that is a state observer.

Thus, observer based stabilization of nonlinear systems has been studied in the past few years. The main contributions, however, concern continuous time systems; this problem has been investigated by several authors, among them [11], [12], [13], [14], [15] and [16]. In [11], using converse Lyapunov stability theory, both local and global asymptotic (resp. exponential) stabilization is obtained, via estimated state feedback. The result presented in [11] was the first separation principle for nonlinear systems in the literature. The applications of this separation principle are restricted in the sense that verifications of the conditions given in the main theory (see [11], Theorem 3.1 and 4.1) heavily depend on the choice of Lyapunov functions. Using the linearization approach, authors in [12] established a nonlinear separation property for the local exponential stabilization problem. In [14] and [16] a global output feedback stabilization is achieved using the high gain observer and the hypothesis that there exists a bounded state feedback which stabilizes the nonlinear system was studied.

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For discrete-time nonlinear systems only few design methods have been established [6], [17] and [18]. Relevant ones have been developed by Byrnes and Lin [6] and Lin [18]. In particular the work in [18], where a global stabilization is achieved via state and output feedback, the proposed technique is judicious, but only systems with Lyapunov stable unforced dynamics are considered, like in the majority of the works in the literature, this may be seen as a conservative condition.

The aim of this work is to relax this condition, by the use of a bounded state feedback control schemes, associated with "Luenberger-type" nonlinear observer, to ensure the global stability of a class of discrete-time nonlinear systems. Thanks to simple Lypunov function, sufficient conditions for stability are deduced and seem to work, without coordinate transformation, for a large class of nonlinear systems, even those with unstable unforced dynamics. This method can be also extended to stochastic systems.

This paper is organized as follow : in section 2 we introduce the problem formulation and the two main results. in Section 3 a numerical example is provided to show the high performances of the proposed method and easiness of the implementation.

II.. Problem formulation and main results

In this paper we consider a class of affine discrete-time nonlinear systems of the form :

$$\begin{cases} x_{k+1} = Ax_k + \sum_{i=1}^{m} g_i(x_k)u_k^i \\ y_k = Cx_k \end{cases} \tag{1}$$

where $x_k \in \mathbf{R}^n$, $y_k \in \mathbf{R}^p$ and $u_k = [u_k^1, u_k^2, \dots, u_k^m]^T \in \mathbf{R}^m$ denote the state, output and input vectors respectively at time instant k. A and C are constant matrices with appropriate dimensions, and $g_i : \mathbf{R}^n \to \mathbf{R}^n$, $1 \leq i \leq m$, are smooth functions. Without loss of generality, we assume that zero is the equilibrium point of the system (1).

The purpose of this work is to design a dynamic compensator

$$\begin{cases} \xi_{k+1} = \phi(\xi_k, y_k) \\ u_k = \vartheta(\xi_k) \end{cases} \tag{2}$$

so that $(x, \xi) = (0, 0)$ is an exponentially stable equilibrium of the closed-loop system (1),(2), even if the free dynamics are unstable.

In what follows, we construct a bounded state feedback which leads to a global separation principle for discrete-time nonlinear systems of the form (1).

Set

$$g(x_k) = [g_1(x_k), g_2(x_k), \ldots, g_m(x_k)]. \tag{3}$$

First, inspired by the well known results on optimal control and the transfer of the Jurdjevic-Quinn control in discrete-time developed in [19], we propose in the following theorem an explicit state feedback law to achieve stabilization of (1).

Theorem 1

Consider a MIMO nonlinear system (1) which satisfies

- H1) $P_k^{1/2}\Lambda_k^T\left(A^T(P_k^{-1}+g(x_k)R^{-1}g(x_k)^T)^{-1}A+Q\right)^{-1} \times \Lambda_k P_k^{1/2} \leq \alpha(1-\delta)I_n$

Then, the equilibrium of the system (1), $x = 0$ is globally exponentially stabilized by the bounded feedback control

$$u_k = u(x_k) = -F_k x_k = -\frac{\gamma}{1+\|L_k x_k\|^2}\ L_k x_k \tag{4}$$

where

$$F_k = F(x_k) = \frac{\gamma}{1+\|L_k x_k\|^2}\ L_k \tag{5}$$

$$L_k = L(x_k) = (R + g(x_k)^T P_k g(x_k))^{-1} g(x_k)^T P_k A \tag{6}$$

$$P_{k+1} = \alpha\left((A - g(x_k)L_k)^T P_k (A - g(x_k)L_k) + L_k^T R L_k + Q\right) \tag{7}$$

$$\Lambda_k = A - g(x_k)F_k, \quad 0 < \delta < 1 \tag{8}$$

where γ is an arbitrarily positive real number. Q and R are positive definite matrices with appropriate dimensions to be chosen as design parameters.

