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Observers for Fault Detection In Networked Systems With Random Delays

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***Abstract*— Motivated by applications in an area of control of networked systems, we consider a fault detection problem for a class of linear systems with random delays. We demonstrate that an observer can be developed to detect the fault occurrence even though the delay value is unknown and is modeled by a random Markov process with a finite number of states.**

I. INTRODUCTION

Advanced real-time control systems may be distributed and consist of multiple electronic modules communicating over a network. For example, automotive vehicles utilize a Controller Area Network (CAN) [8] for communication between different control modules such as engine control module, transmission control module, anti-lock brake module, etc.

Each of the modules on the network can perform multiple computing, actuation and sensing tasks. The computing tasks are usually prioritized (e.g., as foreground or background tasks). Thus the computational delay affecting the operation of a particular controller running in one of these modules may be random and dependent on how many other updates need to be performed at the same time and on their relative priority.

In addition, there are random delays affecting signals transmitted over the network. For example, in a network such as a CAN, the messages are prioritized so that lower priority messages do not interfere with the transmission of the higher priority messages. This ensures the delivery time for the most critical messages, but the lower priority messages may incur a delay before they can be sent if a large number of high priority messages is being transmitted at the same time [8], [12]. See [2] and references therein for a discussion of the effect of network delays on the operation of closed loop control systems. The network and the computational delays, if not properly accounted for, may result in performance deterioration and instability of some of the control loops.

In these and other networked control system applications the delay can be modeled as a random process described by a Markov chain with a finite number of states. Each state in this Markov chain reflects the loading of the network or a processor at a given time [7], [10], [11].

With the motivation to provide redundancy and to take the best advantage of the available computational resources, a *distributed* diagnostics approach can be used for monitoring systems operating over networks,. In this approach, a diagnostics algorithm may be physically running in one of the modules on a network while diagnosing faults related to a part of the system somewhere else on the network. Such an algorithm can rely on measurements that are communicated over a network (possibly with a delay) or on measurements from independent sensors not affected by the delay.

To gain insight into a feasibility of the distributed diagnostics approach here we analyze the fault detection problem for a class of linear systems with a random delay modeled by a continuous-time Markov chain process with a finite number of states. Specifically, motivated by an engine speed control example treated in Section 4, and for the notational simplicity, we consider in detail the following first order system,

$$\dot{y} = a_0 y(t) + a_1 y(t-\eta(t)) + b_F F(t) + bu(t), \qquad (1)$$

where y is the scalar state of the system (available for measurements), and $a_0,\ a_1$ are constant coefficients. The $F(t)$ models an additive constant fault so that

$$\dot{F} = 0, \qquad (2)$$

and $F(t) \neq 0$ corresponds to fault occurrence. The $u(t) \in \mathcal{L}_2$ is a square integral bounded *unmeasured* deterministic disturbance input. The b_F and b are constant coefficients. The random delay $\eta(t)$ is a continuous from the right Markov process with a finite number of states, $\eta(t) \in S = \{\eta_1, \cdots, \eta_m\},\ 0 \leq \eta_j \leq h,\ j = 1, \cdots, m$. In the engine speed control example that we consider in Section 4, y is the engine speed and the term $a_1 y(t-\eta(t))$ in (1) represents the feedback on engine speed measurement which is delayed due to computational and network delays.

By defining new variables,

$$x_1 = y + \frac{b_F \cdot F}{a_0 + a_1}, \quad x_2 = F, \qquad (3)$$

we obtain the following system with a random delay:

$$\begin{aligned} \dot{x}_1 &= a_0 x_1 + a_1 x_1(t-\eta(t)) + bu(t), \\ \dot{x}_2 &= 0. \end{aligned} \qquad (4)$$

The objective is to estimate the fault, $x_2 = F$, from the output measurements,

$$y = x_1 - \frac{b_F x_2}{a_0 + a_1} = Cx, \quad C = [1 \,\vdots\, \frac{-b_F}{a_0 + a_1}]. \qquad (5)$$

We will initially assume that the random delay does not affect the output measurements. We will later extend the results to the case when it does.

