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An anti-windup design for single input adaptive control systems in strict feedback form

Hyun Min Do, Tamer Başar, and Jin Young Choi

***Abstract*— This paper presents an anti-windup design method for improving the performance of single input adaptive control systems in strict feedback form with input saturation. By an appropriate modification of the adaptation laws without saturation, excessive adaptation is prevented during an input saturation. A stability analysis and a proof of asymptotic convergence of the output tracking error are provided. The performance of the proposed design is demonstrated through simulations.**

I. INTRODUCTION

Many real systems have input constraints since actuators have physical upper and lower limits. When applying a control scheme developed without considering input constraints, the performance might not be guaranteed or the system might become unstable because the controller does not work as expected. This controller windup problem may arise also in a nonlinear control scheme or in an adaptive control scheme. Hard constraints may destroy the feedback linearizing effect or the adaptation effect of the controller.

To overcome the windup problem, many studies have been carried out, especially for linear systems [1]. A conventional approach to solve the controller windup problem in LTI systems is the two-step design procedure. First design a linear controller by ignoring the control input nonlinearities, and then add anti-windup bumpless transfer (AWBT) compensation to minimize the adverse effects of any control input nonlinearities on the closed-loop performance. Another approach is to view windup as an inconsistency between the controller output and the states of the controller, and to correct the inconsistency by modifying the controller inputs. This conditioning technique as an anti-windup scheme was originally formulated by Hanus et al [2], [3]. Åström and Wittenmark [4] and Åström and Rundqwist [5] proposed that an observer be introduced into the system to estimate the states of the controller and hence restore consistency between the saturated control signal and the controller state. Another relevant study is by Zheng et al [6], who has proposed a modified internal model control (IMC) based anti-windup scheme.

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H. M. Do is with the School of Electrical Engineering and Computer Science, BK21, ASRI, Seoul National University, Korea hmdo@iccl.snu.ac.kr

T. Başar is with the Coordinated Science Laboratory and Department of Electrical and Computer Engineering, University of Illinois, 1308 W. Main St. Urbana, IL 61801-2307, USA tbasar@decision.csl.uiuc.edu

J. Y. Choi is with the School of Electrical Engineering and Computer Science, BK21, ASRI, Seoul National University, Korea jychoi@iccl.snu.ac.kr

Some approaches have also been introduced to handle constraints in designing nonlinear controllers. Calvet and Arkun [7] have attempted to deal with windup by enforcing feedback linearization, even in the presence of constraints, thereby translating the constraints on the manipulated input into state-dependent constraints on the input to the linearizing feedback loop. An alternative approach is to employ a linear anti-windup scheme in the linear control loop [8], [9], [10]. More recently, Kapoor and Daoutidis [11] proposed a method for designing controller gains and a nonlinear observer-based anti-windup scheme, and Hu and Rangaiah [12] proposed a method to improve the performance of IMC of nonlinear processes with input constraints in the presence of modelling errors and unmeasured disturbances.

The purpose of this paper is to design a method to improve the performance of adaptive control systems in strict feedback form with input saturation. During an input saturation, an adaptation algorithm to reduce an error might not be helpful for controlling a system output. What is worse, a parameter might be excessively adapted and thus the adaptive controller could not work even in the unsaturated region. To minimize the adverse effect of an excessive adaptation, an algorithm to modify adaptive laws is proposed.

The paper is organized as follows. In section II, an adaptive control system in strict feedback form is introduced and the anti-windup design problem is formulated. The anti-windup algorithm and the key stability results are given in section III. Simulation results are described in section IV, and finally, conclusions are drawn in section V.

II. STATEMENT OF THE PROBLEM

Consider the tracking problem associated with the single-input single-output (SISO) system with n-dimensional state $x = (x_1, \cdots, x_n)$:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x) + g(x)u \\ y &= x_1 \end{aligned} \qquad (1)$$

where it is desired to pick a state-feedback control u so that the output y tracks a given reference output trajectory y_d. If the functions f and g are exactly known, then the feedback linearization control law

$$u = \frac{1}{g(x)}\left(-f(x) + v\right) \qquad (2)$$

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where v is some affine state-feedback control, can stabilize the system and make $y(t) \to y_d(t)$ when Assumptions 2.1 and 2.2 below are satisfied [13].

