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Directional Control of a Streamlined Underwater Vehicle by Feedback Passivation

Hye-Young Kim and Craig A. Woolsey

***Abstract*— This paper describes a model-based technique for stabilizing the directional motion of a streamlined underwater vehicle controlled in surge, roll, pitch, and yaw. The six degree of freedom vehicle model allows for a broad range of viscous force and moment representations. The control law, which is derived using feedback passivation, makes steady, streamlined translation in a desired inertial direction globally asymptotically stable.**

I. INTRODUCTION

Conventional streamlined underwater vehicles are underactuated by design. Typically, they include a propulsor and three or more torque actuators, such as tail fins, which provide control in four degrees of freedom: surge, roll, pitch, and yaw. Linear methods can provide good directional control performance over a restricted range of motion, however these approaches are not always well justified, particularly for maneuverable vehicles in dynamic environments. In this paper, we propose a nonlinear control law which globally asymptotically stabilizes streamline translation in a desired direction. The control law is derived using the theory of passive systems and, in particular, the notion of feedback passivation described in [9].

Nonlinear stabilization for streamlined, underactuated underwater vehicles has attracted considerable interest. While results on dynamic stabilization, trajectory tracking, and path following are abundant for vehicles in planar motion, results for motion in three dimensional space are more scarce. In [1], a kinetic energy shaping technique known as interconnection and damping assignment was applied to stabilize a streamlined vehicle using only surge, pitch, and yaw inputs. Internal rotor actuators were proposed for this purpose in [12]; the stabilization result relied on a similar energy shaping technique, the method of controlled Lagrangians. These treatments focused solely on stabilization of the vehicle dynamics, however, ignoring the rotational kinematics. The problem of stabilizing an underactuated vehicle to a particular *path* in inertial space is not a trivial extension of the dynamic stabilization problem. In [3], a nonlinear path following strategy is proposed for an underwater vehicle with surge, pitch, and yaw control. The approach begins with Lyapunov-based control design for the feedback linearized kinematic equations. The full (force and moment) control law is then derived through backstepping. Simulations suggest that the approach is quite effective, although the results stop short of a formal proof of convergence. Another path following strategy is proposed in [2] for a vehicle with surge, roll, pitch, and yaw control. This approach makes use of backstepping and includes an adaptive element to compensate for model uncertainty. The results of [2] and [3], however, only hold locally.

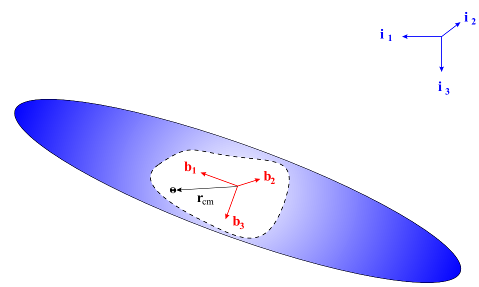

Fig. 1. Spheroidal underwater vehicle.

In this paper, we derive a control law which globally asymptotically stabilizes steady translation of a streamlined underwater vehicle in a particular inertial direction. The technique is based on the method of feedback passivation described in [9]. In this approach, the system is transformed into a feedback interconnection of two passive subsystems. It follows that the interconnection is passive and one may then use output feedback to asymptotically stabilize the system. Intrinsic properties, such as passivity, allow one to derive control algorithms which are globally effective and which work with the natural dynamics rather than supplant them.

II. UNDERWATER VEHICLE EQUATIONS OF MOTION

The underwater vehicle is modeled as a rigid, spheroidal hull, a shape which adequately represents streamlined underwater vehicles. It is assumed that the vehicle mass m is equal to the mass of the fluid displaced by the vehicle; that is, the vehicle is neutrally buoyant. The vehicle is equipped with a single propulsor, which is aligned with the axis of symmetry, and with torque actuators (e.g., tail fins) that provide control moments about three axes. The dynamic model places few restrictions on the expressions for viscous force and moment, allowing viscous models that extend over the full range of vehicle motion.

Vehicle kinematics. The principal axes of the spheroid are taken as the axes of a body-fixed reference frame; these are represented by the unit vectors $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, and $\boldsymbol{b}_3$ in Figure 1. Assuming uniform fluid density, the origin of

H.-Y. Kim is is a former post-doctoral research assistant with the Department of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA.

C. A. Woolsey is with the Department of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA. cwoolsey@vt.edu

the body frame is the vehicle's center of buoyancy, i.e., the center of mass of the displaced fluid. Another reference frame, denoted by the unit vectors $\boldsymbol{i}_1$, $\boldsymbol{i}_2$, and $\boldsymbol{i}_3$, is fixed in inertial space. The location of the body frame with respect to the inertial frame is given by the inertial vector $\boldsymbol{x}$.