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The statistics of the delay process are characterized by the transition functions $P_{ij}(s,t)$ (i.e., the conditional probabilities of the events $\{\eta(t)=\eta_j\}$ given that $\{\eta(s)=\eta_i\}$). They satisfy the Kolmogorov's equations:

$$\begin{aligned} &\frac{\partial P_{ij}}{\partial t} = -c_j P_{ij} + \sum_{k=1}^{m} P_{ik}\lambda_{kj} \\ &P_{ij}(s,s)=1 \text{ if } i=j, \quad P_{ij}(s,s)=0 \text{ if } i\neq j. \end{aligned} \tag{6}$$

The coefficients λ_{kj}, c_j characterize the probabilities of the jumps of the process $\eta(t)$ at the time instant t. Specifically, $\lambda_{kk} = 0$, $\lambda_{kj}(t)\Delta t$ for $k \neq j$ is approximately the probability of transition from η_k to η_j on the time interval $[t, t+\Delta t)$, and $1 - c_j(t)\Delta t$ with $c_j = \sum_{k=1}^{m} \lambda_{jk}(t)$ is approximately the probability of staying in the state η_j during the time interval $[t, t+\Delta t)$.

As it will become clear from the subsequent developments, working with systems having stochastic delays in the state variables requires special care. If we were able to predict the stochastic delay term a priori, we would then be able to write the observer in the traditional form wherein the state estimates are generated by a system model with a known delay, and enhanced with an output injection term. When the delay is unknown, the actual system and the model without an output injection term may behave differently (e.g., one may be stable and the other one may be unstable), thus special conditions are needed to ensure estimation error boundness.

The paper is organized as follows. The observer for the system (4),(5) will be studied in Section 2. In Section 3 we will briefly describe an observer for a more general class of higher order linear systems with a random delay and for the case when output measurements used for fault detection are also affected by the random delay. In Section 4 we will illustrate our fault detection approach with the simulation results for an engine speed control example. Finally, some concluding remarks will be made in Section 5.

II. Fault Detection in First Order Systems

We consider an observer for the system (4) in the form

$$\begin{aligned} &\dot{\hat{x}}_1 = a_0\hat{x}_1 + a_1\sum_{i=1}^{m}\rho_i\hat{x}_1(t-\eta_i) + L_1(\hat{y}-y), \\ &\dot{\hat{x}}_2 = L_2(\hat{y}-y), \\ &\hat{y} = C\hat{x} \end{aligned} \tag{7}$$

where $\sum_{i=1}^{m}\rho_i = 1$, $\rho_i > 0$, $i = 1, \cdots, m$.

This form for the observer can be intuitively motivated by averaging the right-hand side of (4) to its instantaneous expected value with respect to $\eta(t)$, and adding an output injection. In particular, ρ_i can be selected equal to the probability of $\eta(t) = \eta_i$.

If we define,

$$e_1 = \hat{x}_1 - x_1, \quad e_2 = \hat{x}_2 - x_2, \quad e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$

then

$$\begin{aligned} \dot{e}_1 &= a_0e_1 + a_1\sum_{i=1}^{m}\rho_i[\hat{x}_1(t-\eta_i) - x_1(t-\eta(t))] \\ &+ L_1Ce - bu \\ &= a_0e_1 + a_1\sum_{i=1}^{m}\rho_i[\hat{x}_1(t-\eta_i) - x_1(t-\eta_i)] \\ &+ a_1\sum_{i=1}^{m}\rho_i[x_1(t-\eta_i) - x_1(t-\eta(t))] - bu + L_1Ce, \\ \dot{e}_2 &= L_2Ce, \end{aligned}$$

and

$$\dot{e} = A_0e + LCe + \sum_{i=1}^{m}\rho_iA_1e(t-\eta_i) + w(t), \tag{8}$$

where

$$A_0 = \begin{bmatrix} a_0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} a_1 & 0 \\ 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix},$$

and

$$w(t) = \begin{bmatrix} a_1\sum_{i=1}^{m}\rho_i[x_1(t-\eta_i) - x_1(t-\eta(t))] - bu. \\ 0 \end{bmatrix}. \tag{9}$$

Note that the pair (C, A_0) is observable provided $a_0 \neq 0$, $b_F \neq 0$ and $a_0 + a_1 \neq 0$.