Remark

- The main contribution of the proposed approach with respect to the existing result is that we introduce the weighting factor α to control boundness of P_k, and by the way to relax the Lyapunov stability condition of the free dynamics. Design of this parameter will be specified later in the proof.

Proof

First, $\lambda_{max}(.)$ and $\lambda_{min}(.)$ denote the maximum and minimum eigenvalues, respectively.

Consider the Lyapunov function

$$V_k = x_k^T P_k^{-1} x_k \tag{9}$$

A strictly decreasing sequence $\{V_k\}_{k=1,..}$ means that there exists a positive scalar $0 < \delta < 1$ such that

$$\Delta V \ = \ V_{k+1} - V_k \leq -\delta V_k \tag{10}$$

After matrix manipulation, H1) becomes

$$P_k^{1/2}\Lambda_k^T\left(A^T P_k (I + g(x_k)R^{-1}g(x_k)^T P_k)^{-1} A + Q\right)^{-1} \times \Lambda_k P_k^{1/2} \leq \alpha(1-\delta)I_n. \tag{11}$$

We notice that

$$\left(I + g(x_k)R^{-1}g(x_k)^T P_k\right)^{-1} = I - g(x_k) \times \left(I + R^{-1}g(x_k)^T P_k g(x_k)\right)^{-1} R^{-1} g(x_k)^T P_k \tag{12}$$

Using (11) and (12), we obtain

$$P_k^{1/2}\Lambda_k^T\left[A^T P_k\left[I - g(x_k)(I + R^{-1}g(x_k)^T P_k g(x_k))^{-1} \times R^{-1}g(x_k)^T P_k\right] A + Q\right]^{-1} \Lambda_k P_k^{1/2} \leq \alpha(1-\delta)I_n \tag{13}$$

or

$$P_k^{1/2}\Lambda_k^T\left[A^T P_k\left[I - g(x_k)(R + g(x_k)^T P_k g(x_k))^{-1} \times g(x_k)^T P_k\right] A + Q\right]^{-1} \Lambda_k P_k^{1/2} \leq \alpha(1-\delta)I_n \tag{14}$$

which, can be rewritten as

$$P_k^{1/2}\Lambda_k^T\left[A^T P_k A - A^T P_k g(x_k)(R + g(x_k)^T P_k g(x_k))^{-1} \times g(x_k)^T P_k A + Q\right]^{-1} \Lambda_k P_k^{1/2} \leq \alpha(1-\delta)I_n \tag{15}$$

then

$$P_k^{1/2}\Lambda_k^T\left[A^T P_k A - A^T P_k g(x_k) L_k + Q\right]^{-1} \times \Lambda_k P_k^{1/2} \leq \alpha(1-\delta)I_n \tag{16}$$

$$\Rightarrow \ \Lambda_k^T\left[A^T P_k A - A^T P_k g(x_k) L_k + Q\right]^{-1} \times \Lambda_k \leq \alpha(1-\delta)P_k^{-1} \tag{17}$$

After adding and subtracting the same term, we obtain

$$\Lambda_k^T\left[A^T P_k A - A^T P_k g(x_k) L_k - L_k^T g(x_k)^T P_k A + L_k^T g(x_k)^T P_k A + Q\right]^{-1} \Lambda_k \leq \alpha(1-\delta)P_k^{-1} \tag{18}$$

