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Hybrid Variable Structure Path Tracking Control of Articulated Vehicles

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Abstract— **This paper presents a path-tracking hybrid controller for articulated vehicles. It is based on the approximation of the desired path with lines and arcs: suitable controllers are designed for tracking each line and arc and the control objective is attained by switching among the different controllers. Each controller is designed using partial linearization methods and variable structure control theory. The proposed control approach allows the driving point to track any desired path, starting from the set of feasible vehicle configurations, both in forward and backward motion, as confirmed by simulation.**

I. INTRODUCTION

This paper presents a hybrid path-tracking controller which makes an articulated vehicle follow a desired path consisting of lines and arcs, as the vehicle moves forward and/or backward. This means that the driving point is required to converge and track the prescribed path.

The path tracking control of articulated vehicles is a difficult task, especially when they are moving backward, because of the jack-knife effects between the parts of the vehicle. At the same time, path-tracking control is of great interest because it finds application in the field of automatic guidance of a large class of industrial articulated vehicles [1], [2]. In the past years, many approaches have been followed: local linearization [3], fuzzy control [4], neural network [5], genetic algorithms and expert systems [6], as well as Lyapunov methods [7].

In the present work the control strategy consists in approximating the desired path with lines and arcs, design suitable controllers for tracking each line and arc, and then switching among the different controllers according to a prespecified logic. The control laws associated with the various controllers are designed using partial linearization [8], [9], and variable structure control theory [10], [11]. The overall control strategy makes the articulated vehicle follow any chosen path accurately. Indeed, each controller guarantees that the corresponding reference line or arc is tracked asymptotically, starting from the set of feasible vehicle configurations. This set, which will be specified in the paper, does not contain only peculiar configurations, such as, for instance, that characterized by the tractor perpendicular to the trailer, or that for which the trailer is perpendicular to the line to be tracked. Moreover, it is proved that, on the whole, the hybrid controlled nonlinear system, after an arbitrarily short initialization period, becomes equivalent to an asymptotically stable switched linear system.

II. THE ARTICULATED VEHICLE

Consider the articulated vehicle shown in Fig. 1, where a generic posture is depicted. It consists of a tractor equipped with two rear-drive wheels and a front-steering wheel, linked to a trailer with two rear wheels. The symbols in Fig. 1 have the following meaning:

L_1 is the wheelbase of the tractor;
L_2 is the lenght of the semitrailer;
θ is the orientation of the semitrailer with respect to x-axis;
ϕ is the orientation of the tractor with respect to the semitrailer;
α is the steering angle and corresponds to the control variable to be manipulated;
P is the middle point of the semitrailer's rear wheels and it is chosen as "driving point"; its cartesian coordinates are (x_D, y_D).

III. THE HYBRID CONTROL STRATEGY

The aim of the control strategy is to make the articulated vehicle of Fig. 1 follow an arbitrary path. To do this the hybrid control strategy consists in the following steps:

1) approximate the desired path with lines and arcs;
2) describe the position and the dynamics of the articulated vehicle as a function of the distance travelled along the assigned path;

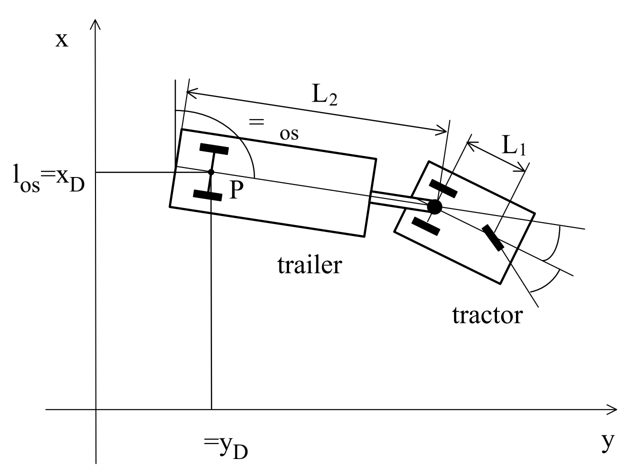

Fig. 1. The articulated vehicle.