We wish to demonstrate that by a proper selection of the gain L in (8) and with the appropriate conditions imposed on a_0, a_1 and b_F the error e can be made integral square bounded. As a first step toward this objective we provide conditions under which $w(t)$ in (9) is integral square bounded.

Specifically, consider the first of the equations in the system (4),

$$\dot{x}_1 = a_0x_1 + a_1x_1(t-\eta(t)) + bu(t). \tag{10}$$

Theorem 1: *If $u \in \mathcal{L}_2$ and $-a_0 > (m + \frac{1}{4}a_1^2)$ in (10) then the state x_1 of (10) is integral square bounded. Furthermore, w in (9) is integral square bounded and so is $\dot{e}$.*

Proof: Consider a functional, V, dependent on $\phi(\cdot) \in C[-h, 0]$ and $\eta(t) \in S = \{\eta_1, \cdots, \eta_m\}$, and defined as follows

$$V = \frac{1}{2}\phi^2(0) + \sum_{j=1}^{m}\int_{-\eta_j}^{0}\phi^2(\tau)d\tau. \tag{11}$$

The infinitesimal generator of V, LV, can be calculated using the techniques described in [1], [5], [6], [9]. Specifically, evaluated at $\eta(t) = \eta_i \in S$ and $x_1(t+\theta) \in C[0, h]$, $-h \leq \theta \leq 0$, $LV = LV_i$, is a sum of the derivative of V along the flow with $\eta(t)$ frozen at η_i and a "jump" term which is a sum of the instantaneous changes that V incurs due to various jumps in $\eta(t)$ from η_i to η_k multiplied by the instantaneous probability rates of the jumps λ_{ik} (see (6)). Since in our case V does not explicitly depends on $\eta(t)$ the "jump" term is zero and

$$
\begin{aligned}
LV_i &= a_0 \cdot x_1^2(t) + a_1 \cdot x_1(t)x_1(t-\eta_i) + b \cdot x_1(t)u(t) \\
&+ \sum_{i=1}^{m} (x_1^2(t) - x_1^2(t-\eta_i)).
\end{aligned}
$$

From the inequality,

$$
|a_1 x_1(t) \cdot x_1(t-\eta_i)| \leq \frac{1}{2\cdot\sigma} \cdot a_1^2 x_1^2(t) + \frac{\sigma}{2} \cdot x_1^2(t-\eta_i),
$$

that holds for any $\sigma > 0$ we obtain that with $0 < \frac{\sigma}{2} \leq 1$,

$$
LV_i \leq (a_0 + m + \frac{a_1^2}{2\sigma})x_1^2(t) + bx_1(t)u(t).
$$

Then, using the Dynkin's formula, we can construct an inequality relating the expected state and input

$$
\begin{aligned}
&\mathcal{E}\left\{\frac{1}{2}x_1^2(t) + \sum_{j=1}^{m}\int_{t-\eta_j}^{t} x_1^2(\tau)d\tau\right\} - V(0) \\
&\leq \int_0^t \mathcal{E}\ LV \\
&\leq \int_0^t (a_0 + m + \frac{a_1^2}{2\sigma}) \cdot \mathcal{E}\ \{x_1^2(\tau)\} + b \cdot \mathcal{E}\ \{x_1(\tau)u(\tau)\}\ d\tau.
\end{aligned}
\tag{12}
$$