A multiplication by I_n, give

$$\Lambda_k^T\left(A^T P_k A - A^T P_k g(x_k) L_k - L_k^T g(x_k)^T P_k A + L_k^T (R + g(x_k)^T P_k g(x_k))(R + g(x_k)^T P_k g(x_k))^{-1} \times\ g(x_k)^T P_k A + Q\right)^{-1} \Lambda_k \leq \alpha(1-\delta)P_k^{-1} \tag{19}$$

which is

$$\Lambda_k^T\left(A^T P_k A - A^T P_k g(x_k) L_k - L_k^T g(x_k)^T P_k A + L_k^T R L_k + L_k^T g(x_k)^T P_k g(x_k) L_k + Q\right)^{-1} \Lambda_k \leq \alpha(1-\delta)P_k^{-1} \tag{20}$$

then

$$\Lambda_k^T \left((A-g(x_k)L_k)^T P_k (A-g(x_k)L_k) \right. \\ \left. + L_k^T R L_k + Q\right)^{-1} \Lambda_k \leq \alpha(1-\delta) P_k^{-1} \quad (21)$$

which is nothing else than,

$$\Lambda_k^T P_{k+1}^{-1} \Lambda_k \leq (1-\delta) P_k^{-1} \quad (22)$$

So, the equation (10) is proved.

Now, we will prove that the matrix P_k is bounded from above and below for all k, i. e. there exists $\underline{\lambda}$ and $\overline{\lambda}$ so that

$$0 < \underline{\lambda} I_n \leq P_k \leq \overline{\lambda} I_n \quad (23)$$

It is easy to verify, from (7), that since $\alpha\, Q$ (Q is fixed by the designer) is positive definite we have $\underline{\lambda} I_n \leq P_k$. The second inequality $P_k \leq \overline{\lambda} I_n$ may be deduced from a good choice of the parameter α. Indeed, the proof is straightforward if we consider the following auxiliary Riccati equation

$$\overline{P}_{k+1} = \alpha\left(A^T \overline{P}_k A + Q\right) \quad (24)$$

which always, under the following value of α

$$\alpha = \begin{cases} 1 & \text{for } \overline{P}_k < \bar{\lambda} I_n \\ \frac{1}{1+\lambda_{max}^2(A)} & \text{otherwise} \end{cases} \quad (25)$$

satisfies

$$\overline{P}_k \leq \bar{\lambda} I_n \quad \text{for all k} \quad (26)$$

On the other hand, when the arbitrary initial matrices are chosen to be

$$P_0 \leq \overline{P}_0 \quad (< \bar{\lambda} I_n) \quad (27)$$

by the use of (6) and (8)

$$\begin{aligned} P_{k+1} &= \alpha\left(A^T\left(P_k - P_k g(x_k)(R + g(x_k)^T P_k g(x_k))^{-1} \right.\right. \\ &\qquad \left.\left. \times g(x_k)^T P_k\right) A + Q\right) \\ &\leq \alpha\left(A^T P_k A + Q\right) \\ &\leq \overline{P}_{k+1} = \alpha\left(A^T \overline{P}_k A + Q\right) \leq \bar{\lambda} I_n \end{aligned} \quad (28)$$

so the boundness of P_k is proved.

Since V_k is a strictly decreasing sequence and P_k is bounded, it follows that

$$\begin{aligned} 0 &\leq \varphi\|x_k\| \leq V_k \leq (1-\delta)^k V_0 \\ \Rightarrow \|x_k\| &\leq \varphi^{-1}(1-\delta)^k x_0^T P_0 x_0 \\ \Rightarrow \|x_k\| &\leq M_1 a^k \quad \text{for k=0,1,\ldots} \end{aligned} \quad (29)$$

where

$$M_1 = \varphi^{-1}\lambda_{max}(P_0)\|x_0\|^2 > 0, \quad 0 < a < 1$$

and

$$0 < \varphi I_n \leq P_k^{-1}$$

Therefore the convergence of the state x_k to zero is ensured.■

We can now establish a global separation principle for a MIMO discrete-time nonlinear system of the form (1) by using the bounded state feedback control strategy proposed in Theorem 1.

Theorem 2

Consider a discrete-time MIMO nonlinear system (1) which satisfies Hypothesis H1). Suppose the pair (A, C) is detectable and the function $g(x_k)$ is globally Lipschitz on $\mathbf{R}^n$. Then for a sufficiently small $\gamma > 0$, a Luenberger-observer-like based output feedback control law

$$\begin{cases} \xi_{k+1} = A\xi_k + g(\xi_k)\hat{u}_k + K(y_k - C\xi_k) \\ \hat{u}_k = u(\xi_k) = -F(\xi_k)\xi_k \end{cases} \quad (30)$$

renders the equilibrium $(x, \xi) = (0, 0)$ of the closed-loop system (1),(30) globally exponentially stable. K is a constant matrix such that $(A - KC)$ is stable.