3) partially linearize this state equations with appropriate state coordinate transformation;
4) design variable structure controllers for tracking each line and each arc;
5) suitably switch among the different controllers to track the desired path.

A. Approximation of the desired path (Step 1)

Consider an arbitrary path P to be tracked: it can be approximated by a suitably chosen sequence $S_P = \{l_i\}$ of n straight lines and/or arcs l_i, $i = 1, ..., n$. Given a path P, it is possible to find an algorithm that computes, given a finite n, the closest path, consisting of lines and arcs, to the original path P. How to do this is an important task, but out of the scope of the present paper.

B. Vehicle dynamics (Step 2)

In the present work, in accordance with [3], and [12], the basic idea is that of describing the vehicle dynamics as a function of the distance λ_i travelled along each line/arc $l_i \in S_P$, i.e., in defining the so-called path tracking offsets dynamics, one for each line/arc l_i. Then the path tracking problem can be viewed as the problem of driving the offsets dynamics asymptotically to zero. The state vector is defined as $x_i = [l_i^{os}\ \phi_i\ \theta_i^{os}]'$ where l_i^{os} is the offset of the driving point from the reference path l_i, and θ_i^{os} is the angular offset between the orientation of the trailer and the desired path l_i.

When the line l_i to be tracked is a straight line, for example the y-axis, (see Fig. 1), $\lambda_i = y_D$, $l_i^{os} = x_D$, and $\theta_i^{os} = \theta$. When l_i is an arc, the distance travelled along the path is given by $\lambda_i = R_i\gamma$, where R_i is the radius of the circle containing arc l_i and $\gamma = \pm\ \mathrm{atan2}\ (y_D, x_D)$, (see Fig. 2). The symbol atan2 denotes the four-quadrant inverse tangent, the + sign hold for paths to be followed in the counterclockwise direction and the - sign for the clockwise direction. The state vector is $x_i = [l_i^{os}\ \phi_i\ \theta_i^{os}]'$ where $l_i^{os} = \sqrt{x_D^2 + y_D^2} - R_i$ and $\theta_i^{os} = \theta - \gamma + \pi/2$.

Under the usual assumption of a planar and slippage-free motion, being α_i the steering angle in each line/arc l_i, the differential equations representing the offsets dynamics of the vehicle are

$$\begin{cases} \frac{dl_i^{os}}{d\lambda_i} = \frac{R_i + l_i^{os}}{R_i} \tan\theta_i^{os} \\ \frac{d\theta_i^{os}}{d\lambda_i} = \frac{R_i + l_i^{os}}{R_i} \left[\frac{1}{L_1} \frac{\tan\alpha_i}{\cos\theta_i^{os}\cos\phi_i} - \frac{1}{L_2} \frac{\tan\phi_i}{\cos\theta_i^{os}} \right] \\ \frac{d\phi_i}{d\lambda_i} = \frac{R_i + l_i^{os}}{R_i L_2} \frac{\tan\phi_i}{\cos\theta_i^{os}} + \frac{1}{R_i} \\ \frac{d\lambda_i}{dt} = \frac{R_i}{R_i + l_i^{os}} \cos\theta_i^{os} \cos\phi_i \end{cases} \tag{1}$$

Note that by setting $R_i = +\infty$ these equations describe the case of a straight-line l_i.

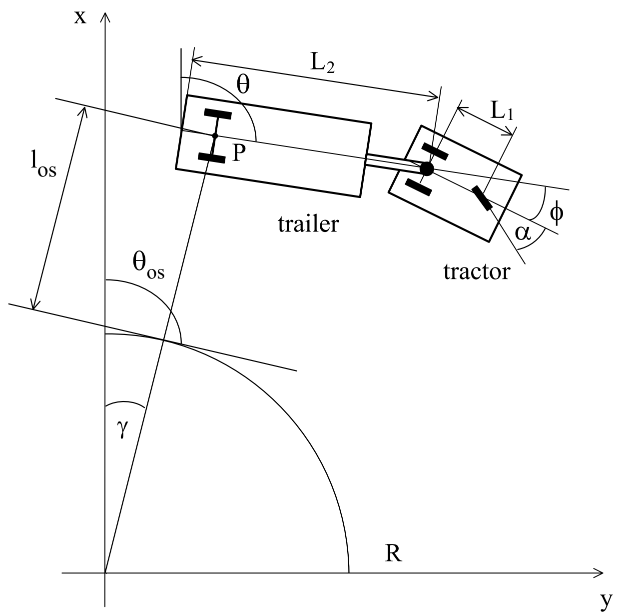

Fig. 2. Deviation of the articulated vehicle from a circular path with radius R.