Assumption 2.1: There exist a positive constant g_l and a positive function $g_l(x)$ such that

$$g(x) \geq g_l(x) \geq g_l > 0. \tag{3}$$

An equivalent assumption and subsequent theory can be developed for the case when $g(x)$ is negative.

Assumption 2.2: The signal $v(t)$ is given by

$$v(t) = y_d^{(n)} - \alpha_n e^{(n-1)}(t) - \ldots - \alpha_1 e(t) \tag{4}$$

where the polynomial $\Lambda(s) = (s^n + \alpha_n s^{n-1} + \cdots + \alpha_1)$ is Hurwitz, and e is the tracking error

$$e(t) := y(t) - y_d(t) \tag{5}$$

where y_d is the desired system output trajectory.

If f and g are partially unknown, the approach described above cannot be directly applied. To solve again the tracking problem in that case, [14] developed an adaptive control scheme, by using a piecewise linear approximation network (PLAN) to estimate the unknown parts of f and g. To describe the results of [14], let f and g be written as

$$f(x) = \bar{f}(x) + \Delta f(x), \quad g(x) = \bar{g}(x) + \Delta g(x) \tag{6}$$

where $\bar{f}(x)$and $\bar{g}(x)$ denote the known parts, and $\Delta f(x)$ and $\Delta g(x)$ denote the unknown parts of f and g, respectively. Let $\Delta\hat{f}(x)$, $\Delta\hat{g}(x)$ be some estimates (approximators) for $\Delta f(x)$ and $\Delta g(x)$, respectively, and introduce

$$\hat{f}(x) := \bar{f}(x) + \Delta\hat{f}(x), \qquad \hat{g}(x) := \bar{g}(x) + \Delta\hat{g}(x). \tag{7}$$

The piecewise linear approximator (PLAN) used in [14] is

$$\begin{aligned} \Delta\hat{f}(x) &= \sum_{i=1}^{N_f} \left(\mathbf{w}_{f_i}^T (x - c_{f_i}) + b_{f_i}\right) \mu_{f_i}(x), & (8) \\ \Delta\hat{g}(x) &= \sum_{i=1}^{N_g} \left(\mathbf{w}_{g_i}^T (x - c_{g_i}) + b_{g_i}\right) \mu_{g_i}(x), & (9) \end{aligned}$$

where c_{f_i} and c_{g_i} indicate the centers, $\mathbf{w}_{f_i}$ and $\mathbf{w}_{g_i}$ indicate the connection weights, and b_{f_i} and b_{g_i} indicate the biases of the i-th local region for the functions f and g, respectively. μ_{f_i} and μ_{g_i} are locally defined influence functions that indicate local regions of applicability for the linear approximations. The linear basis function $\left(w_{f_i}^T (x - c_{f_i}) + b_{f_i}\right)$ is allowed to influence the approximation only in some local neighborhood of c_{f_i}, with the degree of influence determined by $\mu_{f_i}(x)$.

Examples of generating functions for influence functions include truncated radial basis functions and triangular functions (i.e., first-order splines). Truncated radial basis functions are defined by

$$\mu_{f_i}^o(x) = \begin{cases} \exp\left(-\|x - c_i\|\right), & \text{if } \exp\left(-\|x - c_i\|\right) \geq v \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $\|\|$ is a user defined norm and v is a user defined parameter. The locally defined influence function $\mu_{f_i}(x)$ can be obtained by

$$\mu_{f_i}(x) = \frac{\mu_{f_i}^o(x)}{\sum_{j=1}^{N_f} \mu_j^o(x)}. \tag{11}$$

The local linear approximations (8), (9) can be brought into the standard form with unknown parameters θ_f, θ_g, and known basis functions ϕ_f, ϕ_g, as in:

$$\Delta\hat{f}(x) = \phi_f^T(x)\theta_f, \qquad \Delta\hat{g}(x) = \phi_g^T(x)\theta_g, \tag{12}$$

