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Vibration Control of a Cantilever Beam Using Multiple Model Adaptive Control

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Abstract – **Flexible systems with large load variations are very difficult to control. Robust control produces non-optimal results for unmodelled configurations and adaptive control suffers from poor transient responses. The Multiple Model Adaptive Control (MMAC) scheme offers a solution, however, it is highly computationally demanding. In this paper, MMAC with one fixed robust controller and one adaptive controller is applied to attenuate vibrations in cantilever beam structures with large varying parameters. A very simple supervisor scheme is proposed to reduce the computation burden. Simulation results show that the proposed strategy can achieve better transient response than adaptive control and better attenuation than robust control.**

I. INTRODUCTION

A cantilever beam with heavy loads mounted along the beam can be used as a basic representative model for a number of flexible advanced engineering structures [5]. Large sudden variations in the loads often make such a system difficult to control. Although a robust controller may be able to control the system to a certain extent, its performance may not be sufficiently satisfactory [2]. Alternatively, an adaptive controller may be used, however its transient performance may be poorer than desired, thus limiting its application [4]. The MMAC method, which switches between multiple models, was proposed by Narendra and Balakrishnan [1] to improve the transient response of classical adaptive control. One disadvantage with their scheme, however, is that it requires many fixed models to achieve satisfactory performance, thus increasing the computational load. Furthermore, large numbers of fixed models may result in undesirable switching between competitive models, causing deterioration in system performance.

In this paper, a modified MMAC that uses only one fixed robust controller and one adaptive controller, together with a simple switching supervisor scheme, is proposed. Experiments on a cantilever beam system with large varying loads show that the proposed strategy offers improved system performance.

II. VIBRATION SYSTEM MODEL

A cantilever beam (500×50×3mm, 0.5895Kg) with various electromagnetically clamped loads is chosen to demonstrate large parameter variations. The beam is clamped to a concrete block at one end, and varying loads are placed at the free end (L_1) and/or 200mm from the free end (L_2). Shaker-induced disturbances are applied 100mm from the fixed end.

TABLE 1. MODELS AND THEIR NATURAL FREQUENCIES

Model	L_1 (%)	L_2 (%)	f_1 (Hz)	f_2 (Hz)
1	0	0	9.7783	61.277
2	17	0	7.5285	51.392
3	84	0	4.6236	45.548
4	17	84	6.3097	35.577
5	0	84	7.4218	46.910

Modal analysis in ANSYS® is used to find the natural frequencies and to form the beam's transfer function [3]. The analysis shows that the DC gains for the 3rd and higher modes of vibration are very small compared to the 1st and 2nd modes, therefore it is adequate to build a mathematical model of the system based only on the first two modes. The models are 4th-order with very low damping factors (less than 0.03 for Model 1).

The first two natural frequencies of the system are shown in Table 1 for five different loading models, with loads L_1 and L_2 expressed as percentages of the mass of the beam. MATLAB® is then used to transform the five continuous-time models into their discretised counterparts.

III. MULTIPLE MODEL ADPTIVE CONTROL

The MMAC method assumes a set of fixed models that are *a priori* known and an adaptive model that continuously optimises its parameters to suit the plant's current particular parameters. For each model M_i, a controller C_i is designed to satisfy the control objective for M_i. At every sampling instant, a switching scheme based on a minimum-error performance index selects between controllers in order to provide the most appropriate control input to the plant. In parallel, the adaptive controller tunes its parameters to optimise the system accuracy. The performance index can be defined as follows [1]:

$$J_i(k) = \alpha e_i^2(k) + \beta \sum_{j=0}^{k} \exp(-\lambda(k-j)) e_i^2(j)$$
$$(\alpha \geq 0\ \beta, \lambda > 0) \qquad (1)$$

where $e_i(k)=y(k)-\hat{y}_i(k)$ represents the error between the model M_i output $\hat{y}_i(k)$ and the plant output $y(k)$; j is the time index; α and β are the weighting factors for the instantaneous and long term accuracy, respectively. The forgetting factor λ is chosen to ensure the boundedness of $e_i(k)$.