C. Partial linearization (Step 3)

Now, to partially linearize (1) we define the following transformation

$$\begin{cases} \xi_{1i} = l_i^{os} \\ \xi_{2i} = \frac{R_i + l_i^{os}}{R_i} \tan\theta_i^{os} \\ \xi_{3i} = \frac{R_i + l_i^{os}}{R_i^2} \frac{1 + \sin^2\theta_i^{os}}{\cos^2\theta_i^{os}} + \frac{(R_i + l_i^{os})^2}{R_i^2 L_2} \frac{\tan\phi_i}{\cos^3\theta_i^{os}} \end{cases} \tag{2}$$

for $l_i^{os} \in (-\infty, \infty)$, $\phi_i \in (-\pi/2, \pi/2)$, $\theta_i^{os} \in (-\pi/2, \pi/2)$ when the vehicle is moving forward, and $\theta_i^{os} \in (\pi/2, 3\pi/2)$ when it is moving backward [12]. These intervals identify the set of feasible vehicle configurations, which, in the sequel, will be denoted by $\mathcal{F}$.

By differentiating the state $\xi_i = [\xi_{1i}, \xi_{2i}, \xi_{3i}]'$ with respect to λ_i, it yields

$$\begin{cases} \frac{d\xi_{1i}}{d\lambda_i} = \frac{R_i + l_i^{os}}{R_i} \tan\theta_i^{os} \\ \frac{d\xi_{2i}}{d\lambda_i} = \frac{R_i + l_i^{os}}{R_i^2} \frac{1 + \sin^2\theta_i^{os}}{\cos^2\theta_i^{os}} + \frac{(R_i + l_i^{os})^2}{R_i^2 L_2} \frac{\tan\phi_i}{\cos^3\theta_i^{os}} \\ \frac{d\xi_{3i}}{d\lambda_i} = \frac{R_i + l_i^{os}}{R_i^3} \frac{\tan\theta_i^{os}}{\cos^2\theta_i^{os}} (3 + \sin^2\theta_i^{os}) + \\ \quad + \frac{5(R_i + l_i^{os})^2}{R_i^3 L_2} \frac{\tan\phi_i \sin\theta_i^{os}}{\cos^2\theta_i^{os}} + \\ \quad + \frac{2(R_i + l_i^{os})^2}{R_i^2 L_2} \frac{\tan\phi_i \tan\theta_i^{os}}{\cos^3\theta_i^{os}} (1 + \frac{1}{R_i}) + \\ \quad + \frac{2(R_i + l_i^{os})}{R_i^3} \tan\theta_i^{os} + \\ \quad + \frac{(R_i + l_i^{os})^3}{R_i^3 L_2^2} \frac{(3\sin^2\phi_i \sin\theta_i^{os} - \frac{\tan\phi_i}{\cos\theta_i^{os}})}{\cos^3\theta_i^{os}\cos^2\phi_i} + \\ \quad + \frac{(R_i + l_i^{os})^3}{R_i^3 L_2 L_1} \frac{\tan\alpha_i}{\cos^3\theta_i^{os}\cos^2\phi_i} \end{cases} \tag{3}$$