Let $\boldsymbol{R}$ denote the proper rotation matrix which transforms free vectors from the body frame to the inertial frame. From Euler's theorem on rotations, $\boldsymbol{R}$ may be represented as a rotation through an angle $\theta \in [0, 2\pi)$ about a fixed axis defined by a unit vector $\boldsymbol{\chi}$ [8]. The matrix $\boldsymbol{R}$ may be parameterized using unit quaternions as follows. Define

$$\boldsymbol{q} = \begin{bmatrix} q_0 \\ \boldsymbol{q}_\chi \end{bmatrix} := \begin{bmatrix} \cos(\frac{\theta}{2}) \\ \boldsymbol{\chi} \sin(\frac{\theta}{2}) \end{bmatrix}.$$

Then

$$\boldsymbol{R}(\boldsymbol{q}) = \mathbb{I} - 2\left(q_0 \mathbb{I} - \hat{\boldsymbol{q}}_\chi\right) \hat{\boldsymbol{q}}_\chi,$$

where $\mathbb{I}$ is the identity matrix and where the operator $\hat{\cdot}$ denotes the 3×3 skew-symmetric matrix satisfying $\hat{\boldsymbol{a}}\boldsymbol{b} = \boldsymbol{a} \times \boldsymbol{b}$ for vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. Note that the identity rotation corresponds to the unit quaternion $\boldsymbol{q} = [1,\ 0,\ 0,\ 0]^T$.

The unit quaternions are a redundant parameterization. Generically, any two quaternions $\boldsymbol{q}$ and $-\boldsymbol{q}$ describe the same rotation. To avoid this difficulty, one might choose a three-parameter representation, such as the modified Rodrigues parameters described in [10]. Any three-parameter representation, however, is necessarily singular at some point. While this may or may not be a meaningful obstruction, in practice, we proceed using unit quaternions.

Let the body frame vector $\boldsymbol{v} = [v_1,\ v_2,\ v_3]^T$ represent the translational velocity of the body frame origin with respect to inertial space. Let $\boldsymbol{\omega} = [\omega_1,\ \omega_2,\ \omega_3]^T$ represent the body angular velocity. Following [4], we define

$$\boldsymbol{\eta} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{q} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\nu} = \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{pmatrix}.$$

The kinematic equations are

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \quad \text{where} \quad \boldsymbol{J}(\boldsymbol{\eta}) = \begin{pmatrix} \boldsymbol{R}(\boldsymbol{q}) & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{4\times 3} & \frac{1}{2}\boldsymbol{Q}(\boldsymbol{q}) \end{pmatrix}$$

and where

$$\boldsymbol{Q}(\boldsymbol{q}) = \begin{bmatrix} -\boldsymbol{q}_\chi^T \\ q_0 \mathbb{I} + \hat{\boldsymbol{q}}_\chi \end{bmatrix}.$$

In this paper, we consider only directional control. We therefore consider only the attitude kinematics

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{Q}(\boldsymbol{q})\boldsymbol{\omega}. \tag{1}$$

Once the global directional control problem is addressed, one may consider path following as a natural extension.

Vehicle dynamics. Let the diagonal matrix $\boldsymbol{M}_{11} = \mathrm{diag}(m_1, m_2, m_3)$ represent the sum of $m\mathbb{I}$ and the added mass matrix, which is diagonal for a spheroid. For a *prolate* spheroid, whose 1-axis is the axis of revolution, $m_1 < m_2 = m_3$. Let the matrix $\boldsymbol{M}_{22}$ represent the sum of the rigid body inertia and the added inertia matrix. Finally, suppose that the center of mass is located by the body frame vector $\boldsymbol{r}_{\mathrm{cm}}$ and let $\boldsymbol{M}_{12} = -m\hat{\boldsymbol{r}}_{\mathrm{cm}}$. The generalized momentum is

$$\boldsymbol{\mu} = \boldsymbol{M}\boldsymbol{\nu}$$

or

$$\begin{pmatrix} \boldsymbol{P} \\ \boldsymbol{\Pi} \end{pmatrix} = \begin{pmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{12}^T & \boldsymbol{M}_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{pmatrix}. \tag{2}$$

In expression (2), $\boldsymbol{P}$ is the translational momentum of the body/fluid system and $\boldsymbol{\Pi}$ is the angular momentum about the center of buoyancy.

The dynamic equations are

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{\mathrm{ext}} \tag{3}$$

where

$$\boldsymbol{C}(\boldsymbol{\nu}) = -\boldsymbol{C}(\boldsymbol{\nu})^T = \begin{pmatrix} \boldsymbol{0}_{3\times 3} & -\hat{\boldsymbol{P}} \\ -\hat{\boldsymbol{P}} & -\hat{\boldsymbol{\Pi}} \end{pmatrix}$$

is the "Coriolis and centripetal matrix" in [4] and where

$$\boldsymbol{g}(\boldsymbol{\eta}) = \begin{pmatrix} \boldsymbol{0} \\ -\boldsymbol{r}_{\mathrm{cm}} \times mg\left(\boldsymbol{R}(\boldsymbol{q})^T \boldsymbol{i}_3\right) \end{pmatrix}$$

is the generalized gravitational force. While there is no true gravity force, because the vehicle is neutrally buoyant, there is a gravity torque about the center of buoyancy. Equation (3) may be rewritten in terms of the generalized momentum:

$$\dot{\boldsymbol{\mu}} = -\boldsymbol{C}\left(\boldsymbol{M}^{-1}\boldsymbol{\mu}\right) \boldsymbol{M}^{-1}\boldsymbol{\mu} - \boldsymbol{g}(\boldsymbol{\eta}) + \boldsymbol{\tau}_{\mathrm{ext}} \tag{4}$$