where

$$\begin{aligned} \theta_f &= [\mathbf{w}_{f_1}^T, b_{f_1}, \ldots, \mathbf{w}_{f_{N_f}}^T, b_{f_{N_f}}]^T, & (13) \\ \theta_g &= [\mathbf{w}_{g_1}^T, b_{g_1}, \ldots, \mathbf{w}_{g_{N_g}}^T, b_{g_{N_g}}]^T, & (14) \\ \phi_f(x) &= [(x - c_{f_1})^T \mu_{f_1}(x), \mu_{f_1}(x), \ldots, \\ &\quad \left(x - c_{f_{N_f}}\right)^T \mu_{f_{N_f}}(x), \mu_{f_{N_f}}(x)]^T, & (15) \\ \phi_g(x) &= [(x - c_{g_1})^T \mu_{g_1}(x), \mu_{g_1}(x), \ldots, \\ &\quad \left(x - c_{g_{N_g}}\right)^T \mu_{g_{N_g}}(x), \mu_{g_{N_g}}(x)]^T. & (16) \end{aligned}$$

Combining (7) and (12), $\hat{f}(x)$ and $\hat{g}(x)$ are then represented by

$$\begin{aligned} \hat{f}(x) &= \bar{f}(x) + \phi_f^T(x)\theta_f, & (17) \\ \hat{g}(x) &= \bar{g}(x) + \phi_g^T(x)\theta_g, & (18) \end{aligned}$$

which show explicit linear dependence on the unknown parameters θ_f and θ_g. The above PLAN is a universal approximator [14]. Therefore, if the networks (i.e., N_f and N_g) are made large enough, they can approximate the functions $f(x)$ and $g(x)$ to any degree of accuracy ϵ. Due to the linearity of the approximators in the approximating parameters, and the assumption that ϕ_f and ϕ_g are basis functions, there exists a unique θ_f^* such that

$$\theta_f^* = \arg\min_{\theta} \int \|\Delta f(x) - \phi_f^T(x)\theta_f\|_2^2 dx, \tag{19}$$

with $\Delta f(x) - \phi_f^T(x)\theta_f^* = \epsilon_f(x; N_f)$ and $\|\epsilon_f(x; N_f)\| < \epsilon_f(N_f)$ for a given N_f and approximation given by (12) [14], [15]. Similarly, there exists a unique θ_g^*, associated with the approximation to $\Delta g(x)$. Combining this result with (6) and (7) yields

$$\begin{aligned} \hat{f}(x) - f(x) &= \tilde{\theta}_f^T \phi_f(x) - \epsilon_f(x; N_f), & (20) \\ \hat{g}(x) - g(x) &= \tilde{\theta}_g^T \phi_g(x) - \epsilon_g(x; N_g), & (21) \end{aligned}$$

where $\tilde{\theta}_f := \theta_f - \theta_f^*$, $\tilde{\theta}_g := \theta_g - \theta_g^*$. Then the adaptive control of [14] is given by

$$u_{ad}(t) = \frac{v(t) - \hat{f}(x)}{\hat{g}(x)} \tag{22}$$

where f and g in (2) have been replaced by $\hat{f}$ and $\hat{g}$, respectively, with the counterpart of Assumption 2.1 in place (with g replaced by $\hat{g}$).

This is not necessarily an effective controller because of the approximation error ϵ. To compensate this error, a sliding control will be added. An ideal sliding control with time varying gain is defined as

$$u_{sl}(t) = -k_{sl}(t)\mathrm{sgn}(e_1(t)) \tag{23}$$

where e_1 represents a sliding surface, which is defined as

$$e_1(t) = \beta_n e^{(n-1)}(t) + \cdots + \beta_1 e(t) \tag{24}$$

or in the s-domain $E_1(s) = \Psi(s)E(s)$, where $\Psi(s) = \left(\beta_n s^{n-1} + \cdots + \beta_1 s\right)$. Here the sliding gain is given by

$$k_{sl} = \frac{\bar{\epsilon}_f + \bar{\epsilon}_g|\bar{u}|}{g_l}, \tag{25}$$

where $\bar{\epsilon}_f$ and $\bar{\epsilon}_g$ are upper bounds on $|\epsilon_f(x; N_f)|$ and $|\epsilon_g(x; N_g)|$, respectively.