The resulting linear system, by viewing $\tan(\alpha_i) = \nu_i$ as the control variable, is

$$\begin{cases} \dot{\xi}_{1i} = \xi_{2i} \\ \dot{\xi}_{2i} = \xi_{3i} \\ \dot{\xi}_{3i} = F_i + G_i\nu_i \end{cases} \tag{4}$$

being

$$
\begin{aligned}
F_i = & \frac{R_i+l_i^{os}}{R_i^3}\frac{\tan\theta_i^{os}}{\cos^2\theta_i^{os}}(3+\sin^2\theta_i^{os})+ \\
& +\frac{5(R_i+l_i^{os})^2}{R_i^3 L_2}\frac{\tan\phi_i\sin\theta_i^{os}}{\cos^2\theta_i^{os}}+ \\
& +\frac{2(R_i+l_i^{os})^2}{R_i^2 L_2}\frac{\tan\phi_i\tan\theta_i^{os}}{\cos^3\theta_i^{os}}(1+\frac{1}{R_i})+ \\
& +\frac{2(R_i+l_i^{os})}{R_i^3}\tan\theta_i^{os}+ \\
& +\frac{(R_i+l_i^{os})^3}{R_i^3 L_2^2}\frac{(3\sin^2\phi_i\sin\theta_i^{os}-\frac{\tan\phi_i}{\cos\theta_i^{os}})}{\cos^3\theta_i^{os}\cos^2\phi_i}
\end{aligned} \tag{5}
$$

$$
G_i = \frac{(R_i+l_i^{os})^3}{R_i^3 L_2 L_1}\frac{1}{\cos^3\theta_i^{os}\cos^2\phi_i} \tag{6}
$$

D. Design of the control laws (Step 4)

The control law is designed relying on the variable structure control methodology. To do this, introduce the linear scalar functions

$$
s_i(\xi_i) = f_{1i}\xi_{1i} + f_{2i}\xi_{2i} + \xi_{3i}, \tag{7}
$$

and the sliding manifolds $s_i(\xi_i) = 0$, one for each portion l_i of the desired path, and where f_{1i}, f_{2i} are chosen so that the polynomial $\varphi(\lambda) = \lambda^2 + f_{2i}\lambda + f_{1i}$ is Hurwitz. Note that, as usual in variable structure control, [10], [11], the sliding manifold $s_i(\xi_i) = 0$ is selected so that when the state of system (2) is restricted to lay on it, the system dynamics exhibits the desired behavior.

Then, according to a variable structure control strategy, define the control law

$$
\nu_i = -K_i \mathrm{sign}(s_i(\xi_i)), \qquad K_i > 0 \tag{8}
$$

where K_i is such that the "reaching" condition for the sliding manifold $s_i(\xi_i)$, by regarding λ_i as time scale, is fulfilled [13], i.e.

$$
K_i = \frac{k_i}{|G_i|}, \qquad k_i > |f_{1i}\xi_{2i} + f_{2i}\xi_{3i} + F_i| \tag{9}
$$

Condition (9) guarantees that the sliding manifold $s_i(\xi_i) = 0$ is reached in finite time [13]. Once in sliding mode $s_i(\xi_i) = 0$, i.e., $\xi_{3i} = -f_{1i}\xi_{1i} - f_{2i}\xi_{2i}$. Then, system (4) becomes equivalent to the reduced order linear

$$
\left\{ \begin{array}{l} \dot{\xi}_{1i} = \xi_{2i} \\ \dot{\xi}_{2i} = -f_{1i}\xi_{1i} - f_{2i}\xi_{2i} \end{array} \right. \tag{10}
$$

which can be rewritten, in compact form, as

$$
\dot{\bar{\xi}}_i = A_i \bar{\xi}_i \tag{11}
$$

where $\bar{\xi}_i = [\xi_{1i}, \xi_{2i}]'$ and $A_i = \begin{bmatrix} 0 & 1 \\ -f_{1i} & -f_{2i} \end{bmatrix}$.

As a result, once in sliding mode, $\xi_i \to 0$ as $\lambda_i \to \infty$, and consequently also $l_i^{os} \to 0$, and $\theta_i^{os} \to 0$, i.e., the vehicle tracks the desired portion of the path.