Proof

define e_k, the state estimation error vector, so that

$$e_k = x_k - \xi_k$$

The closed-loop system (1),(30) can be expressed as

$$\begin{cases} e_{k+1} = (A-KC)e_k + (g(x_k) - g(\xi_k))\,\hat{u}_k \\ \xi_{k+1} = A\xi_k + g(\xi_k)\hat{u}_k + KCe_k \end{cases} \quad (31)$$

Since the pair (A, C) is detectable, so there exists a positive definite matrix S such that

$$(A-KC)^T S (A-KC) - S = -I.$$

Let us define

$$X(e_k) = e_k^T S e_k \quad (32)$$

Then

$$\begin{aligned} \Delta X_k &= X(e_{k+1}) - X(e_k) \\ &= -e_k^T I_n e_k + 2e_k^T(A-KC)^T S\left(g(x_k) - g(\xi_k)\right)\hat{u}_k \\ &\quad + \hat{u}_k^T\left(g(x_k) - g(\xi_k)\right)^T S(g(x_k) - g(\xi_k))\,\hat{u}_k \\ &\leq -\|e_k\|^2 + 2e_k^T(A-KC)^T S\left(g(x_k) - g(\xi_k)\right)\hat{u}_k \\ &\quad + \hat{u}_k^T(g(x_k) - g(\xi_k))^T S\left(g(x_k) - g(\xi_k)\right)\hat{u}_k \end{aligned} \quad (33)$$

From $\|\hat{u}_k\| \leq \gamma$ and the Lipschitz condition of $g(.)$, we deduce that

$$\Delta X_k \leq -\|e_k\|^2\left(1 - 2\gamma\beta\|(A-KC)S\| - \gamma^2\beta^2\|S\|\right) \quad (34)$$

Where β is the Lipschitz constant associated with $g(\cdot)$.

Obviously, it is possible to choose $\gamma > 0$ so that for some ε, $0 < \varepsilon < 1$

$$\Delta X_k = e_{k+1}^T S e_{k+1} - e_k^T S e_k \leq -\varepsilon e_k^T S e_k \leq 0 \quad (35)$$

This implies

$$\lambda_{min}(S)\|e_k\|^2 \leq e_k^T S e_k \leq (1-\varepsilon) e_{k-1}^T S e_{k-1} \leq \dots \\ \leq (1-\varepsilon)^k e_0^T S e_0 \leq (1-\varepsilon)^k \lambda_{max}(S)\|e_0\|^2 \quad (36)$$

therefore

$$\|e_k\|^2 \leq D z^k \quad \text{for k=0,1,}\dots \quad (37)$$

with

$$D = \frac{\lambda_{max}(S)}{\lambda_{min}(S)}\|e_0\|^2 > 0 \quad \text{and} \quad 0 < z < 1.$$

On the other hand, recall that by theorem 1, we satisfied the following relationship

$$\|A\xi_k + g(\xi_k)_k \hat{u}_k\|^2 \leq M_2 \ a^{k+1}, \quad (38)$$

where

$$M_2 = \varphi^{-1}\lambda_{max}(P_0)\|\xi_0\|^2 > 0 \quad \text{and} \ 0 < a < 1, \ \text{k=0,1,}\dots \quad (39)$$

Without loss of generality, we can deduce from (31) and (38) that

$$\begin{aligned} \|\xi_{k+1}\| &\leq \|A\xi_k + g(\xi_k)\hat{u}_k\| + \|KCe_k\| \\ &\leq M_2 \ a^{k+1} + D z^k \|KC\| \leq \dots \\ &\leq \frac{M_2}{1-a} + \frac{D\|KC\|}{1-z} \end{aligned} \quad (40)$$

From (37) and (40), we conclude that all trajectories of the closed-loop system (31) are bounded.

To show that $(e,\xi) = (0,0)$ is a global exponentially stable equilibrium of (31), we let (e_k,ξ_k) be a trajectory of system (31) with the initial value (e_0,ξ_0). Let m^0 denote its ω-limit set (i.e.

$$m^0 = \lim_{k\to\infty} \phi^k(x),$$

where $\phi^k(x)$ is a series extract from the solution of the system (31)). Clearly, m^0 is nonempty, compact, and invariant because (e_k,ξ_k) is bounded $\forall k$. In addition, it follows from (37) that $\lim_{k\to\infty} e_k = 0$. Therefore, any point in m^0 must be a pair of the form $(0,\xi_k)$.

