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Energy Optimal Reconfiguration for Large Scale Formation Flying

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***Abstract*— An efficient method for energy optimal reconfiguration of formation flying involving multiple spacecraft is presented. The idea is to introduce a set of way-points through which the spacecraft are required to pass combined with certain parameterization of the trajectories. The resulting energy optimal with collision avoidance constraints problem is formulated as a parameter optimization problem in terms of the way-points parameters. A numerical algorithm is proposed and used for scenarios with multiple spacecraft.**

I. INTRODUCTION

Formation flying spacecraft refers to a set of spatially distributed spacecraft flying in formation, capable of interacting and cooperating with one another. Future space missions, like the Terrestrial Planet Finder (TPF), Terrestrial Planet Imager (TPI), and Space Technology–3 (ST3), will use formation flying extensively.

Formation flying control requires autonomous fleet reconfiguration for which a path planner is needed to compute spacecraft maneuvers such that collisions are avoided and eventually some performance index (fuel, energy, maneuver time, etc.) is optimized. Collision avoidance and trajectory generation problems have been the subject of extensive research in air traffic control, robotics, formation flying [1], [2], [3], [4], [5], [6], [7], [8]. These problems are generally difficult to solve because the set of feasible solutions is non-convex, possibly infinite dimensional and defined using an infinite number of constraints [1], [6]. Several methods have been suggested to generate solvable approximations in which the trajectories are restricted to a set of basis functions and in which the constraints are imposed at a finite number of points in time [1], [3], [5], [8]. This generally results in very large feasibility problems, whose numerical solution is difficult.

This paper presents a method for the generation of energy optimal, collision free, reconfiguration trajectories for formation flying in deep space (gravity free environment). The idea is to parameterize the trajectories using piecewise cubic polynomials (which are energy optimal for each *individual* spacecraft) and to use a set of way-points through which these trajectories pass. An efficient numerical algorithm which exploits the mathematical structure of the problem is used to select these way-points locations and velocities such that collisions are avoided. Examples show that this algorithm is very efficient even for a large number of spacecraft.

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This investigation is based on the assumption that piecewise linear, continuous accelerations are implemented. Current and future research will address the problem of how this methodology can be implemented using on–off controls that are characteristic to the thrusters currently on board of spacecraft.

II. STATEMENT OF THE PROBLEM

The spacecraft are modeled as identical *point masses*, of unitary mass, acted upon *only* by internally generated forces (e. g. by thrusters) used to control their motion. We assume that the maneuver time is the same for all spacecraft (i. e. synchronous reconfiguration), let it be called T. Hence, if N is the number of spacecraft, r_l, v_l, a_l, $l = 1, \dots, N$, the position, velocity, and acceleration vectors with respect to an inertial reference frame, respectively, of spacecraft l, the equations of motion and the terminal conditions are:

$$\dot{r}_l(t) = v_l(t),\ \dot{v}_l(t) = a_l(t),$$

$$r_l(0) = r_{l_0},\ r_l(T) = r_{l_T},\ v_l(0) = v_{l_0},\ v_l(T) = v_{l_T}, \quad (1)$$

where $l = 1, \dots, N$, r_{l_0}, r_{l_T}, v_{l_0}, v_{l_T} are the initial and final conditions and t is the time.

We remark that a double integrator has been used to represent each spacecraft dynamics, ignoring the orbital forces. This simplification has been adopted in view of the following observation. Consider a deep space Earth-trailing formation flying mission and assume that the spacecraft are only a few kilometers apart. Then it can be shown that the differential orbital force between two spacecraft is negligible (on the order of 10^{-23} [9]). Because the reconfiguration scenarios that we are interested in occur on relatively short time scales ignoring the orbital forces is well justified.

The collision avoidance constraints are specified in terms of the *forbidden spheres* associated with the spacecraft: any two forbidden spheres do not intersect:

$$\| r_l(t) - r_m(t) \|^2 \geq (R_l + R_m)^2,$$

$$l = 1, \dots, N-1,\ m = l+1, \dots, N,\ t \in [0, T], \quad (2)$$

where R_l is the radius of the forbidden sphere associated with spacecraft l. The objective is to find $a_l(t),\ t \in [0, T]$, $l = 1, \dots, N$, such that the μ energy expended,

$$J_\mu = \sum_{l=1}^{N} \mu_l \int_0^T a_l^T(t) a_l(t)\ dt \quad (3)$$

is minimized and collisions are avoided. The weights $\mu_l > 0$ add up to 1: $\sum_{l=1}^{N} \mu_l = 1$.