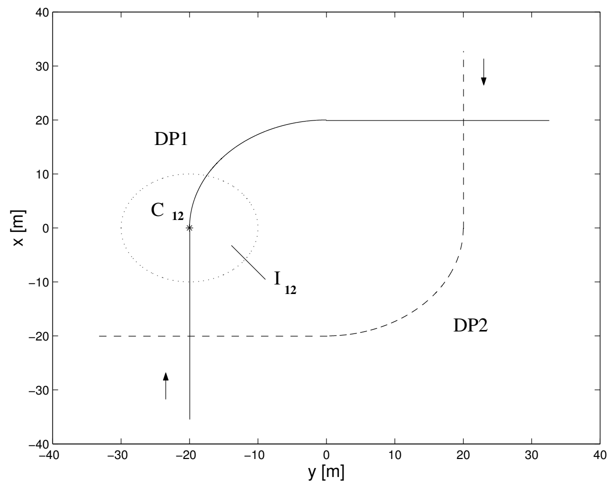

Fig. 3. The desired paths.

E. Choice of the switching strategy (Step 5)

Given a path to be tracked, approximated by a suitable sequence $S_P = \{l_i\}$ of straight lines and arcs, up to now it has been showed how to design a sequence of controllers $C_P = \{\nu_i\}$ each of them guaranteeing, starting from any configuration in $\mathcal{F}$, the tracking of the corresponding line/arc l_i. It is now necessary to define how to switch between the different control laws ν_i: in fact, it is well known that an hybrid strategy where the controller switches between different control laws can result in an overall unstable closed-loop system even if each control law is designed so as to guarantee stability. To this end, the following switching strategy can be defined:

Switching time conditions

Let I be a finite set of indexes (it contains the integers from 1 to n, n being the number of the l_i's composing the reference path), and let $A = \{A_i : i \in I\}$ be the closed bounded set of the real 2×2 matrices A_i in (11), which, by virtue of the choice of the control laws ν_i, are stable and such that there exist two finite, nonnegative and positive numbers, respectively, for which

$$
\exp(A_1 t) \le \exp(a_i - \lambda_i t), \qquad t \ge 0
$$

Starting from an initial configuration in $\mathcal{F}$, assume that the line/arc l_1 path tracking controller ν_1 is first used.

When the vehicle, controlled by the ν_1 control law, enters a convenient region I_{12} containing the crossing point C_{12} between line/arc l_1 and line/arc l_2 (see Fig. 3), one needs to switch to the line/arc l_2 path tracking controller, paying attention to the fact that the switching time instant t_{12} satisfies the following conditions:

$$
\begin{array}{lll} \tau_{12} < t_{12} < T_{\max} & \quad and \quad & |\alpha_1 - \alpha_2| \le \varepsilon \\ or & & \\ t_{12} = T_{\max} & & \end{array} \tag{12}
$$

where ε and $T_{\max}$ are project parameters and τ_{12} is given by $\max\{\tau_0, T_1, T_2\}$, being T_1, T_2 the reaching times for the sliding manifolds $s_1(\xi_1) = 0$, and $s_2(\xi_2) = 0$, respectively, and τ_0 is a dwelling time such that

$$\tau_0 = \sup_{i \in I} \left\{ \frac{a_i}{\lambda_i} \right\} \tag{13}$$

.

When the vehicle, following a generic part l_i of the reference path, controlled by the ν_i control law, enters a convenient region I_{ij} containing the crossing point C_{ij} between line/arc l_i and line/arc l_j, one needs to switch to the line/arc l_j path tracking controller, paying attention to the fact that the switching time instant t_{ij} satisfies the following conditions:

$$\begin{array}{lll} \tau_{ij} < t_{ij} < T_{\max} & \quad and \quad & |\alpha_i - \alpha_j| \leq \varepsilon \\ or & & \\ t_{ij} = T_{\max} & & \end{array} \tag{14}$$

where the dwelling time τ_{ij} is given by $\max\{\tau_0, T_j\}$, being T_j the reaching time for the sliding manifold $s_j(\xi_j) = 0$. Note that condition $t_{ij} \geq T_i$ is not needed because, after the first switch, it is automatically satisfied by the choice of the switching strategy.

The above conditions have the following meaning: condition $t_{ij} > \tau_{ij}$ assures that the interval between any two consecutive switching times is no smaller than the so-called "dwelling time" τ_0, and that the reaching condition for the sliding manifold $s_j(\xi_j) = 0$ is satisfied. Then, system (1) becomes equivalent to the reduced order linear (11) with stable matrix A_i. It is indeed well known, see [14], that the switch among stable linear systems may result in a stable controlled system provided that the switching is "slow", i.e., when a switch has occurred, a new one can occur only after a suitable dwelling time.