After straightforward algebraic manipulations, with σ as previously constrained, and with σ_1 chosen such that

$$
\kappa \triangleq -\left(a_0 + m + \frac{a_1^2}{2\sigma} + b^2 \cdot \frac{1}{2\cdot\sigma_1}\right) > 0, \tag{13}
$$

we obtain

$$
\begin{aligned}
\mathcal{E}\int_0^t x_1^2(\tau)\ d\tau &\leq \frac{\sigma_1}{2\cdot\kappa} \cdot \int_0^t \mathcal{E}u^2(\tau)\ d\tau + \frac{V(0)}{\kappa} \\
&= \beta_1 \cdot \int_0^t \mathcal{E}u^2(\tau)\ d\tau + \beta_2.
\end{aligned}
$$

Note that if $a_0 + m + \dfrac{a_1^2}{4} < 0$ (our assumption) and since $0 < \sigma \leq 2$, σ_1 can always be chosen to satisfy (13).
To summarize, we have shown that the parameters σ, σ_1 can be chosen, under the previous mentioned constraints, to bound the state such that

$$
\mathcal{E}\int_0^t x_1^2(\tau)\ d\tau \leq \beta_1 \cdot \mathcal{E}\int_0^t u^2(\tau)\ d\tau + \beta_2 < \infty.
$$

Since $x_1(t)$ is integral square bounded then so are $x_1(t-\eta_j)$, $j = 1, \cdots, m$. Since

$$
\mathcal{E}x_1^2(t-\eta(t)) \leq \mathcal{E}\max_{j=1,\cdots,m} x_1^2(t-\eta_j) \leq \mathcal{E}\sum_{j=1}^{m} x_1^2(t-\eta_j),
$$

$x_1^2(t-\eta(t))$ is also integral square bounded. From (9) it thus follows that $w(t)$ is integral square bounded, and from (8) that $\dot{e}$ also is. □

Theorem 1 demonstrated the integral square boundness of x_1. Note that in the presence of fault the state of (1), y, is not, in general, square integral bounded. The state of (8), x_1, remains, however, integral square bounded thanks to the transformation (3).

To show that the estimation error e due to the observer (7) is integral square bounded it is now sufficient to study the properties of the system (8) with m *fixed* (non-random) delays and with a stochastic input, $w(t)$, that, under the assumptions of Theorem 1, is integral square bounded. Methods for stability analysis of stochastic systems with multiple fixed time delays are described in detail in Chapter 5 of the book [3]. See also references [4], [1].

Specifically, if we let

$$
\bar{A}_0 = A_0 + LC, \quad \bar{A}_j = \rho_j A_1,\ j = 1, \cdots, m,
$$

so that

$$
\dot{e} = \bar{A}_0 e + \sum_{j=1}^{m} \bar{A}_j e(t-\eta_j) + w(t), \tag{14}
$$

then the following result holds:

Theorem 2: *If w in (9) is integral-square bounded, where x_1 is the solution of (10), then e in (14) is integral square bounded provided there exist $P > 0$, $R > 0$, $Q > 0$ such that*

$$
\bar{A}_0^{\mathrm{T}} P + P\bar{A}_0 + \sum_{j=1}^{m} P\bar{A}_j R^{-1}\bar{A}_j^{\mathrm{T}} P + PQ^{-1}P + m \cdot R < 0. \tag{15}
$$

Proof: The proof follows by considering a functional V, dependent on $e(t+s)$, $-h \leq s \leq 0$, and defined as

$$
V = e^{\mathrm{T}}(t)Pe(t) + \sum_{j=1}^{m}\int_{t-\eta_j}^{t} e^{\mathrm{T}}(s)Re(s)ds,
$$

and by showing that

$$
\begin{aligned}
LV_i \leq\ & e^{\mathrm{T}}(t)[\bar{A}_0^{\mathrm{T}} P + P\bar{A}_0 + \sum_{j=1}^{m} P\bar{A}_j R^{-1}\bar{A}_j^{\mathrm{T}} P \\
&+ PQ^{-1}P + m \cdot R]e(t) + w^{\mathrm{T}}(t)Qw(t).
\end{aligned}
\tag{16}
$$