The overall control law is then given by

$$u(t) = u_{ad}(t) + u_{sl}(t) \tag{26}$$

When applying the control above to the real system, however, there may be some limitations (and thereby performance degradation) because all physical systems are subject to actuator saturation. Our objective in this paper is to develop an anti-windup scheme to overcome this controller windup problem.

III. Anti-windup design and analysis

A. Anti-windup algorithm

If the control input is saturated at maximum or minimum input levels, it might be impossible to follow the desired system output $y_d(t)$. This windup phenomenon makes the tracking error $e(t)$ increase and thereby the parameters change excessively. The excessive adaptation due to an actuator saturation is not helpful in reducing the error and it makes a delayed system response worse when the control input returns to an unsaturated region. To overcome this problem, we extend the anti-windup method in [12]. The main idea is to redefine the error term in the adaptive law so as to tame excessive adaptation.

If we denote a constrained input as $\bar{u}$, then

$$\bar{u} = sat(u) = \begin{cases} u_{max} & \text{if } u \geq u_{max} \\ u & \text{if } u_{min} < u < u_{max} \\ u_{min} & \text{if } u \leq u_{min} \end{cases}. \tag{27}$$

Now let us first rewrite (26) in explicit form, using (22), (23) and (27):

$$\bar{u} = \frac{v(t) - \hat{f}(x)}{\hat{g}(x)} - k_{sl}(t)\mathrm{sgn}(e_1(t)) \tag{28}$$

where v and e_1 can be represented as

$$\begin{aligned} v &= y_d^{(n)} - \alpha_n e^{(n-1)} - \ldots - \alpha_1 e & (29)\\ &= (y_d^{(n)} + \alpha_n y_d^{(n-1)} + \ldots + \alpha_1 y_d) \\ &\quad -(\alpha_n y^{(n-1)} + \ldots + \alpha_1 y) & (30)\\ &= \Lambda(s)y_d - (\Lambda(s) - s^n)y, & (31)\\ e_1 &= \beta_n e^{(n-1)} + \ldots + \beta_1 e & (32)\\ &= \Psi(s)(y - y_d). & (33)\end{aligned}$$

Using (31) and (33), (28) can be rewritten as

$$\begin{aligned}&\Lambda(s)y_d - k_{sl}(t)\hat{g}(x)\mathrm{sgn}\left(\Psi(s)(y - y_d)\right) \\ &\quad = \hat{f}(x) + \hat{g}(x)\bar{u} + (\Lambda(s) - s^n)y. \end{aligned} \tag{34}$$

Now, let us consider equation (34) in a different way. Assume $\bar{u}$ is given and y_d is a variable. Then we can find y_d which will be achieved using a given control input $\bar{u}$, that is, by setting $\bar{u}$ as u_{max} or u_{min} and solving the equation (34) when a saturation happens, we can get a modified desired output which can be achieved within the unsaturated input range. The excessive adaptation can be prevented if this modified desired output is used in adaptive laws. Define the modified desired output as $\bar{y}_d$ and thereby $\bar{e}$ as $y - \bar{y}_d$ and $\bar{e}_1$ as $\Psi(s)\bar{e}$. The error equation by considering the input saturation can be represented as

$$\begin{aligned} \Lambda(s)\bar{e} &= \Lambda(s)y - \Lambda(s)\bar{y}_d & (35)\\ &= s^n y + (\Lambda(s) - s^n)y - \Lambda(s)\bar{y}_d & (36)\\ &= f(x) + g(x)u - \hat{f}(x) - \hat{g}(x)\bar{u} \\ &\quad -k_{sl}(t)\hat{g}(x)\mathrm{sgn}(\Psi(s)\bar{e}) & (37)\\ &= -\tilde{\theta}_f^T\phi_f + \epsilon_f + \left(-\tilde{\theta}_g^T\phi_g + \epsilon_g\right)\bar{u} \\ &\quad -k_{sl}(t)\hat{g}(x)\mathrm{sgn}(\Psi(s)\bar{e}). & (38)\end{aligned}$$

If a saturation does not happen, $\bar{y}_d$ is equal to y_d and thereby $\bar{e}$ to e and $\bar{e}_1$ to e_1. For the proofs that follow, the following assumption will be required.