The 2nd term on the right-hand side of (1) poses high computation demand that increases with the numbers of models. If $\beta = 0$, the computation burden reduces significantly, however rapid switching between controllers may occur, resulting in poor system performance. To minimise this problem, the number of fixed models may be

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reduced to one by designing the fixed controller such that its bandwidth covers both modes of vibration for all five models. A robust pole-placement control strategy [2] is adopted for the design of the fixed controller, and the desired poles are selected based on the first two natural frequencies of Model 1 (which is chosen as the nominal model since it has the widest bandwidth and highest vibration amplitude). The adaptive controller is also based on the pole-placement method, and a standard Recursive Least Square method is used to estimate the plant parameters. For both the robust controller and the adaptive controller, the damping factors are set to 1 and the three auxiliary poles are placed at 0 to ensure fast transient response. With this strategy, the performance index (1) can be simplified to:

$$J_i(k)=\mathrm{abs}(e_i) \qquad (2)$$

In order to further eliminate unnecessary switching between these two models, a bias function is added to the adaptive model:

$$\text{if abs}(e_a)<\delta \text{ then } e_a = 0, \text{ if abs}(e_a) \geqslant \delta \text{ then } e_a = E \qquad (3)$$

where e_a is the adaptive model error, and $\delta > 0$ and $E > 0$ are chosen empirically. Function (3) will force the system to choose the fixed model while the adaptive model is in its transient phase, and then to switch to and stay with the adaptive model once it has come out of its transient phase.

IV SIMULATIONS

SIMULINK®-based implementations are used to assess the performance of the proposed scheme. Four cases are studied: (i) adaptive control alone, (ii) robust control alone, (iii) MMAC with one fixed model and performance index (1), and (iv) MMAC with one fixed model and performance index (2)-(3). In (iii), $\alpha = \beta = 1$ and $\lambda = 0.1$, and in (iv) $\delta = 0.01$ and $k = 1$. The plant is changed every 3 seconds according to the sequence of models 4→5→1→3→2. The disturbance is a single sinusoid; 17.8N, and its frequency is switched according to the natural frequency of the current model. The responses for the four case studies are shown in Figure 1 and the corresponding switching diagrams for (iii) and (iv) are shown in Figure 2. Comparison between responses for (i) and (ii) shows that the adaptive control outperforms the robust control, especially when the plant is Model 4 or Model 5; however, large transient response is observed for (i). Response for (iii) shows that the MMAC has improved transient response over the adaptive control. The constraint on the minimum time between plant transitions for MMAC is set by the switching speed of the selector. Further comparison between responses for (iii) and (iv) reveals that the MMAC with the simple performance index (2)-(3) achieves almost the same performance as the original MMAC with performance index (1). In addition, Figure 2 demonstrates that switching activity for (iv) is significantly reduced relative to that for (iii).

V CONCLUSION

Preliminary simulation results based on the cantilever beam with varying loads show that the MMAC with only one fixed robust controller and one adaptive controller can improve the transient performance of adaptive control. It has further been shown that the MMAC switching scheme can be significantly simplified without impairing the system performance. This is advantageous in applications where the controller's computational power is limited.

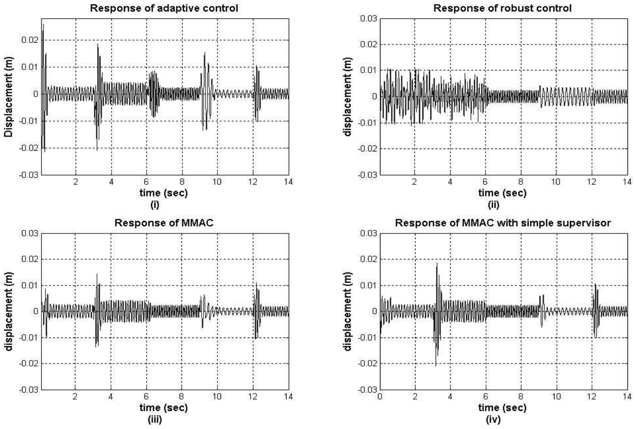

Fig.1 Responses for cases (i)-(iv).

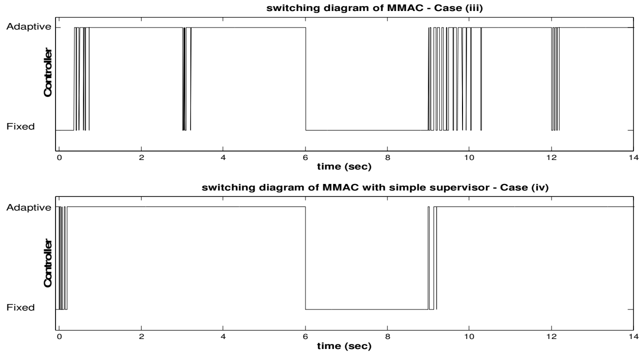

Fig 2. Switching diagrams for cases (iii) and (iv).

VI. REFERENCES

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