To demonstrate (16) the following inequalities are used

$$
\begin{aligned}
&e^{\mathrm{T}}(t)P\bar{A}_j e(t-\eta_j) + e(t-\eta_j)^{\mathrm{T}}\bar{A}_j^{\mathrm{T}} Pe(t) \leq \\
&e^{\mathrm{T}}(t)P\bar{A}_j R^{-1}\bar{A}_j^{\mathrm{T}} Pe(t) + e^{\mathrm{T}}(t-\eta_j)Re(t-\eta_j),
\end{aligned}
$$

and

$$
e^{\mathrm{T}}(t)Pw(t) + w^{\mathrm{T}}(t)Pe(t) \leq e^{\mathrm{T}}(t)PQ^{-1}Pe(t) + w^{\mathrm{T}}(t)Qw(t).
$$

The inequality (16), $V \geq 0$ and the Dynkin's formula now imply the desired result. □

Note that the condition (15) is also implied by the following Linear Matrix Inequality on P, which is obtained using the Schur's complement:

$$
\begin{bmatrix}
-\bar{A}_0^{\mathrm{T}} P - P\bar{A}_0 - mR & P \\
P^{\mathrm{T}} & \left(Q^{-1} + \sum_{j=1}^{m}\bar{A}_j R^{-1}\bar{A}_j^{\mathrm{T}}\right)^{-1}
\end{bmatrix} > 0.
$$

Remark 1: Less restrictive, delay-dependent sufficient mean square stability conditions from [3], modified for the case when $w(t)$ is present, can also be used. These conditions take the following form: For $m_0 \in [0, m]$,

$$\sum_{j=1}^{m_0} \eta_j ||\bar{A}_j|| < 1,$$

and for $\tilde{A} = \sum_{j=1}^{m_0} \bar{A}_j$ and for some $R_j > 0$, $G_j > 0$, $S_{ij} > 0$, $Q > 0$, there exists a positive definite solution, P, of the matrix Riccati equation,

$$(\bar{A}_0 + \tilde{A})^{\mathrm{T}} P + P(\bar{A}_0 + \tilde{A}) + \sum_{j=m_0+1}^{m} (R_j + P\bar{A}_j R_j^{-1} \bar{A}_j^{\mathrm{T}} P)$$
$$+ \sum_{j=1}^{m_0} \eta_j (G_j + (\bar{A}_0 + \tilde{A})^{\mathrm{T}} P \bar{A}_j G_j^{-1} \bar{A}_j^{\mathrm{T}} P (A_0 + \tilde{A})^{\mathrm{T}})$$
$$+ \sum_{j=m_0+1}^{m} \sum_{k=1}^{m_0} \eta_k (S_{jk} + \bar{A}_j^{\mathrm{T}} P \bar{A}_k S_{jk}^{-1} \bar{A}_k^{\mathrm{T}} P \bar{A}_j) = -Q.$$

These conditions can also be expressed in terms of a Linear Matrix Inequality on P.

Remark 2: Note that under the assumptions of Theorem 1 and Theorem 2 and with $u \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$, it follows that $\mathcal{E}x_1^2(t) \to 0$, $\mathcal{E}||e(t)||^2 \to 0$ as $t \to \infty$. For example, let $V = x_1^2$. Then from (10),

$$LV_i \leq 2a_0 x_1(t)^2 + a_1 x_1^2(t-\eta_i) + a_1 x_1(t)^2 + x_1(t)^2 + b^2 u(t)^2.$$

The boundness of $\mathcal{E}x_1^2(t)$ (i.e., $\mathcal{E}x_1^2(t) \leq C_1$ for some $C_1 > 0$) follows from (12) given the integral square boundness of x_1 that we have already proven. Hence LV_i is bounded and there exists $\bar{C} > 0$ such that $\mathcal{E}LV \leq \mathcal{E}\max\{LV_1, \cdots, LV_m\} \leq \bar{C}$. Applying the Dynkin's formula to V shows that $|\mathcal{E}x_1^2(t_2) - \mathcal{E}x_1^2(t_1)| \leq \bar{C}|t_2 - t_1|$ so $\mathcal{E}x_1^2(t)$ is Lipshitz continuous. Hence, it is also uniformly continuous. The Barbalat's lemma applied to $\mathcal{E}x_1^2(t)$ now shows that $\mathcal{E}x_1^2(t) \to 0$ as $t \to \infty$. Similar arguments can be applied to show that $\mathcal{E}||e(t)||^2 \to 0$ as $t \to \infty$.