Let $(0,\overline{\xi}) \in m^0$ and $(0,\overline{\xi}_k)$ be the corresponding trajectory. Obviously, this trajectory is characterized by the following equation

$$\overline{\xi}_{k+1} = A\overline{\xi}_k + g(\overline{\xi}_k)u(\overline{\xi_k}) \quad (41)$$

which has been proved to be globally exponentially stable at $\xi = 0$.

In, other words, the global exponential behavior of the closed loop system (31) at $(e,\xi) = (0,0)$ is completely determined by the flow on the invariant manifold governed by system (41) [20]. Since the latter is globally exponentially stable, so is the closed-loop system (31).■

As an immediate consequence, we have the following global separation principle for MIMO Bilinear systems

Corollary

Consider the following MIMO discrete-time bilinear system

$$\begin{cases} x_{k+1} = Ax_k + \sum_{i=1}^m B_i x_k u_k^i \\ y_k = Cx_k \end{cases} \quad (42)$$

Suppose assumption H1) holds with $g_i(x_k) = B_i x_k$. Suppose the pair (A,C) is detectable. Then a dynamic compensator of the form (30), with $g_i(x_k) = B_i x_k$, renders the system (42) globally exponentially stable.

III.. ILLUSTRATIVE EXAMPLE

In this section, the proposed control method is applied to an example with very interesting properties, in order to illustrate the high performances of the proposed approach.

Consider a nonlinear system described by

$$\Sigma \begin{cases} x_1(k+1) = 1.5x_1(k) + f_1(x_k)u(k) \\ x_2(k+1) = f_2(x_k)u(k) \\ y(k) = x_1(k) \end{cases} \quad (43)$$

where

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad g(x_k) = \begin{pmatrix} f_1(x_k) \\ f_2(x_k) \end{pmatrix} \quad \text{and} \quad C = [1 \quad 0]$$

We have one eigenvalue of A out of the open unit circle. Thus the free dynamics are unstable. We can see also that the pair (A,C) is detectable.

Computing the condition H1), with

$$R = r, \quad Q = \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix} \text{ and } P_k = \begin{bmatrix} p_{1k} & p_{2k} \\ p_{2k} & p_{3k} \end{bmatrix}$$

we get the following matrix

$$\Lambda_k^T \left(A_k^T (P_k^{-1} + g(x_k) R^{-1} g(x_k)^T)^{-1} A_k + Q\right)^{-1} \Lambda_k \\ - \alpha(1-\delta) P_k^{-1} = \begin{bmatrix} H_k & 0 \\ 0 & J_k \end{bmatrix} \quad (44)$$

where

$$J_k = -\frac{\alpha\delta}{p_{1k}}$$

and

$$H_k = (r \ + f_1^2(x_k) \ p_{1k} + f_2^2(x_k) \ p_{3k}) \\ \times \frac{\left(1.5 - 1.5\frac{f_1^2(x_k)\gamma p_{1k}}{r + f_1^2(x_k)p_{1k} + f_2^2(x_k)p_{3k} + 1.5|x_1(k)|f_1(x_k)p_{1k}}\right)^2}{2.25p_{1k}r + 2.25f_2^2(x_k)p_{1k}p_{3k} + qr + qf_2^2(x_k)p_{3k} + qf_1^2(x_k)p_{1k}} \\ + 2.25\frac{\gamma^2 f_1^2(x_k) f_2^2(x_k) p_{1k}^2}{\left(r + f_1^2(x_k)p_{1k} + f_2^2(x_k)p_{3k} + 1.5|x_1(k)|f_1(x_k)p_{1k}\right)^2 q} \\ - \frac{\alpha\delta}{p_{1k}} \quad (45)$$

It's clear that choosing $r = 1$ and a large q, allow us to verify Hypothesis H1).

Therefore, we conclude from theorem 2, that the system Σ is globally exponentially stabilized (GAS) by the dynamic output feedback (30), as presented in the following figures.

Where $f_1(x_k) = x_1(k)$, $f_2(x_k) = x_1(k)$, $q = 10^5$, $P_0 = 100 \times I_2$, $x_0 = [15 \;\; 13]^T$ and $\xi_0 = -[15 \;\; 20]^T$.