We remark that the diameter of the collision avoidance region (forbidden sphere) is a reflection of how far the actual spacecraft is away from being a point mass. Making the point mass approximation is especially useful when there is no interest in the orientation of the spacecraft (hence we only want to perform a translational reconfiguration). If orientation is important during the maneuver, 6 DOF models have to be employed.

III. SOLUTION APPROACH

Consider first the case of one spacecraft ($N = 1$, $\mu_1 = 1$). Let $\{(t_j, w_j, v_j) \in \Re \times \Re^3 \times \Re^3,\ j = 1, \ldots, M + 2\}$ be a sequence of way-points specifying time, position, and velocity, with $t_j < t_{j+1}\ \forall j = 1, \ldots, M + 1$. Let $r(t)$, $t_1 \le t \le t_{M+2}$ denote C^1 trajectories going through these way-points: $r(t_j) = w_j$, $\dot r(t_j) = v_j$. The *unique* trajectory that minimizes J_μ is given by [4]:

$$r(t) = \frac{1}{6}c_j(t^3 - t_j^3) + \frac{1}{2}d_j(t^2 - t_j^2) -$$
$$(\frac{1}{2}c_j t_j^2 + d_j t_j - v_j)(t - t_j) + r_j,\ t_j \le t \le t_{j+1} \quad (4)$$

for $j = 1, \ldots, M + 1$, where

$$c_j = \frac{-12(w_{j+1} - w_j) + 6(v_{j+1} + v_j)(t_{j+1} - t_j)}{(t_{j+1} - t_j)^3} \quad (5)$$

$$d_j = \frac{v_{j+1} - v_j}{t_{j+1} - t_j} +$$
$$\frac{t_{j+1} + t_j}{(t_{j+1} - t_j)^3}(6(w_{j+1} - w_j) - 3(v_{j+1} + v_j)(t_{j+1} - t_j)). \quad (6)$$

Consider now $N > 1$ spacecraft and assume that for the l-th spacecraft $M_l \ge 0$ *intermediate* way-points are introduced. Our method assumes that each spacecraft follows a trajectory described by (4). Hence the resulting trajectories are energy optimal for each *individual spacecraft*. The way-points parameters can be determined using various considerations (e.g. satisfaction of the collision avoidance constraints).

We classify the way-points as *intermediate* and *end* points (i. e. initial and final points of the trajectories). We assume that the end points locations and velocities are fixed.

In order to simplify the problem, we introduce the dimensionless time $\zeta = t/T$, $0 \le \zeta \le 1$. The maneuver duration, T, can be fixed or determined aposteriori to enforce saturation constraints on the controls, a_l, as shown in [8].

Using (4) the position vector of the l-th spacecraft, r_l, can be expressed in terms of ζ as follows:

$$r_l(\zeta) = A_{l_j} U_{l_j}, \quad (7)$$

where

$$A_{l_j} = \left[\ M_1 \quad M_2 \quad M_3 \quad M_4\ \right], \quad (8)$$

$$M_1 = (1 + \frac{2(\zeta^3 - \zeta_{l_j}^3)}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3} +$$
$$\frac{-3(\zeta_{l_{j+1}} + \zeta_{l_j})(\zeta^2 - \zeta_{l_j}^2) + 6\zeta_{l_j}\zeta_{l_{j+1}}(\zeta - \zeta_{l_j})}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3})I_3, \quad (9)$$

$$M_2 = (\frac{\zeta^3 - \zeta_{l_j}^3 - (2\zeta_{l_{j+1}} + \zeta_{l_j})(\zeta^2 - \zeta_{l_j}^2)}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2} +$$
$$\frac{(2\zeta_{l_j}\zeta_{l_{j+1}} + \zeta_{l_{j+1}}^2)(\zeta - \zeta_{l_j})}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2})I_3, \quad (10)$$

$$M_3 = (\frac{-2(\zeta^3 - \zeta_{l_j}^3) + 3(\zeta_{l_{j+1}} + \zeta_{l_j})(\zeta^2 - \zeta_{l_j}^2)}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3} -$$
$$\frac{6\zeta_{l_j}\zeta_{l_{j+1}}(\zeta - \zeta_{l_j})}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3})I_3, \quad (11)$$