The generalized force $\boldsymbol{\tau}_{\mathrm{ext}}$ in (3) and (4) represents external forces and moments which do not derive from scalar potential functions. These forces and moments include, for example, control torques, forces due to lift and drag, and propulsive forces. We decompose $\boldsymbol{\tau}_{\mathrm{ext}}$ into control inputs and viscous terms:

$$\boldsymbol{\tau}_{\mathrm{ext}} = \boldsymbol{\tau}_{\mathrm{c}} + \boldsymbol{\tau}_{\mathrm{v}} = \begin{pmatrix} \boldsymbol{F}_{\mathrm{c}} \\ \boldsymbol{T}_{\mathrm{c}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{F}_{\mathrm{v}} \\ \boldsymbol{T}_{\mathrm{v}} \end{pmatrix}.$$

Let $\boldsymbol{\beta}_i$ represent the i^{th} basis vector for $\mathbb{R}^3$; for example, $\boldsymbol{\beta}_1 = [1,\ 0,\ 0]^T$. Because the vehicle's single thruster is aligned with the axis of symmetry, $\boldsymbol{F}_{\mathrm{c}} = F_{\mathrm{c}}\,\boldsymbol{\beta}_1$, where F_{c} represents (scalar) thrust.

The terms $\boldsymbol{F}_{\mathrm{v}}$ and $\boldsymbol{T}_{\mathrm{v}}$ are generally quite complicated. We make some simple and relatively general modeling assumptions. For example, we make the following assumption about the viscous moment.

Assumption 2.1:

$$\begin{aligned} \boldsymbol{T}_{\mathrm{v}} \cdot \boldsymbol{\omega} \leq 0 \quad &\text{when} \quad \boldsymbol{\omega} \neq \boldsymbol{0} \\ \boldsymbol{T}_{\mathrm{v}} = \boldsymbol{0} \quad &\text{when} \quad (\boldsymbol{\omega}, \boldsymbol{v}) = (\boldsymbol{0}, v_{\mathrm{d}}\boldsymbol{\beta}_1) \quad \forall\ v_{\mathrm{d}} \in \mathbb{R} \end{aligned}$$

In words, we assume that the viscous moment opposes angular rate, in general, and that it vanishes for pure translation along the symmetry axis.

We impose a bit more structure on the model of the viscous force. By definition, drag opposes vehicle velocity and lift acts orthogonally to the velocity vector. Considering

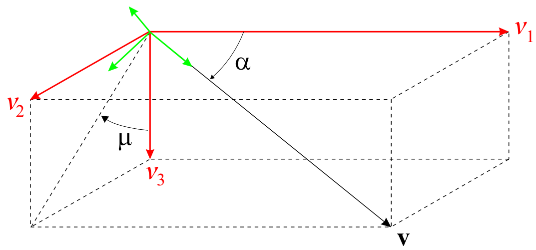

Fig. 2. Hydrodynamic angles for an axisymmetric body.

axial symmetry, we assume that the lift force acts in the plane determined by the velocity vector and the vehicle's longitudinal axis. This assumption fails to capture out-of-plane forces due to asymmetric fluid flow, however it is consistent with standard modeling assumptions.

To express $\boldsymbol{F}_{\rm v}$, we first introduce some hydrodynamic angles. Let

$$\mu = \begin{cases} \arctan\left(\frac{v_2}{v_3}\right) & v_2 \neq 0 \text{ and/or } v_3 \neq 0 \\ 0 & v_2 = v_3 = 0 \end{cases} \qquad (5)$$

The "4-quadrant" arctangent is used in the definition of μ; identifying π with $-\pi$, we have $\mu \in (-\pi, \pi]$. Rotating through the angle μ about the $\boldsymbol{b}_1$ axis defines an intermediate reference frame in which the velocity vector has components only in the "$1-3$" plane. Moreover, the 3-axis component of velocity, in this intermediate frame, is non-negative. Let

$$\alpha = \arctan\left(\frac{\sqrt{v_2^2 + v_3^2}}{v_1}\right) \qquad (6)$$

where, once again, the 4-quadrant arctangent is used. Note that $\alpha \in [0, \pi]$. Rotating through the angle α about the intermediate 2-axis defines a new reference frame in which the 1-axis is aligned with the velocity vector, as shown in Figure 2. Following the terminology of [4], we refer to this reference frame as the "current" frame.

It is standard practice to express lift and drag in terms of non-dimensional coefficients. To non-dimensionalize the forces, we define F_0 as the product of dynamic pressure and a reference area S:

$$F_0 = \frac{1}{2}\rho\|\boldsymbol{v}\|^2 S.$$

Here, ρ is the fluid density. In the current frame, the viscous force takes the form

$$\begin{pmatrix} -\mathcal{D} \\ 0 \\ -\mathcal{L} \end{pmatrix} = -F_0 \begin{pmatrix} C_D(\alpha) \\ 0 \\ C_L(\alpha) \end{pmatrix},$$

In general, the dimensionless coefficients C_D and C_L depend on Reynolds number as well as angle of attack. For convenience, we ignore Reynolds number effects in the remaining discussion; these effects can be incorporated, if necessary, with little impact on the assumptions and conclusions.