Assumption 3.1: $\Psi(s)$ is a Hurwitz polynomial and $\Psi(s)\Lambda^{-1}(s)$ is minimal and strictly positive real.

The dynamic equation for the modified sliding surface $\bar{e}_1$ is

$$\begin{aligned}\bar{e}_1 \;=\; \frac{\Psi(s)}{\Lambda(s)}\Big[&-\tilde{\theta}_f^T\phi_f + \epsilon_f + \left(-\tilde{\theta}_g^T\phi_g + \epsilon_g\right)\bar{u} \\ &-k_{sl}(t)\hat{g}(x)\mathrm{sgn}(\bar{e}_1)\Big].\end{aligned} \tag{39}$$

The adaptation law for θ_f is given by

$$\frac{d\theta_f}{dt} = \Gamma_f \bar{e}_1(t)\phi_f(x) \tag{40}$$

where Γ_f is a matrix of positive adaptation rates. The adaptation for θ_g will be constrained within the following convex set S to satisfy Assumption 2.1 with g replaced by $\hat{g}$:

$$\mathcal{S} = \{\theta_g \mid \tilde{g} := g_l - \hat{g}(\theta_g, x) \leq 0\}. \quad (41)$$

Using the projection method [16], the adaptive law for θ_g can be expressed as follows:

$$\frac{d\theta_g}{dt} = \begin{cases} \Gamma_g \bar{e}_1(t)\phi_g(x)\bar{u} & \text{if } \theta_g \in \mathcal{S}^o \text{ or} \\ & (\theta_g \in \bar{\mathcal{S}} \text{ and } \bar{u}\bar{e}_1 \geq 0) \\ 0 & \text{otherwise} \end{cases}, \quad (42)$$

where Γ_g is a positive adaptation rate, $\mathcal{S}^o$ is the interior of $\mathcal{S}$, $\bar{\mathcal{S}}$ is the boundary of $\mathcal{S}$, and $\theta_g(0)$ is chosen to be in $\mathcal{S}$. In this adaptive law, whenever $\theta_g \in \bar{\mathcal{S}}$ we have $\dot{\theta}_g^T \nabla_{\theta_g}\tilde{g} = -\Gamma_g \bar{u}\bar{e}_1\phi_g^T\phi_g \leq 0$. This implies that vector $\dot{\theta}_g$ points either in the opposite or vertical direction to $\nabla_{\theta_g}\tilde{g}$, that is, it points toward inside of $\mathcal{S}$ or along the tangent plane of $\bar{\mathcal{S}}$ at point θ_g. Since $\theta_g(0) \in \mathcal{S}$, it follows that θ_g will never leave $\mathcal{S}$, i.e., $\theta_g(t) \in \mathcal{S} \ \ \forall t \geq 0$.

B. Stability analysis

In this subsection, we provide a stability analysis for the overall adaptive control system with the anti-windup algorithm, and prove asymptotic convergence of the output tracking error. Since $\Psi(s)/\Lambda(s)$ is strictly proper by Assumption 3.1, (39) may be represented by the state-space dynamic equation

$$\begin{aligned} \dot{\xi} &= A\xi + B\Big[-\tilde{\theta}_f^T\phi_f + \epsilon_f + \left(-\tilde{\theta}_g^T\phi_g + \epsilon_g\right)\bar{u} \\ &\quad -k_{sl}(t)\hat{g}(x)\text{sgn}(\bar{e}_1)\Big] \\ \bar{e}_1 &= C\xi. \end{aligned} \quad (43)$$