$$M_4 = (\frac{\zeta^3 - \zeta_{l_j}^3 - (\zeta_{l_{j+1}} + 2\zeta_{l_j})(\zeta^2 - \zeta_{l_j}^2)}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2}$$
$$+ \frac{\zeta_{l_j}(2\zeta_{l_{j+1}} + \zeta_{l_j})(\zeta - \zeta_{l_j})}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2})I_3, \quad (12)$$

$$U_{l_j} = \left[\ w_{l_j}^T \quad u_{l_j}^T \quad w_{l_{j+1}}^T \quad u_{l_{j+1}}^T\ \right]^T, \quad (13)$$

$\zeta_{l_j} \le \zeta \le \zeta_{l_{j+1}}$, $u_{l_j} = v_{l_j}T$, and w_{l_j}, v_{l_j}, and ζ_{l_j} are the position, velocity, and dimensionless time of the j-th way-point of the l-th spacecraft, respectively.

Consider now two spacecraft, l and m. Let ζ be fixed, $\zeta \in [\zeta_{l_j}\ \zeta_{l_{j+1}}]$ and $\zeta \in [\zeta_{m_k}\ \zeta_{m_{k+1}}]$. Then, using (7), the distance square between spacecraft l and m trajectories, d_{lm}^2, can be expressed as a *time varying quadratic form* in the way-points locations and velocities:

$$d_{lm}^2(\zeta) = \| r_l(\zeta) - r_m(\zeta) \|^2 = U^T A_{lm}(\zeta) U, \quad (14)$$

where

$$A_{lm}(\zeta) = (A_l - A_m)^T (A_l - A_m), \quad (15)$$

$$A_l = \left[\ 0 \quad \ldots \quad 0 \quad A_{l_j} \quad 0 \quad \ldots \quad 0\ \right], \quad (16)$$

$$A_m = \left[\ 0 \quad \ldots \quad 0 \quad A_{m_k} \quad 0 \quad \ldots \quad 0\ \right], \quad (17)$$

$$U = \left[\ w_{1_1}^T \quad u_{1_1}^T \quad \ldots \quad w_{l_j}^T \quad u_{l_j}^T \quad \ldots \quad w_{N_{M_N+2}}^T \quad u_{N_{M_N+2}}^T\ \right]^T. \quad (18)$$

Using (3) and (7) J_μ can be expressed as a *quadratic* form in U:

$$J_\mu = \sum_{l=1}^{N} \mu_l J_l = U^T B U,\ B = diag(\mu_1 B_1\ \ldots\ \mu_N B_N), \quad (19)$$

where $B_l \ge 0$ is associated with the energy of the l-th spacecraft, J_l:

$$J_l = \sum_{j=1}^{M_l+1} U_{l_j}^T B_{l_j} U_{l_j} = U_l^T B_l U_l, \quad (20)$$

$$B_{l_j} = \frac{4}{T^3}(\zeta_{l_{j+1}} - \zeta_{l_j})(C^T C +$$
$$3(\zeta_{l_{j+1}}^2 + \zeta_{l_{j+1}}\zeta_{l_j} + \zeta_{l_j}^2) D^T D + \frac{3}{2}(\zeta_{l_{j+1}} + \zeta_j)(D^T C + C^T D)), \quad (21)$$

$$C = [\frac{-3(\zeta_{l_{j+1}} + \zeta_{l_j})}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3} I_3 \; \frac{-2\zeta_{l_{j+1}} - \zeta_{l_j}}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2} I_3$$

$$\frac{3(\zeta_{l_{j+1}} + \zeta_{l_j})}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3} I_3 \; \frac{-\zeta_{l_{j+1}} - 2\zeta_{l_j}}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2} I_3], \tag{22}$$

$$D = [\frac{2}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3} I_3 \; \frac{1}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2} I_3$$

$$\frac{-2}{(\zeta_{l_{j+1}} - \zeta_{l_j})^3} I_3 \; \frac{1}{(\zeta_{l_{j+1}} - \zeta_{l_j})^2} I_3], \tag{23}$$

$$U_l = \left[\begin{array}{ccccc} w_{l_1}^T & u_{l_1}^T & \ldots & w_{l_{M_l+2}}^T & u_{l_{M_l+2}}^T \end{array} \right]^T, \tag{24}$$