To transform the viscous force from the current frame to the body frame, we define the proper rotation matrix

$$\begin{aligned} \boldsymbol{R}_{\rm BC}(\mu, \alpha) &= e^{-\mu\widehat{\boldsymbol{\beta}}_1} e^{-\alpha\widehat{\boldsymbol{\beta}}_2} \\ &= \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ \sin\mu\sin\alpha & \cos\mu & \sin\mu\cos\alpha \\ \cos\mu\sin\alpha & -\sin\mu & \cos\mu\cos\alpha \end{pmatrix}. \end{aligned}$$

Then

$$\begin{aligned} \boldsymbol{F}_{\rm v} &= -\boldsymbol{R}_{\rm BC}(\mu, \alpha) \begin{pmatrix} \mathcal{D} \\ 0 \\ \mathcal{L} \end{pmatrix} \\ &= -F_0 \boldsymbol{R}_{\rm BC}(\mu, \alpha) \begin{pmatrix} C_D(\alpha) \\ 0 \\ C_L(\alpha) \end{pmatrix}. \end{aligned}$$

We make the following assumptions about the form of the drag and lift coefficients.

Assumption 2.2:

- $C_D(\alpha)$ is a continuous, even function which is positive for all α.
- $C_L(\alpha)$ is a continuous, odd function which is positive (negative) when $e^{i\alpha}$ lies in the first or third (second or fourth) quadrant of the complex plane.

For a simple, prolate spheroid, the assumption that C_D is even and positive is an empirical fact. The assumption regarding the form of C_L is consistent with intuition for a prolate spheroid in a steady flow. These assumptions are satisfied, for example, by the drag and lift coefficients

$$\begin{aligned} C_D(\alpha) &= C_{D_0} + C_{D_1}(1 - \cos(2\alpha)) \\ C_L(\alpha) &= \frac{1}{2} C_{L_\alpha} \sin(2\alpha). \end{aligned}$$

Constant parameters C_{L_α}, C_{D_0}, and C_{D_1} can be approximated from steady aerodynamic data as presented, for example, in [5] and [6].

In reality, a spheroid moving at a large angle of attack is subject to complicated, unsteady forces [11]. While these effects certainly impact the dynamics, they are difficult to model accurately. Such effects are typically ignored in control design with the understanding that well-designed model-based feedback can provide suitable system performance even when the model is imperfect.

Error dynamics. Our goal is to stabilize the motion

$$\boldsymbol{q}_{\rm e} = \boldsymbol{q}_{\rm d}, \quad \boldsymbol{\omega}_{\rm e} = \boldsymbol{0}, \quad \boldsymbol{v}_{\rm e} = v_{\rm d}\,\boldsymbol{\beta}_1$$

where $\boldsymbol{q}_d$ is the unit quaternion representing some constant desired attitude and $v_{\rm d} > 0$ is a constant desired speed. Let $\bar{\boldsymbol{q}}_{\rm d}$ represent the quaternion conjugate of $\boldsymbol{q}_{\rm d}$ and define the attitude error quaternion

$$\boldsymbol{e} = \bar{\boldsymbol{q}}_{\rm d} * \boldsymbol{q},$$

where $*$ denotes quaternion multiplication; see [8]. Thus, when $\boldsymbol{q} = \boldsymbol{q}_{\rm d}$, we have $\boldsymbol{e} = \boldsymbol{e}_{\rm d} = [1,\ 0,\ 0,\ 0]^T$. To shift the equilibrium to the origin, define the attitude error vector

$$\tilde{\boldsymbol{e}} = \boldsymbol{e} - \boldsymbol{e}_{\rm d}.$$

Next, define the generalized velocity error

$$\tilde{\boldsymbol{\nu}} = \begin{pmatrix} \tilde{\boldsymbol{v}} \\ \tilde{\boldsymbol{\omega}} \end{pmatrix} = \boldsymbol{\nu} - \boldsymbol{\nu}_{\mathrm{e}} \qquad \text{where} \qquad \boldsymbol{\nu}_{\mathrm{e}} = \begin{pmatrix} \boldsymbol{v}_{\mathrm{e}} \\ \boldsymbol{\omega}_{\mathrm{e}} \end{pmatrix}.$$

Correspondingly, the generalized momentum error is

$$\tilde{\boldsymbol{\mu}} = \begin{pmatrix} \tilde{\boldsymbol{P}} \\ \tilde{\boldsymbol{\Pi}} \end{pmatrix} = \boldsymbol{M}\tilde{\boldsymbol{\nu}}.$$

The error dynamics are

$$\begin{aligned} \dot{\tilde{\boldsymbol{e}}} &= \frac{1}{2}\boldsymbol{Q}(\tilde{\boldsymbol{e}} + \boldsymbol{e}_{\mathrm{d}})\tilde{\boldsymbol{\omega}} & (7) \\ \dot{\tilde{\boldsymbol{\mu}}} &= -\boldsymbol{C}\left(\boldsymbol{M}^{-1}\left(\tilde{\boldsymbol{\mu}} + \boldsymbol{\mu}_{\mathrm{d}}\right)\right)\left(\boldsymbol{M}^{-1}\left(\tilde{\boldsymbol{\mu}} + \boldsymbol{\mu}_{\mathrm{d}}\right)\right) \\ &\quad + \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{r}_{\mathrm{cm}} \times mg\left(\boldsymbol{R}^T \boldsymbol{i}_3\right) \end{pmatrix} + \boldsymbol{\tau}_{\mathrm{c}} + \boldsymbol{\tau}_{\mathrm{v}}, & (8) \end{aligned}$$

where it is understood that the appropriate shift to error coordinates is to be made in the rotation matrix $\boldsymbol{R}$ and in the generalized viscous force $\boldsymbol{\tau}_{\mathrm{v}}$. Our goal is to stabilize the equilibrium at the origin for equations (7) and (8).