From Assumption 3.1, the triple (A, B, C) satisfies the conditions of the Lefschetz-Kalman-Yakubovich lemma [16], and hence there exist positive definite symmetric matrices P and L, a vector q, and a scalar $\nu > 0$ satisfying $PA + A^TP = -qq^T - \nu L$ and $PB = C^T$. If we take the Lyapunov candidate function as

$$V = \frac{1}{2}\left(\xi^TP\xi + \tilde{\theta}_f^T\Gamma_f^{-1}\tilde{\theta}_f + \tilde{\theta}_g^T\Gamma_g^{-1}\tilde{\theta}_g\right), \quad (44)$$

its derivative along trajectories of the system is given by

$$\begin{aligned} \dot{V} &= -\frac{1}{2}\xi^T(qq^T + \nu L)\xi + (\epsilon_f + \epsilon_g\bar{u})\bar{e}_1 \\ &\quad -k_{sl}(t)\hat{g}(x)\text{sgn}(\bar{e}_1)\bar{e}_1 - \tilde{\theta}_f^T\phi_f\bar{e}_1 - \tilde{\theta}_g^T\phi_g\bar{u}\bar{e}_1 \\ &\quad +\tilde{\theta}_f^T\Gamma_f^{-1}\dot{\theta}_f + \tilde{\theta}_g^T\Gamma_g^{-1}\dot{\theta}_g. \end{aligned} \quad (45)$$

By applying the adaptive laws (40) and (42), the last four terms can be eliminated for $\theta_g \in \mathcal{S}$. In case where $\theta_g(0) \in \bar{\mathcal{S}}$ and $\bar{u}\bar{e}_1 < 0$, the term of $\tilde{\theta}_g^T\phi_g\bar{u}\bar{e}_1$ cannot be eliminated since $\dot{\theta}_g = 0$. Based on the assumption that $\theta_g^* \in \mathcal{S}$, we have $\theta_g^T\nabla_{\theta_g}\tilde{g} \geq 0$ (i.e., $-\theta_g^T\phi_g \geq 0$), when $\theta_g \in \bar{\mathcal{S}}$. Since $\bar{u}\bar{e}_1 < 0$ from the adaptive law, the term $-\tilde{\theta}_g^T\phi_g\bar{u}\bar{e}_1$ is nonpositive. Then, by applying the sliding control of (23), we obtain

$$\dot{V} \leq -\alpha\xi^2 + |\epsilon_f + \epsilon_g\bar{u}||\bar{e}_1| - k_{sl}(t)\hat{g}(x)|\bar{e}_1| \quad (46)$$

where $\alpha = \lambda_{min}(qq^T + \nu L) > 0$. The sliding gain (25) yields

$$k_{sl}(t)\hat{g}(x) = \frac{\hat{g}(x)}{g_l}(\bar{\epsilon}_f + \bar{\epsilon}_g|\bar{u}|) \geq |\epsilon_f + \epsilon_g\bar{u}|. \quad (47)$$

Finally, we get

$$\dot{V} \leq -\alpha\xi^2 \leq 0. \quad (48)$$

This implies that $V, \bar{e}_1, \tilde{\theta}_f, \tilde{\theta}_g \in L_\infty$. This yields directly $\theta_f, \theta_g \in L_\infty$. From (48), furthermore, $\xi \in L_2$. Then, $|\bar{e}_1| \leq |C||\xi|$ implies $\bar{e}_1 \in L_2$. Since $\Psi(s)$ is a Hurwitz polynomial, $\bar{e} \in L_\infty \cap L_2$. Also, it is clear that $\dot{\xi} \in L_\infty$ from (43). This means ξ is uniformly continuous. Therefore from Barbalat's Lemma [16] and the L_2 property of ξ, we conclude that ξ, $\bar{e}_1$, and $\bar{e}$ converge to zero asymptotically. It means that the system output converges to the modified desired output without an actuator saturation. We now summarize this result in the following theorem:

Theorem 3.1: The system described by (1) with control law given by (26) with (22) and (23), and parameter adaptation laws given by (40) and (42), and the anti-windup algorithm given by (34) under Assumptions 2.1, 2.2, and 3.1, is stable in the sense that

1) $\tilde{\theta}_f$, $\tilde{\theta}_g$, θ_f, $\theta_g \in L_\infty$;
2) ξ, $\bar{e}_1$, $\bar{e} \in L_\infty \cap L_2$;
3) ξ, $\bar{e}_1$, $\bar{e}$ converge to zero asymptotically.