Assume that w_{l_1}, u_{l_1}, $w_{l_{M_l}+2}$, $u_{l_{M_l}+2}$, and ζ_{l_j}, $l = 1, \ldots, N$, $j = 1, \ldots, M_l + 2$, are given. We introduce the vector of optimization variables,

$$x = [w_{1_2}^T \; u_{1_2}^T \; \ldots \; w_{N_{M_N}+1}^T \; u_{N_{M_N}+1}^T]^T, \tag{25}$$

such that J_μ and d_{lm}^2 are expressed as

$$J_\mu = x^T Q x + b^T x + c, \tag{26}$$

$$d_{lm}^2(\zeta) = x^T Q_{lm}(\zeta) x + b_{lm}^T(\zeta) x + c_{lm}(\zeta), \tag{27}$$

with $Q > 0$, b, c, $Q_{lm}(\zeta)$, $b_{lm}(\zeta)$, $c_{lm}(\zeta)$ easy to compute from B, $A_{lm}(\zeta)$ and the given end points parameters.

In this framework the energy optimal collision avoidance problem (an approximation of the original one) becomes:

$$\min_x J_\mu \text{ subject to: } d_{lm}^2(\zeta) \geq (R_l + R_m)^2,$$

$$l = 1, \ldots, N-1, \; m = l+1, \ldots, N, \; \zeta \in [0, \; 1]. \tag{28}$$

IV. NUMERICAL SOLUTION

In the following we propose a *sequential* algorithm to approach (28), motivated by the fact that *the collision avoidance constraints are critical*. Hence we first solve the collision avoidance problem and then minimize J_μ while making sure that the collision avoidance constraints are satisfied.

A. Collision Avoidance Problem Solution

The collision avoidance problem is to find x such that

$$d_{lm}^2(\zeta) \geq (R_l + R_m)^2,$$

$$l = 1, \ldots, N-1, \; m = l+1, \ldots, N, \; \zeta \in [0, \; 1]. \tag{29}$$

For this problem's solution we proceed as follows. At the current iteration step, knowing x, for each pair of spacecraft, $(l, \; m)$, we calculate the *global* minimum of $d_{lm}^2(\zeta)$, $0 \leq \zeta \leq 1$ (this is easy since $d_{lm}^2(\zeta)$ is a piecewise polynomial of degree 6 in ζ). Let d_{lm*}^2, denote those global minima which violate the constraints ($d_{lm*}^2 < (R_l + R_m)^2$) and ζ_{lm*} be the corresponding dimensionless times.

Next we build a penalty function based only on the violating constraints:

$$P(x) = \sum_{l,m} ((R_l + R_m)^2 - d_{lm*}^2) =$$

$$\sum_{l,m} ((R_l + R_m)^2 - x^T Q_{lm}(\zeta_{lm*}) x - b_{lm*}(\zeta_{lm*})^T x - c_{lm*}(\zeta_{lm*})) \tag{30}$$

where only the violating pairs, (l, m), appear in the sum.

We assume that a change in x is made along a direction $g \neq 0$:

$$x_+ = x + \alpha g. \tag{31}$$

Then $P(x_+)$ can be expressed as:

$$P(x_+) = P + \alpha^2 g^T H g + \alpha g^T \nabla P, \tag{32}$$

where $P = P(x)$, $H \leq 0$ is half of the Hessian of $P(x)$ and ∇P is the gradient of $P(x)$.

Next we solve $P(x_+) = 0$, yielding

$$\alpha = \frac{-g^T \nabla P + / - \sqrt{(g^T \nabla P)^2 - 4 g^T H g \; P}}{2 g^T H g} \; if \; g^T H g < 0 \tag{33}$$

$$\alpha = -\frac{P}{g^T \nabla P} \; if \; g^T H g = 0, \tag{34}$$

and select α of *minimum absolute value*.

At the next step we update the penalty function based on the *current* violating constraints and iterate until the algorithm converges (e. g. no constraints are violated), the number of iterations allowed is exceeded, or the norm of x variation between two consecutive steps is smaller than the allowed tolerance.