Remark 3: Instead of a deterministic term bu in (1), (10) it is possible to consider a stochastic disturbance term of the form $b \cdot x_1 \cdot d\zeta$, so that

$$dx_1 = a_0 x_1 dt + a_1 x_1(t - \eta(t))dt + bx_1 d\zeta,$$

where ζ is the standard Wiener process independent of $\eta(t)$. This term may, for example, model a stochastic uncertainty in the coefficient a_0. The presence of the stochastic disturbance terms results in an additional term, $\frac{b^2}{2}x_1^2$ in LV_i so the condition of Theorem 1 has to be modified to $-a_0 > (m + \frac{1}{4}a_1^2 + \frac{b^2}{2})$. For a treatment of stability of systems with random delays and Wiener process inputs, see [5].

III. EXTENSIONS

The basic developments, starting with the transformation (3), defining the observer (7) and obtaining results similar to the ones given in Theorem 1 and Theorem 2 can be also extended to the multi-dimensional case where a_0, a_1, b and b_F in (1) are matrices of appropriate dimensions. The condition for the integral square boundness of x_1 (now a vector state) and $w(t)$ in Theorem 1 can be imposed, in a delay-independent form, as follows:

$$a_0^{\mathrm{T}} P + P a_0 + P a_1 R^{-1} a_1^{\mathrm{T}} P + b^{\mathrm{T}} P Q^{-1} P b + mR < 0. \quad (17)$$

Note that in the first-order case, the condition $-a_0 > (m + \frac{1}{4}a_1^2)$ guaranteed another condition $-(a_0 + m + \frac{a_1^2}{2\sigma} + b^2 \cdot \frac{1}{2\sigma_1}) > 0$ for $0 < \sigma \leq 2$, $\sigma_1 > 0$, which we used in the proof of Theorem 1. The latter condition with the least restrictive value of $\sigma = 2$ is a special case of (17) where $P = 1/2$, $R = 1$ and $Q = \frac{1}{2}\sigma_1$. The conditions of Theorem 2 remain unchanged with the appropriate definitions of $\bar{A}_0$, $\bar{A}_j$, $j = 1, \cdots, m$, and if $w(t)$ in (14) is appropriately modified for the higher order case.

Consider now the case when the sensor measurements that we use for fault monitoring are delayed. For example, this situation may arise when the same signals transmitted over a network are used both for feedback control and for diagnostics. In this case, equations (4) and (5) become

$$\begin{aligned} \dot{x}_1 &= a_0 x_1(t) + a_1 x_1(t - \eta(t)) + bu(t), \\ \dot{x}_2 &= 0, \\ y(t) &= Cx(t - \eta(t)). \end{aligned} \quad (18)$$

The observer for (18) takes the form

$$\begin{aligned} \dot{\hat{x}}_1 &= a_0 \hat{x}_1(t) + \sum_{j=1}^{m} a_1 \rho_j \hat{x}_1(t - \eta_j) + L_1(\hat{y} - y), \\ \dot{\hat{x}}_2 &= L_2(\hat{y} - y), \\ \hat{y}(t) &= C \sum_{j=1}^{m} \rho_j \hat{x}(t - \eta_j). \end{aligned} \quad (19)$$

In this case, the observer error equation (14) applies with

$$\bar{A}_0 = \begin{bmatrix} a_0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_j = \rho_j LC + \rho_j \begin{bmatrix} a_1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \; j = 1, \cdots, m,$$

and

$$w(t) = L \sum_{j=1}^{m} \rho_j C(x(t - \eta_j) - x(t - \eta(t)))$$
$$+ \begin{bmatrix} \sum_{j=1}^{m} a_1 \rho_j (x_1(t - \eta_j) - x_1(t - \eta(t))) - bu \\ 0 \end{bmatrix}.$$