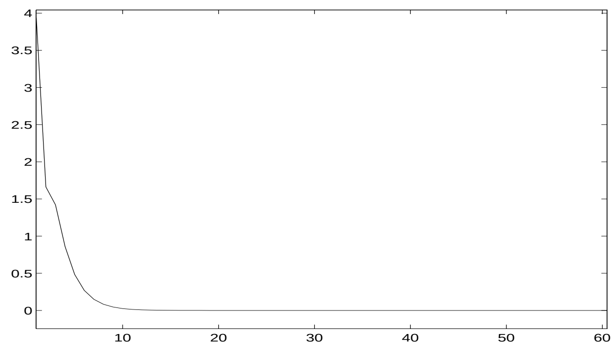

Fig. 1. $V(x_k)$ with respect to sampling time k.

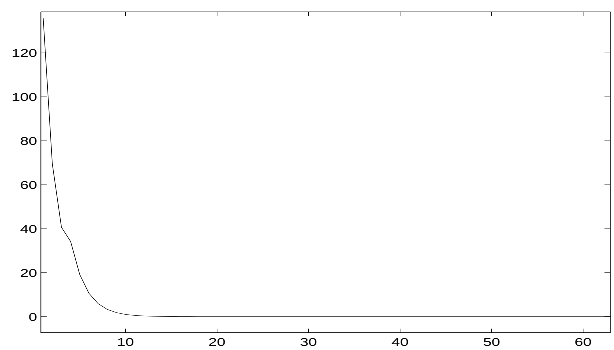

Fig. 2. $X(x_k)$ with respect to sampling time k.

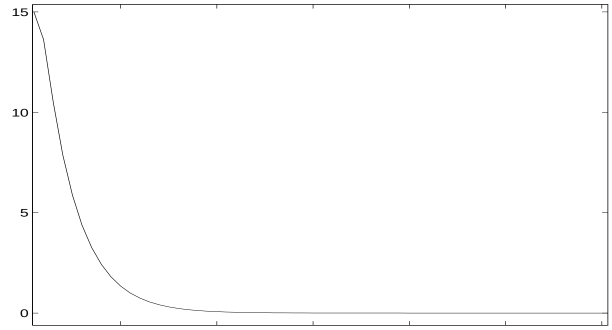

Fig. 3. $x_1(k)$ with respect to sampling time k.

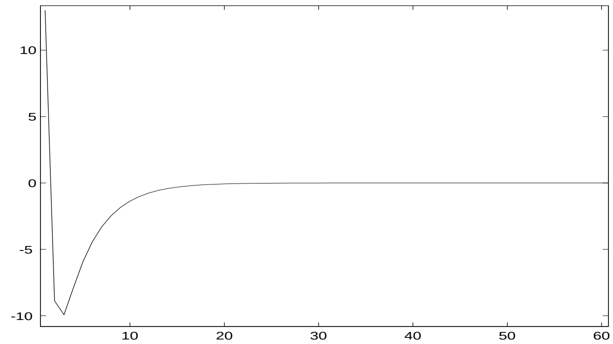

Fig. 4. $x_2(k)$ with respect to sampling time k.

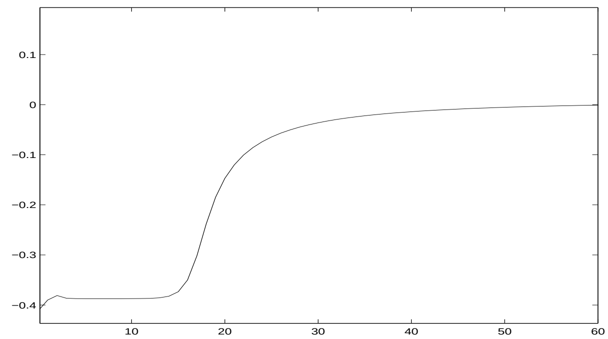

Fig. 5. $\hat{u}_k$ with respect to sampling time k.

IV.. Conclusion

In this paper, we have presented bounded state feedback control schemes which globally exponentially stabilizes a class of discrete-time affine nonlinear systems, whose free dynamics can be unstable. The nonlinear system, if it is to be stabilized, must satisfy a stability condition, which is established in terms of matrix inequality. The Luenberger-observer-like output feedback control law based on a Riccati-like equation stabilizes a class of MIMO non-linear discrete-time systems. The crucial point to establish this separation principle was the use of a bounded feedback. The simulation results show the high performances of our approach.

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