We remark that our algorithm implicitly assumes that the time dependency of the constraints is negligible for small variations of x. This is one of the reasons for choosing the solution of minimum absolute value of $P(x_+) = 0$. By doing so we expect the number of iterations to be small.

In this paper we choose $g = \nabla P$, the gradient of $P(x)$, because it provides the fastest variation in $P(x)$. The resulting algorithm is coined DIG (from Distance and Gradient) and it is described next.

DIG Algorithm Description

1) **Initialization:** Set

$$x = -0.5 Q^{-1} b \tag{35}$$

which is the solution of the *unconstrained* μ energy optimal problem.

2) **Trajectories assessment:** For all $(l, \; m)$ pairs compute the global minima of $d_{lm}^2(\zeta)$, $\zeta \in [0, 1]$, which violate the constraints, d_{lm*}^2, and the corresponding ζ, ζ_{lm*}. If there is no violation exit.

3) **Direction of movement calculation:** Compute the gradient of the penalty function:

$$g = \nabla P = -\sum_{l,m} (2 Q_{lm}(\zeta_{lm*}) x + b_{lm}(\zeta_{lm*})) \tag{36}$$

where only the violating pairs appear in the sum.
If $g = 0$ then slightly perturb x randomly and go back to step 2, else calculate H and P:

$$H = -\sum_{l,m} Q_{lm}(\zeta_{lm*}), \; P = \sum_{l,m} ((R_l + R_m)^2 - d_{lm*}^2). \tag{37}$$

4) **Prediction:** Predict the next value of x:

$$x_+ = x + \alpha g \tag{38}$$

where

$$\alpha = \frac{-\parallel g \parallel^2 + \sqrt{\parallel g \parallel^4 - 4g^T Hg \; P}}{2g^T Hg} \quad if \;\; g^T Hg < 0 \tag{39}$$

and

$$\alpha = -\frac{P}{\parallel g \parallel^2} \;\; if \;\; g^T Hg = 0. \tag{40}$$

5) **Return:** Set

$$x = x_+ \tag{41}$$

and return to step 2.

The process terminates when all spacecraft are separated (successful termination, through step 2), the variation in x, αg, is too small, or the number of iterations allowed is exceeded.

B. Minimization of J_μ

After the spacecraft have been separated through successful application of DIG we can minimize J_μ taking care that the collision avoidance constraints are satisfied. The following gradient based algorithm (JG) can be used.

Algorithm JG

1) **Direction of movement (d) calculation:**

$$d = -\frac{\parallel \nabla J_\mu \parallel^2}{2\nabla J_\mu^T Q \nabla J_\mu} \nabla J_\mu, \;\; \nabla J_\mu = 2Qx + b. \tag{42}$$

Set the step size, $p = 1$.

2) **Tentative step:**

$$x_t = x + pd \tag{43}$$

3) **Calculation of the step size, p:** Determine if the collision avoidance constraints are violated at x_t. If they are satisfied set $x = x_t$ and go back to step 1 else set $p = p/2$ and go back to step 2.
Finally, if the number of iterations is greater than the maximum allowed or the variations in x or in J_μ are less than prescribed tolerances, exit.

Other minimization algorithms (e. g. Newton based) can be implemented and other methods to change p can be designed.

V. EXAMPLES

The first example of the application of the above methodology is to the rest-to-rest "swapping cube maneuver", in which 8 spacecraft placed at the corners of a cube of side length 10 m must swap places simultaneously. The following data is used: $R_l = 1$ m, $\mu_l = 0.125$, $l = 1, \ldots, 8$.

The unconstrained energy optimal trajectories are straight lines cubic parameterized by time. If these are used the spacecraft collide at the center of the cube, all 28 collision avoidance constraints being simultaneously violated.

The number of intermediate way-points per spacecraft is chosen using the following heuristic method. Because the energy optimal unconstrained trajectories reach the critical point, the center of the cube, at the same time, $\zeta = 0.5$, we introduce one intermediate way-point per spacecraft, $M_l = 1$, with $\zeta_{l_2} = 0.5$, $l = 1, \ldots, 8$. We remark that, for most of the missions currently under consideration, the introduction of one way-point per spacecraft may be sufficient in this formulation of the problem, since one way-point introduces 6 optimization variables (position and velocity vectors). Furthermore, for reconfiguration maneuvers which are not synchronous, in which each spacecraft has associated with it a different maneuver time, T_l, no intermediate way-points may be necessary.