III. Feedback Passivation of Nonlinear Cascade Systems

With an appropriate choice of $\boldsymbol{T}_{\mathrm{c}}$ and F_{c}, the system described in Section II can be transformed into a nonlinear cascade which can be treated using the method of "feedback passivation" in [9]. In this section, we review some basic concepts from the theory of passive systems.

Consider a system H described by the state and output equations

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}), \end{aligned} \qquad (9)$$

where $\boldsymbol{u}$ is a vector of inputs and $\boldsymbol{y}$ is a vector of outputs of the same dimension. We assume that $\boldsymbol{f}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}$, so that the system has an equilibrium at the origin. We also assume that $\boldsymbol{h}(\boldsymbol{0}, \boldsymbol{0}) = \boldsymbol{0}$.

Definition 3.1: The system (9) is *passive* if there exists a positive semi-definite function $S(\boldsymbol{x})$, with $S(\boldsymbol{0}) = 0$, such that for every input $\boldsymbol{u}(t)$

$$S(\boldsymbol{x}(\tau)) - S(\boldsymbol{x}(0)) \leq \int_0^\tau \boldsymbol{y}(t) \cdot \boldsymbol{u}(t)\, dt \qquad (10)$$

for all $\tau \geq 0$. The function $S(\boldsymbol{x})$ is called a *storage function*.

Note that if S is C^1, then condition (10) may be written $\dot{S} \leq \boldsymbol{y} \cdot \boldsymbol{u}$. Making an analogy between the storage function S and a physical system's total energy, the condition for passivity requires that the rate of increase of stored energy does not exceed the input power. A simple but important observation is that the negative feedback interconnection of two passive systems is also passive. The sum of the two systems' storage functions is a storage function for the feedback interconnected system.

If a storage function $S(\boldsymbol{x})$ for the system (9) is positive *definite*, then it is a Lyapunov function when $\boldsymbol{u} = \boldsymbol{0}$. Thus, the equilibrium at the origin is stable in the absence of any input. Choosing $\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{y}$ with $\boldsymbol{K} > 0$ makes $\dot{S} \leq 0$ and one may examine asymptotic stability using Lasalle's invariance principle.

Now, consider a nonlinear cascade system of the form

$$\begin{aligned} \dot{\boldsymbol{z}} &= \boldsymbol{f}_z(\boldsymbol{z}) + \tilde{\boldsymbol{\psi}}(\boldsymbol{z}, \boldsymbol{\xi}) & (11) \\ \dot{\boldsymbol{\xi}} &= \boldsymbol{f}_\xi(\boldsymbol{\xi}) + \boldsymbol{g}(\boldsymbol{\xi})\boldsymbol{u} & (12) \end{aligned}$$

and suppose that this system has an equilibrium at the origin. Following [9], we make two assumptions.

Assumption 3.2: The equilibrium $\boldsymbol{z} = \boldsymbol{0}$ of $\dot{\boldsymbol{z}} = \boldsymbol{f}_z(\boldsymbol{z})$ is globally stable and a C^2, positive definite function $S_z(\boldsymbol{z})$ is known which satisfies

$$L_{\boldsymbol{f}_z} S_z = \frac{\partial S_z}{\partial \boldsymbol{z}} \cdot \boldsymbol{f}_z \leq 0.$$

Assumption 3.3: There exists an output $\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{\xi})$ such that

- the system (12) is passive with respect to input $\boldsymbol{u}$ and output $\boldsymbol{y}$, with a C^1, positive definite, radially unbounded storage function $S_\xi(\boldsymbol{\xi})$, and
- one may write

$$\tilde{\boldsymbol{\psi}}(\boldsymbol{z}, \boldsymbol{\xi}) = \boldsymbol{\psi}(\boldsymbol{z}, \boldsymbol{\xi})\boldsymbol{h}(\boldsymbol{\xi}) = \boldsymbol{\psi}(\boldsymbol{z}, \boldsymbol{\xi})\boldsymbol{y}.$$

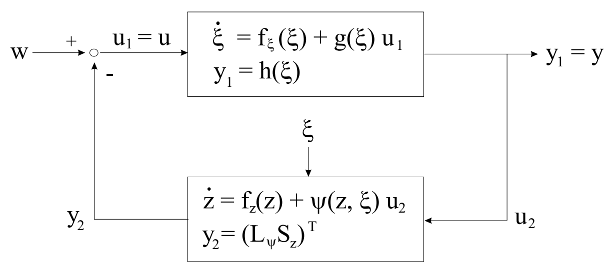

Fig. 3. Passive feedback interconnection

The following theorem, given in [9], says that the complete system (11) and (12) may be transformed into a feedback interconnection of two passive subsystems.