IV. SIMULATIONS

The performance of the anti-windup algorithm developed is now illustrated through simulations. The plant and all configurations used in the simulation are identical to those used in [14]. The nonlinear system is given by

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= f(x) + g(x)u(t) \\ y &= x_1, \end{aligned} \quad (49)$$

where

$$f(x) = 4\left(\frac{\sin(4\pi x_1)}{\pi x_1}\right)\left(\frac{\sin(\pi x_2)}{\pi x_2}\right)^2 \quad (50)$$

$$g(x) = 2 + \sin(3\pi(x_1 - 0.5)). \quad (51)$$

We use the same reference trajectory as in [14], which is generated from a third-order system with a bandwidth of 10 rad/s driven by a 0.4 Hz square wave with unit amplitude, and 0.5 mean. The filters are selected to satisfy Assumptions 3.1 as $\Psi(s) = (s+15)$ and $\Lambda(s) = (s+10)^2$. All differential equations have been implemented by using *ode23*$(\cdot)$ function in *Matlab*. The controller parameters and regression vectors for parameter adaptation have been updated every 0.01 s. The known portion of f and g are

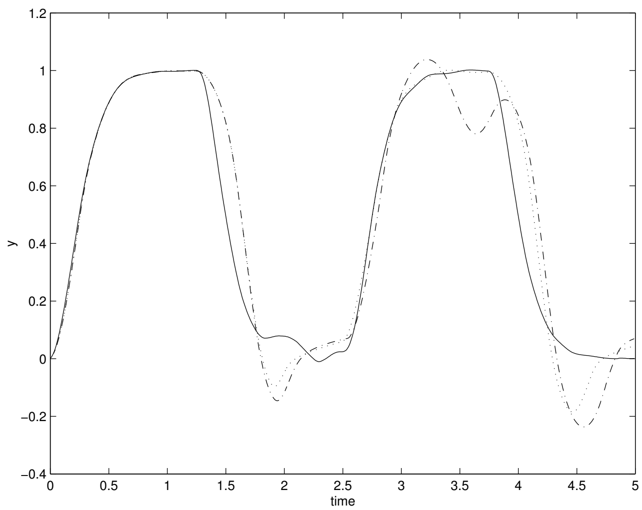

Fig. 1. The output trajectories. The solid line indicates the unsaturated case. The dashdot line indicates the saturated case without an anti-windup algorithm. The dotted line indicates the saturated case with the anti-windup algorithm.

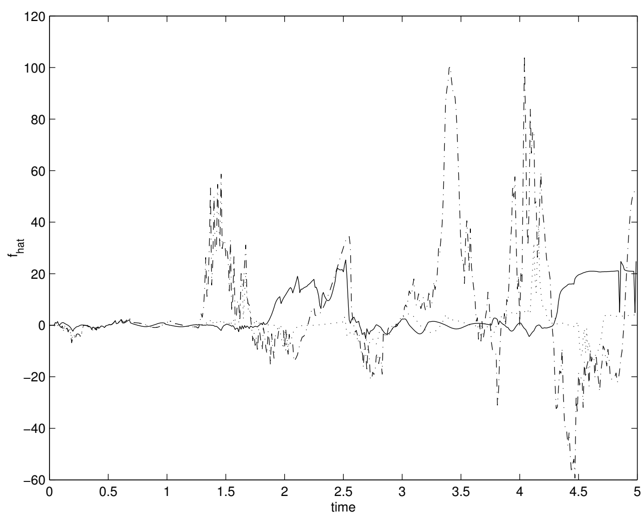

Fig. 2. The $\hat{f}$ trajectories. The solid line indicates the unsaturated case. The dashdot line indicates the saturated case without an anti-windup algorithm. The dotted line indicates the saturated case with the anti-windup algorithm.

given by $\bar{f} = 0$ and $\bar{g} = 2$ and the lower bound on g is set to $g_l = 1$. The adaptation rate matrices are set to $\Gamma_f = 750I$, and Γ_g is a block diagonal matrix with the diagonal element of $[750\ 0;\ 0\ 10]$. The approximation region was chosen by $[-0.41.4] \times [-3.43.4]$. Then the number of mode candidates for f and g on the approximation region are $N_f = 306$ and $N_g = 9$, respectively.