Then, if $t_{ij} \geq \tau_{ij}$, the steering angle α_i for tracking line l_i using the path tracking control law ν_i, and the steering angle α_j for tracking line l_j using the path tracking control law ν_j are calculated contemporarily. The switch between the two controllers occurs only when $|\alpha_i - \alpha_j| \leq \varepsilon$ in order to guarantee continuity of the steering angle and to avoid abrupt swerves.

It could happen that within a prespecified waiting time $T_{\max}$, it is not possible to satisfy this continuity condition on the steering angle. In this case, at $t_{ij} = T_{\max}$ the controller switches from the control law ν_i to the control law ν_j even if $|\alpha_i - \alpha_j| > \varepsilon$. This condition is needed to assure that the vehicle does not deviate too much from the desired path. After the switching the vehicle is controlled by the ν_j control law for tracking line l_j until conditions for a new switch arise.

IV. Stability and convergence analysis

The stability of the origin of the overall hybrid closed-loop system (1), (8) is now investigated.

Proposition 1: Given, for $i = 1, \ldots, n$, system (1), controlled by (8), the switching time instants complying with the switching time conditions (12), (14) with τ_0 as in (13). Then, for any $x_i \in \mathcal{F}$, and for $t \geq T_1$, the overall hybrid controlled system is equivalent in Filippov's sense [10], to a switched linear system of type $\dot{x} = A_\sigma x$, and its state transition matrix satisfies

$$\|\Phi(t, \mu)\| \leq \exp(a - \lambda(t - \mu)), \qquad \forall\, t \geq \mu \geq T_1$$

where the symbol $\|\cdot\|$ denotes the norm and

$$a = \sup\{a_i\} \qquad \lambda = \inf\left\{\lambda_i - \frac{a_i}{\tau_0}\right\}$$

with $\lambda \in (0, \lambda_i]$, and $i \in I$.

Proof:

The result can be proved by a straightforward application of Lemma 2 in [15]. Indeed, because of the choice of the switching time conditions which are at the basis of the switching logic, and of the choice of the sliding mode based control laws, the evolution of the overall controlled system in time can be viewed as a sequence of time evolutions of linear autonomous systems with matrices A_i. Then, starting from the reaching time instant of the sliding manifold $s_1(\xi_1) = 0$, namely T_1, the overall hybrid controlled system is equivalent in Filippov's sense, to a switched linear system of type $\dot{x} = A_\sigma x$, which, by virtue of the assumptions, satisfies Lemma 2 in [15].

The direct consequence of this result is that, from the reaching time instant T_1 on, the matrix A_σ of the equivalent system is exponentially stable with a decay rate λ. Relying on this fact, the main result can be proved.

Proposition 2: Given, for $i = 1, \ldots, n$, system (1), controlled by (8), then, for any $x_1 \in \mathcal{F}$, the final portion l_n of the reference path is reached in finite time, and the origin of the ξ_n state space is an asymptotically stable equilibrium point of system (1) with $i = n$.

Proof:

This result follows from the fact that, by assumption, $x_1 \in \mathcal{F}$, and, for $0 \leq t < T_1$, the reaching condition is satisfied by virtue of the choice of the control law ν_1. For $t \geq T_1$, the result of Proposition 1 holds. The final portion of the reference path is reached in finite time because of the choice of the switching time conditions, which enable the commutation between the control laws ν_i and ν_j only if $s_j(\xi_j) = 0$, and because of the fact that the reaching of each sliding manifold naturally occurs in finite time. Then, A_n is asymptotically stable, and ξ_n is asymptotically steered to the origin of the state space.