Theorem 3.4: The feedback transformation

$$\boldsymbol{u} = \boldsymbol{w} - (L_\psi S_z)^T = \boldsymbol{w} - \left(\frac{\partial S_z}{\partial \boldsymbol{z}}\boldsymbol{\psi}(\boldsymbol{z}, \boldsymbol{\xi})\right)^T \qquad (13)$$

renders the system (11) and (12) passive from the input $\boldsymbol{w}$ to the output $\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{\xi})$. A storage function is

$$S(\boldsymbol{\xi}, \boldsymbol{z}) = S_\xi(\boldsymbol{\xi}) + S_z(\boldsymbol{z}). \qquad (14)$$

Figure 3 illustrates Theorem 3.4. Because the feedback interconnected system is passive (with input $\boldsymbol{w}$, output $\boldsymbol{y}$, and storage function $S(\boldsymbol{\xi}, \boldsymbol{z})$), the equilibrium at the origin is stable when $\boldsymbol{w} = \boldsymbol{0}$. Moreover, under a zero-state detectability assumption described in [9], choosing

$$\boldsymbol{w} = -\boldsymbol{K}\boldsymbol{y}$$

with $\boldsymbol{K} > 0$ makes the origin globally asymptotically stable. Alternatively, rather than require zero-state detectability, one may use Lasalle's invariance principle to investigate asymptotic stability.

IV. CONTROL DESIGN

By defining the thrust and control moments appropriately, equations (7) and (8) may be written in the nonlinear cascade form (11) and (12).

Proposition 4.1: The feedback control law

$$F_{\mathrm{c}} = \bar{F} := F_0 \left(\cos\alpha C_D(\alpha) - \sin\alpha C_L(\alpha)\right) - k_{v_1}\tilde{v}_1 \quad (15)$$

and

$$\boldsymbol{T}_{\mathrm{c}} = -\left(\boldsymbol{r}_{\mathrm{cm}} \times mg\left(\boldsymbol{R}^T\boldsymbol{i}_3\right) + \boldsymbol{P}\times(v_{\mathrm{d}}\boldsymbol{\beta}_1)\right) + \boldsymbol{u}, \quad (16)$$

with $k_{v_1} > 0$, renders the subsystem (8) passive with input $\boldsymbol{u}$, output $\boldsymbol{y} = \boldsymbol{\omega}$, and storage function

$$S_\xi = \frac{1}{2}\tilde{\boldsymbol{\mu}}^T\boldsymbol{M}^{-1}\tilde{\boldsymbol{\mu}} \quad (17)$$

Proof: Under the given choice of feedback,

$$\begin{aligned}\dot{S}_\xi &= \tilde{\boldsymbol{\nu}}\cdot\dot{\tilde{\boldsymbol{\mu}}}\\ &= \tilde{\boldsymbol{v}}\cdot(\boldsymbol{F}_{\mathrm{c}}+\boldsymbol{F}_{\mathrm{v}}) - (v_{\mathrm{d}}\boldsymbol{\beta}_1)\cdot(\boldsymbol{P}\times\boldsymbol{\omega})\\ &\quad +\boldsymbol{\omega}\cdot(-\boldsymbol{P}\times(v_{\mathrm{d}}\boldsymbol{\beta}_1)+\boldsymbol{u}) + \boldsymbol{\omega}\cdot\boldsymbol{T}_{\mathrm{v}}\\ &= \tilde{\boldsymbol{v}}\cdot(\boldsymbol{F}_{\mathrm{c}}+\boldsymbol{F}_{\mathrm{v}}) + \boldsymbol{\omega}\cdot\boldsymbol{u} + \boldsymbol{\omega}\cdot\boldsymbol{T}_{\mathrm{v}}.\end{aligned}$$

The last term is non-positive by Assumption 2.1. The first term satisfies

$$\begin{aligned}\tilde{\boldsymbol{v}}\cdot(\boldsymbol{F}_{\mathrm{c}}+\boldsymbol{F}_{\mathrm{v}}) &= -\begin{pmatrix}0\\ \tilde{v}_2\\ \tilde{v}_3\end{pmatrix}\cdot\boldsymbol{R}_{\mathrm{BC}}(\mu,\alpha)\begin{pmatrix}\mathcal{D}\\ 0\\ \mathcal{L}\end{pmatrix}\\ &\quad +\tilde{v}_1\left(F_{\mathrm{c}} - \begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\boldsymbol{R}_{\mathrm{BC}}(\mu,\alpha)\begin{pmatrix}\mathcal{D}\\ 0\\ \mathcal{L}\end{pmatrix}\right)\\ &= -\left(\tilde{v}_2\sin\mu + \tilde{v}_3\cos\mu\right)\left(\sin\alpha C_D(\alpha)+\cos\alpha C_L(\alpha)\right)F_0\\ &\quad +\tilde{v}_1\left(F_{\mathrm{c}} - F_0\left(\cos\alpha C_D(\alpha) - \sin\alpha C_L(\alpha)\right)\right).\end{aligned}$$

Consider the first term. By definition of μ,

$$\tilde{v}_2\sin\mu + \tilde{v}_3\cos\mu = \sqrt{\tilde{v}_2^2+\tilde{v}_3^2} \geq 0.$$