Application of our methodology in which DIG is followed by JG yielded a collision free solution in 19 iterations. The maneuver time, $T = 11.5$ s, has been determined aposteriori as shown in [8] such that the maximum absolute value of the components of the accelerations does not exceed 1 m/s^2. The resulting trajectories are shown in Fig. 1 and the time histories of the distances in Fig. 2 indicating that the minimum allowed distance (2 m) is not violated. The accelerations time histories, given in Fig. 3, show that they are continuous (even though the method allows for discontinuous accelerations).

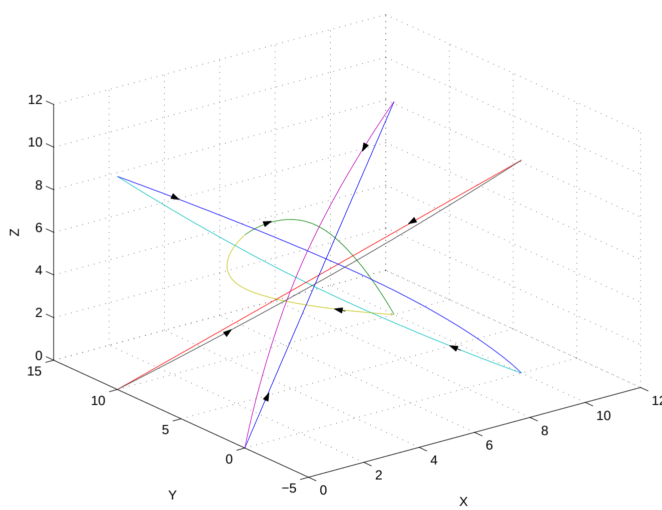

Fig. 1. Collision Free Trajectories for the Swapping Cube Maneuver

The cost along the unconstrained trajectories is 2.37 whereas the one along the constrained, collisions free, trajectories is 3.22, showing that 35.9 $\%$ additional energy is required to avoid collisions.

We remark that if an additional parameter, the step size, s, is introduced in the DIG algorithm such that, at each step we have

$$x_+ = x + s\alpha g \tag{44}$$

better performance can be obtained for various values of s (i.e. less μ energy), however at the expense of computation

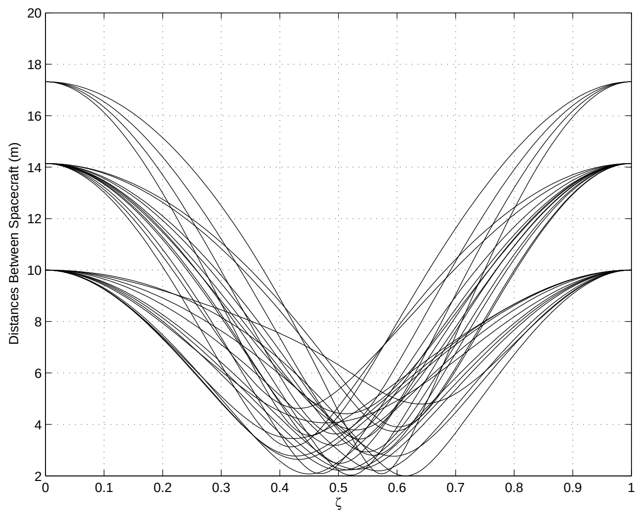

Fig. 2. Distances Time Histories for the Swapping Cube Maneuver

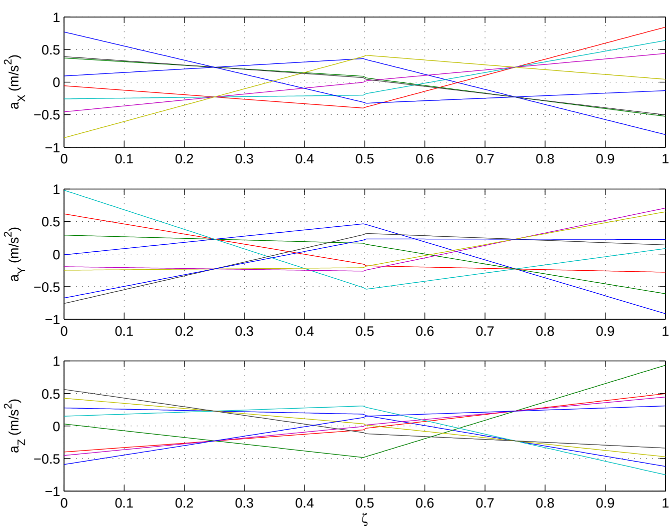

Fig. 3. Accelerations Time Histories for the Swapping Cube Maneuver

time (i.e. increase of the number of iterations). Fig. 4 indicates the influence of s on J_μ and the number of iterations. If $s = 0.4$ is used the additional energy used to avoid collisions is only 25.4 $\%$ greater than the unconstrained one. However 35 iterations are needed in this case.