V. Simulation example

As an example, in this section, an articulated vehicle is considered consisting of a tractor with wheelbase $L_1 = 5m$, towing a trailer of length $L_2 = 5m$, the absolute value of the longitudinal velocity is $v = 1m/s$. In Fig. 3 two simple paths to be tracked are depicted: they can be approximated by two straight lines and one arc. In Fig. 4 the simulated trajectories of the driving point P_2 are shown, demonstrating

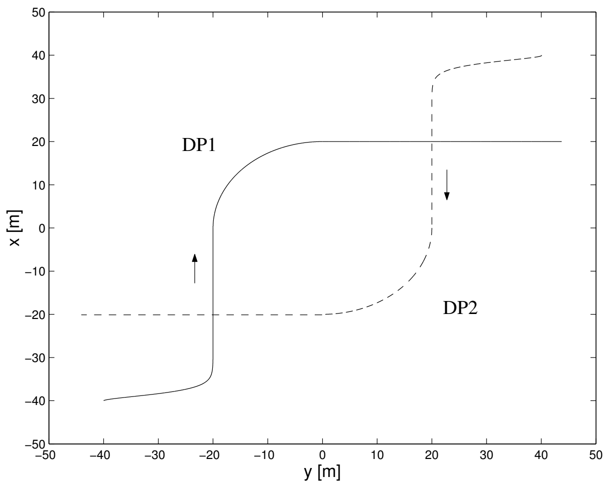

Fig. 4. Trajectories of the articulated vehicle.

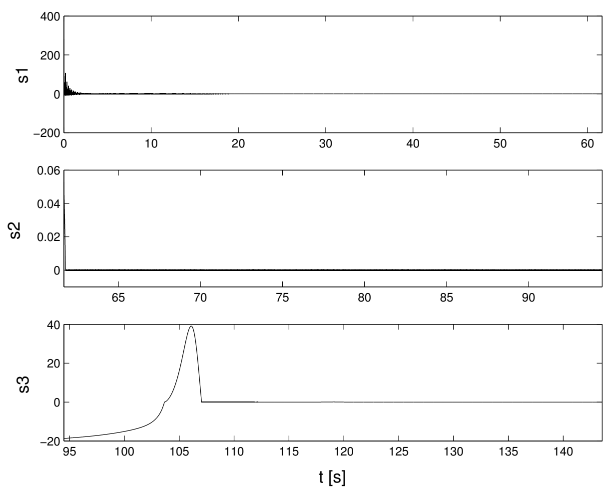

Fig. 5. Time evolution of the s_i's in forward motion.

the tracking properties of the designed control strategy. The solid line represents the trajectory of the driving point (starting from the initial configuration $x_2 = -40m$, $y_2 = -40m$, $\theta_1 = \pi/3$, $\theta_2 = \pi/4$, when the vehicle is moving forward and it is following the desired path $DP1$; the dashed line represents the trajectory of the driving point (starting from the initial configuration $x_2 = 40m$, $y_2 = 40m$, $\theta_1 = \pi/3$, $\theta_2 = \pi/4$, when the vehicle is moving backward and it is following the desired path $DP2$. In Fig. 5 and Fig. 6 the evolution of the s_i's versus time is depicted in both cases. Fig. 7, Fig. 8 and Fig. 9 show the evolutions of l_{os}, ϕ, θ_{os} versus time, when the vehicle is moving forward, following the x-axis (Fig. 7), a part of a circle (Fig. 8) and the y-axis (Fig. 9) respectively. Since this variables (l_{os}, ϕ, θ_{os}), which represent the difference between the simulated trajectory and the desired path, are steered to zero, it is possible to conclude about the good tracking properties of the proposed control strategy.

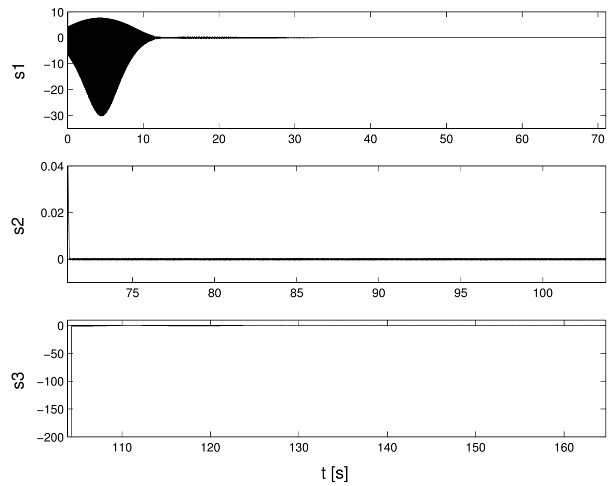

Fig. 6. Time evolution of the s_i's in backward motion.