Also, given the assumptions on the form of C_D and C_L and the fact that $\alpha\in[0,\pi]$, we have

$$\sin\alpha C_D(\alpha) + \cos\alpha C_L(\alpha) \geq 0$$

(and strictly positive for $\alpha\in(0,\pi)$). Therefore, the first term is non-positive. Now consider the second term and let $F_{\mathrm{c}} = \bar{F}$, as stated in the proposition. The rate of change of S_ξ becomes

$$\begin{aligned}\dot{S}_\xi &= -\sqrt{\tilde{v}_2^2+\tilde{v}_3^2}\left(\sin\alpha C_D(\alpha)+\cos\alpha C_L(\alpha)\right)F_0\\ &\quad -k_{v_1}\tilde{v}_1^2 + \boldsymbol{\omega}\cdot\boldsymbol{u} + \boldsymbol{\omega}\cdot\boldsymbol{T}_{\mathrm{v}}\\ &\leq \boldsymbol{\omega}\cdot\boldsymbol{u}.\end{aligned}$$

This inequality proves the proposition. □

It follows by Lyapunov's direct method that the origin of (8) is stable when $\boldsymbol{u} = \boldsymbol{0}$. We have completed the first step of a two-step design procedure in which we first stabilize the vehicle dynamics and then the attitude.

The modified equations of motion, under the feedback control laws (15) and (16), are

$$\dot{\tilde{\boldsymbol{e}}} = \frac{1}{2}\boldsymbol{Q}(\tilde{\boldsymbol{e}}+\boldsymbol{e}_{\mathrm{d}})\tilde{\boldsymbol{\omega}} \quad (18)$$

$$\begin{pmatrix}\dot{\tilde{\boldsymbol{P}}}\\ \dot{\tilde{\boldsymbol{\Pi}}}\end{pmatrix} = \begin{pmatrix}\left(\tilde{\boldsymbol{P}}+\boldsymbol{P}_{\mathrm{d}}\right)\times\tilde{\boldsymbol{\omega}} + \boldsymbol{F}_{\mathrm{v}} + \bar{F}\boldsymbol{\beta}_1\\ \left(\tilde{\boldsymbol{\Pi}}+\boldsymbol{\Pi}_{\mathrm{d}}\right)\times\tilde{\boldsymbol{\omega}} + \left(\tilde{\boldsymbol{P}}+\boldsymbol{P}_{\mathrm{d}}\right)\times\tilde{\boldsymbol{v}} + \boldsymbol{T}_{\mathrm{v}} + \boldsymbol{u}\end{pmatrix} \quad (19)$$

Define

$$\boldsymbol{z} = \tilde{\boldsymbol{e}} \qquad \text{and} \qquad \boldsymbol{\xi} = \tilde{\boldsymbol{\mu}}.$$

Then equations (18) and (19) are in the nonlinear cascade form (11) and (12) with

$$\boldsymbol{f}_z(\boldsymbol{z}) = \boldsymbol{0} \qquad \text{and} \qquad \tilde{\boldsymbol{\psi}}(\boldsymbol{z},\boldsymbol{\xi}) = \frac{1}{2}\boldsymbol{Q}(\tilde{\boldsymbol{e}}+\boldsymbol{e}_{\mathrm{d}})\tilde{\boldsymbol{\omega}}.$$

and with

$$\begin{aligned}\boldsymbol{f}_\xi(\boldsymbol{\xi}) &= \begin{pmatrix}\left(\tilde{\boldsymbol{P}}+\boldsymbol{P}_{\mathrm{d}}\right)\times\tilde{\boldsymbol{\omega}} + \boldsymbol{F}_{\mathrm{v}} + \bar{F}\boldsymbol{\beta}_1\\ \left(\tilde{\boldsymbol{\Pi}}+\boldsymbol{\Pi}_{\mathrm{d}}\right)\times\tilde{\boldsymbol{\omega}} + \left(\tilde{\boldsymbol{P}}+\boldsymbol{P}_{\mathrm{d}}\right)\times\tilde{\boldsymbol{v}} + \boldsymbol{T}_{\mathrm{v}}\end{pmatrix}\\ \boldsymbol{g}(\boldsymbol{\xi}) &= \begin{pmatrix}\boldsymbol{0}_{3\times3}\\ \mathbb{I}\end{pmatrix}\end{aligned}$$

Equation (18) satisfies Assumption 3.2 with

$$\begin{aligned}S_z(\boldsymbol{z}) = \frac{k_e}{2}\tilde{\boldsymbol{e}}\cdot\tilde{\boldsymbol{e}} &= \frac{1}{2}k_e\left[(1-e_0)^2 + \boldsymbol{e}_\chi\cdot\boldsymbol{e}_\chi\right]\\ &= k_e(1-e_0)\end{aligned}$$

and $k_e > 0$. The choice of S_z is inspired by [7], where the problem of rigid body attitude control is considered. Equations (18) and (19) also satisfy Assumption 3.3 with $\boldsymbol{y} = \boldsymbol{\xi} = \boldsymbol{\omega}$, $\psi(\boldsymbol{z},\boldsymbol{\xi}) = \frac{1}{2}\boldsymbol{Q}(\tilde{\boldsymbol{e}}+\boldsymbol{e}_{\mathrm{d}})$, and with S_ξ given in (17). By Theorem 3.4, the feedback control law