Figure 1 depicts the system output trajectories. Comparing with the unsaturated case, the performance of the saturated case is worse because of the input constraint. However, the proposed anti-windup method improves the error by reducing the excessive adaptation effect. This excessive adaptation phenomenon can definitely be observed

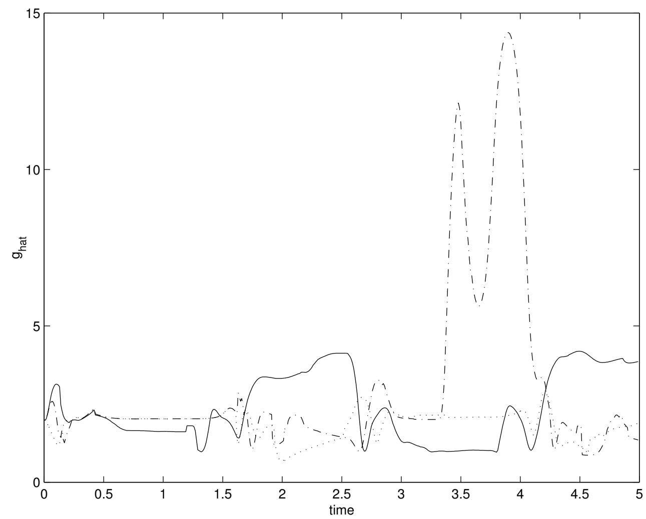

Fig. 3. The $\hat{g}$ trajectories. The solid line indicates the unsaturated case. The dashdot line indicates the saturated case without an anti-windup algorithm. The dotted line indicates the saturated case with the anti-windup algorithm.

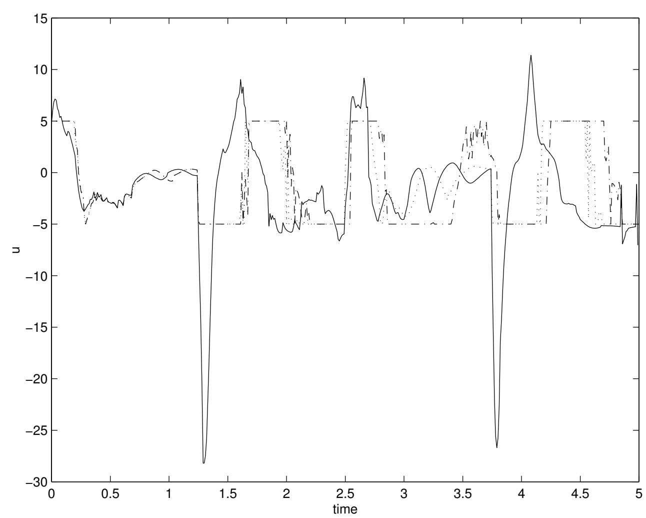

Fig. 4. The control input trajectories. The solid line indicates the unsaturated case. The dashdot line indicates the saturated case without an anti-windup algorithm. The dotted line indicates the saturated case with the anti-windup algorithm.

in Figures 2 and 3. Without an anti-windup algorithm, $\hat{f}$, $\hat{g}$ vary largely during an input saturation because the error due to an input saturation is large. This large variation makes the control input large and thus stay in the saturated region. It makes the system response slow. Figure 4 depicts the control input trajectories. From this, we can see that the saturated input escapes from the saturated region earlier with the proposed anti-windup method than the saturated case without an anti-windup algorithm. Through these simulations, it is thus shown that the proposed method can improve the performance in the presence of input saturation.

V. Conclusion

In this paper, we have presented an anti-windup method for single input adaptive control systems in strict feedback form to improve the performance when there is constraint on the input. To prevent an excessive adaption of parameters in the presence of input saturation, we have modified the desired output trajectory in the adaptation law, to assure that the control input stays in the unsaturated region. Stability analysis has shown that the system output asymptotically converges to the modified desired output trajectory without input saturation. Simulation results show that the proposed method improves the performance in the presence of an input saturation constraint. It seems possible that with some effort this approach can be extended to adaptive control systems in strict feedback form with multiple control inputs.

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