The next example involves 16 spacecraft equidistantly placed on a circle of radius 10 m. The reconfiguration maneuver considered is a rest-to-rest one, coined "swapping circle": each spacecraft must swap its place with the one opposite to it with respect to the center of the circle. If the unconstrained energy optimal trajectories are used, the spacecraft collide at the center of the circle, all 90 collision avoidance constraints being simultaneously violated.

We introduce one intermediate way-point per spacecraft with $\zeta_{l_2} = 0.5$ and assume $T = 20$ sec. Fig. 5 shows the variation of J_μ and of the number of iterations with the

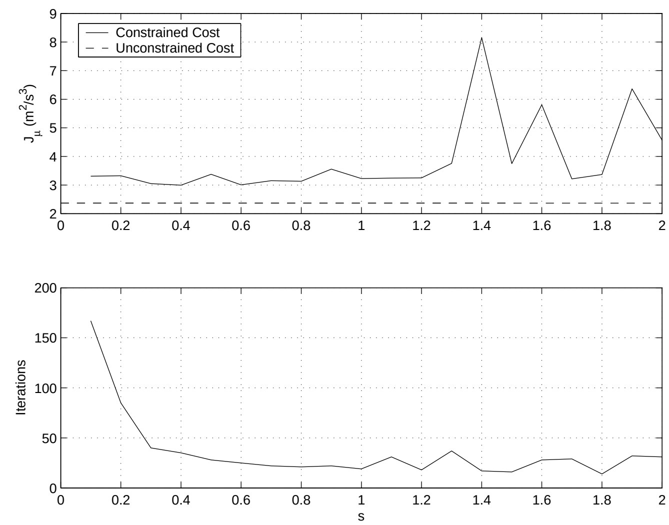

Fig. 4. DIG Step Size Influence on the Convergence Properties for the Swapping Cube Maneuver

step size, s. For $s = 1$, 39 iterations are necessary and the final value of the cost is $J_\mu = 1.26$ (the unconstrained cost is 0.6). The corresponding collision free trajectories are given in Fig. 6, while the distances and accelerations time histories are shown in Fig. 7 and 8, respectively. The accelerations are continuous and all distances are greater or equal to 2 m, the minimum allowed distance.

If $s = 0.2$ is used the final value of J_μ is 1.15, however obtained after 240 iterations. For a faster solution, with a good final value of the cost, $s = 1.5$ can be used, yielding $J_\mu = 1.48$ after 28 iterations.

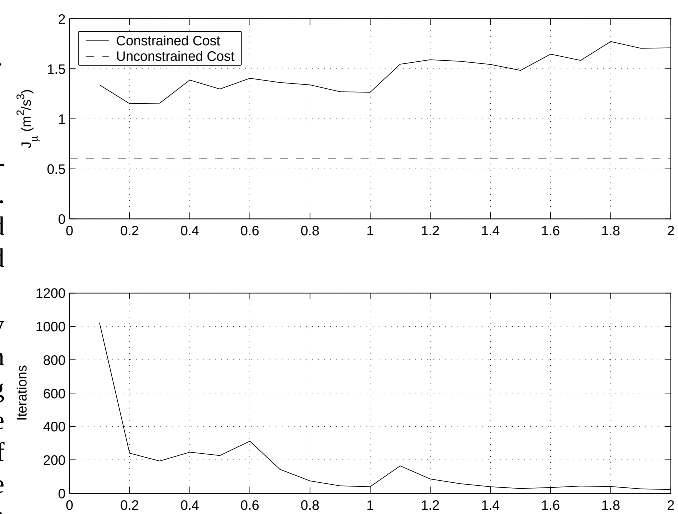

Fig. 5. DIG Step Size Influence on the Convergence Properties for the Swapping Circle Maneuver