$$\begin{aligned}\boldsymbol{u} &= \boldsymbol{w} - (L_\psi S_z)^T\\ &= \boldsymbol{w} - \left(\frac{\partial S_z}{\partial \boldsymbol{z}}\cdot\left(\frac{1}{2}\boldsymbol{Q}(\tilde{\boldsymbol{e}}+\boldsymbol{e}_{\mathrm{d}})\right)\right)^T\\ &= \boldsymbol{w} - \frac{1}{2}k_e\boldsymbol{e}_\chi\end{aligned}$$

renders the system passive from the new input $\boldsymbol{w}$ to the output $\boldsymbol{y} = \boldsymbol{\omega}$ with respect to the storage function

$$S(\boldsymbol{\xi},\boldsymbol{z}) = S_\xi(\boldsymbol{\xi}) + S_z(\boldsymbol{z}).$$

The rate of change of the storage function is

$$\begin{aligned}\dot{S} &= -\sqrt{\tilde{v}_2^2+\tilde{v}_3^2}\left(\sin\alpha C_D(\alpha)+\cos\alpha C_L(\alpha)\right)F_0\\ &\quad -k_{v_1}\tilde{v}_1^2 + \boldsymbol{T}_{\mathrm{v}}\cdot\boldsymbol{\omega} + \boldsymbol{y}\cdot\boldsymbol{w}\\ &\leq \boldsymbol{y}\cdot\boldsymbol{w}.\end{aligned}$$

Suppose we choose $\boldsymbol{w} = -\boldsymbol{K}\boldsymbol{y}$ with $\boldsymbol{K} > 0$. Then

$$\begin{aligned}\dot{S} &= -\sqrt{\tilde{v}_2^2+\tilde{v}_3^2}\left(\sin\alpha C_D(\alpha)+\cos\alpha C_L(\alpha)\right)F_0\\ &\quad -k_{v_1}\tilde{v}_1^2 + \boldsymbol{T}_{\mathrm{v}}\cdot\boldsymbol{\omega} - \boldsymbol{\omega}^T\boldsymbol{K}\boldsymbol{\omega} \leq 0.\end{aligned}$$

Because S is radially unbounded, this proves that the origin is globally stable. To conclude global asymptotic stability, we apply Lasalle's invariance principle. Define the set

$$E = \left\{ (\tilde{\boldsymbol{e}}, \tilde{\boldsymbol{\mu}}) \mid \dot{S} = 0 \right\}$$

and let $\mathcal{M}$ be the largest invariant set contained in E. By Lasalle's principle, trajectories converge to the set $\mathcal{M}$ as $t \to \infty$. If $\mathcal{M} = \{\mathbf{0}\}$, then the equilibrium is globally asymptotically stable. Now, $\dot{S} \equiv 0$ implies that $\tilde{\boldsymbol{\nu}} \equiv \mathbf{0}$. Thus,

$$\dot{\tilde{\boldsymbol{\nu}}} = \mathbf{0} \qquad \Rightarrow \qquad \dot{\tilde{\boldsymbol{\mu}}} = \mathbf{0}.$$

Within the set $\mathcal{M}$,

$$\dot{\tilde{\boldsymbol{\mu}}} = \begin{pmatrix} 0 \\ -\frac{1}{2} k_e \boldsymbol{e}_\chi \end{pmatrix}.$$

Thus, $\boldsymbol{e}_\chi = \mathbf{0}$ so $e_0 = 1$ and $\tilde{\boldsymbol{e}} = \mathbf{0}$. Global asymptotic stability follows from Lasalle's invariance principle.

V. Conclusions and Future Work

This paper has presented a control strategy which globally asymptotically stabilizes translation of a streamlined underwater vehicle in a desired inertial direction. The approach relies on the technique of feedback passivation, in which the system is transformed through feedback into a passive system. Using known results from the theory of passive systems, one may then define output feedback to provide asymptotic stability. The main result uses very general modeling assumptions for the viscous force.

The problem of directional control is a simplification of a more practical problem: nonlinear path following. Intuitively, having globally asymptotically stabilized the direction of travel, one might close an "outer loop" to stabilize motion along a straight path. The more challenging problem of following a parameterized curve in space remains as future work. This problem was partially addressed in [3], where the error kinematics were expressed in terms of the Serret-Frenet frame associated with the desired path. The hydrodynamic model was limited, however, because it neglected added mass and assumed small angles of attack and sideslip. Our hope is that the present results on feedback passivation will serve as a preliminary step toward global nonlinear path following for streamlined underwater vehicles.

Several other issues remain to be addressed, including the effect of non-zero mean disturbances, such as a net buoyant force or cross currents. There are also the issues of actuator dynamics and actuator limits. Regarding the actuators, it may be feasible to eliminate roll control for a vehicle with a low center of gravity. Some streamlined underwater vehicles use control planes that are slaved to provide only pitch and yaw actuation. Thus, the problem has practical relevance. An increasing number of vehicles use vectored thrusters, which offer improved agility but do not provide roll control. For agile vehicles, it is especially important to develop control schemes which perform well over a broad operating envelope.

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