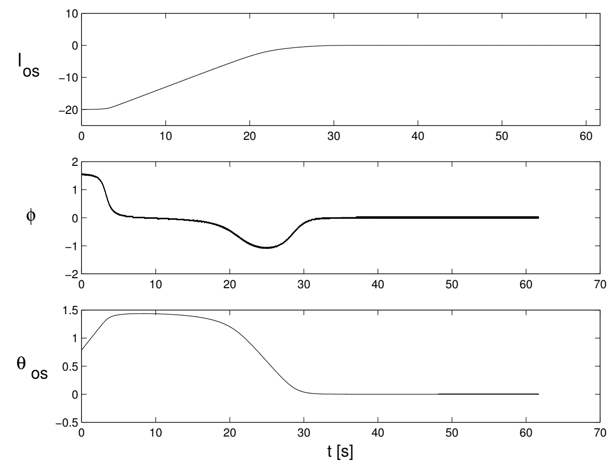

Fig. 7. Time evolution of l_{os}, ϕ, θ_{os} in forward motion, following the x-axis.

VI. CONCLUSIONS

In this paper, a hybrid variable structure control strategy for articulated vehicles has been designed. The proposed controller allows the vehicle to follow any path, while it is moving forward or backward. The stability features of the proposed hybrid control approach have been analyzed. With respect to other proposals appeared in the literature, the advantages of the presented control strategy are that the controllers associated with the various lines and arcs of the reference path are very simple, and that they have good robustness features versus matched bounded uncertainties, since they inherit the robustness properties of variable structure control. However, an extensive discussion of this issue is out of the scope of the present paper, this robustness feature being a typical attribute of sliding mode based controllers.

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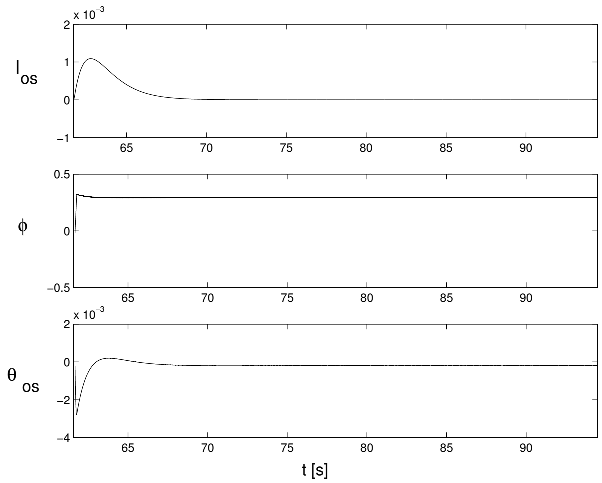

Fig. 8. Time evolution of l_{os}, ϕ, θ_{os} in forward motion, following a part of a circle.

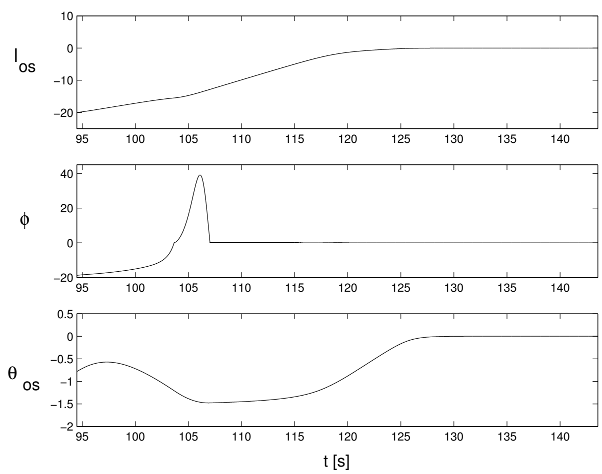

Fig. 9. Time evolution of l_{os}, ϕ, θ_{os} in forward motion, following the y-axis.

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