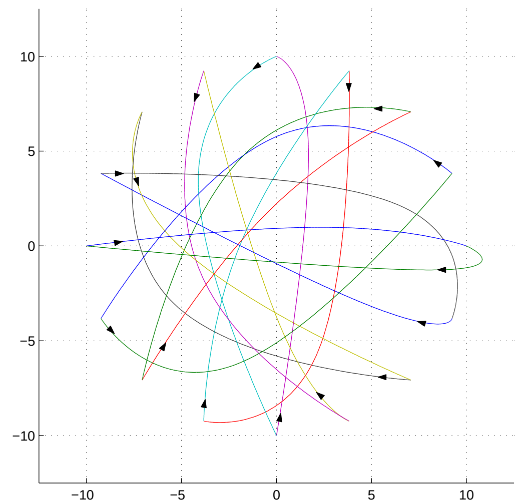

Fig. 6. Collision Free Trajectories for the Swapping Circle Maneuver

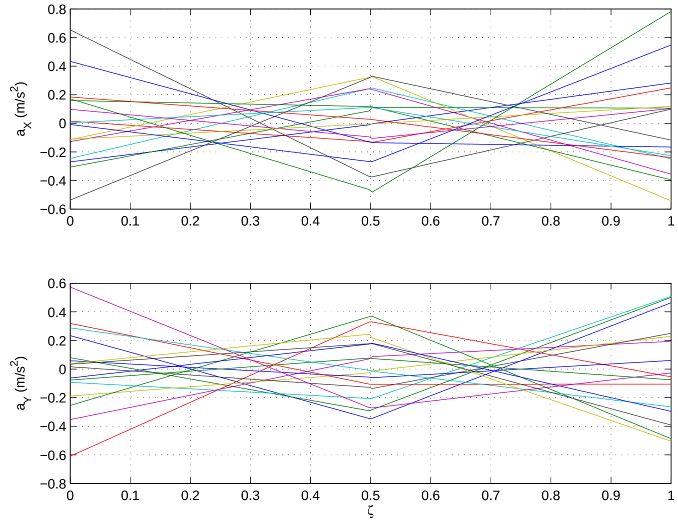

Fig. 8. Accelerations Time Histories for the Swapping Circle Maneuver

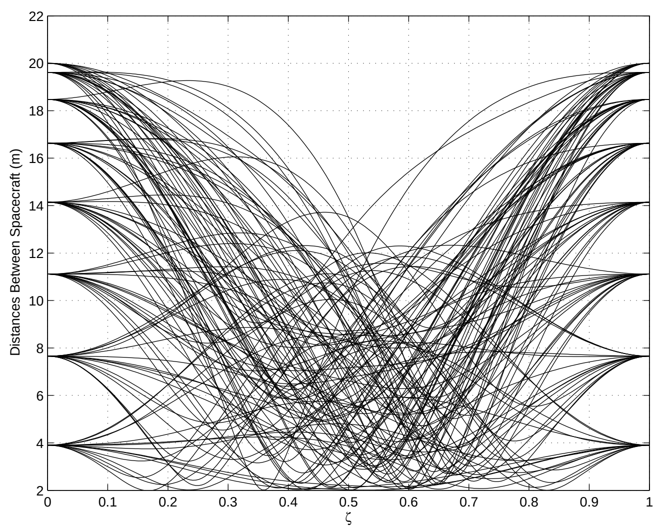

Fig. 7. Distances Time Histories for the Swapping Circle Maneuver

VI. CONCLUSIONS

An efficient energy optimal and collision avoidance reconfiguration technique for formation flying is proposed. The methodology is characterized by two important features: first, the spacecraft trajectories are parameterized using piecewise cubic polynomials, which are energy optimal for each *individual* spacecraft for a given sequence of waypoints, and second, the collision avoidance constraints are approximated in such a way that closed form solutions for the approximated problem are obtained. This combination leads to an algorithm whose application to complicated collision avoidance problems yields very fast solutions. Moreover, using these solutions, reconfiguration can be conducted very efficiently (i. e. with low energy). An important parameter which can improve the performance of the method is the step size used in the collision avoidance solver.

Future work will analyze the extension of this procedure to problems involving equalization of energy constraints as well as the development of analogous techniques for fuel optimal problems.

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