Conditions in Theorem 1 guarantee the integral square boundness of x_1 in (18). Under these conditions, and since $x = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}]^{\mathrm{T}}$, $\dot{x}_2 = 0$ so $x_2(t - \eta(t)) = x_2(t - \eta_j)$, $w(t)$ is guaranteed to be integral square bounded. To show the integral square boundness of e in (14) the sufficient conditions delineated in Remark 1 can be used. Note that (15) may not hold in this case because of the form of $\bar{A}_0$ and due to it staying unaffected by the output injection.

IV. Engine Speed Control Example

We consider a speed control system for a small-size internal combustion engine. The dynamics of the engine speed around neutral idle are described by a first order equation,

$$\dot{\omega}(t) = \frac{1}{J_t} \cdot (T_i(t) - C_f\omega - T_a),$$

where ω is the engine speed in rad/sec, $J_t = 0.1$ is the lumped engine and accessory inertia, $C_f = 0.3$ is the loss torque coefficient, T_i is the combustion torque produced by the engine, and T_a is the accessory load torque. The fault is assumed to occur as a rapid, large accessory load change resulting from a short or from an inadvertent accesory motor failure. It is modeled as a change in the accessory torque, i.e.,

$$T_a = T_{a0} + \Delta T_a(t),$$

where T_{a0} is assumed to be accurately estimated (the nominal value of $T_{a0} = 2.0$ was used in simulations). Thus,

$$\begin{aligned} \dot{\omega}(t) &= \frac{1}{J_t} \cdot (T_i(t) - C_f\omega - T_a) \qquad (20) \\ &= R_0 + \Delta R + k_1 T_i + k_2\omega, \end{aligned}$$

where

$$R_0 = -\frac{T_{a0}}{J_t}, \quad \Delta R = -\frac{\Delta T_a}{J_t}, \quad k_1 = \frac{1}{J_t}, \quad k_2 = -\frac{C_f}{J_t}.$$

The objective of an engine speed controller is to maintain ω near a set point ω_d. The desired indicated torque $T_{i,d}$ to support the steady-state operation with $\omega = \omega_d$ is given by

$$T_{i,d} = \frac{-k_2\omega_d - R_0}{k_1}.$$

Defining,

$$T_i = T_{i,d} + \Delta T_i,$$

and $y = \omega - \omega_d$ we have

$$\dot{y}(t) = \Delta R + k_1 \Delta T_i + k_2 \cdot y(t).$$

The nominal feedback law provides a proportional speed error compensation,

$$\Delta T_i = \frac{k_f}{k_1} y(t),$$

where $k_f = -1.5$. Although we do not treat integral compensation explicitly in this example, it can be easily included using the results of Section III.

It is assumed that the engine speed feedback compensation term is delayed due to excessive loading on the network over which the commands are transmitted to the engine actuators. Furthermore, this delay may be exacerbated by a computational delay in the engine control module which concurrently performs multiple other computations besides those required for engine speed regulation. To summarize, there is a random delay affecting the feedback term ΔT_i so that

$$\begin{aligned} \Delta T_i &= \frac{k_f}{k_1} \cdot (\omega(t - \eta(t)) - \omega_d) \\ &= \frac{k_f}{k_1} \cdot y(t - \eta(t)). \end{aligned}$$

Now we have

$$\begin{aligned} \dot{y}(t) &= \Delta R + k_f \cdot y(t - \eta(t)) + k_2 y(t), \\ b_F &= 1, \\ b &= 0, \\ F(t) &= \Delta R, \\ a_0 &= k_2, \\ a_1 &= k_f, \end{aligned}$$

yielding equation of the form (1),

$$\dot{y}(t) = a_0 y(t) + a_1 y(t - \eta(t)) + b_F F(t) + bu(t).$$

The delay, $\eta(t)$, is modeled as a Markov chain taking on one of two values, $\eta(t) \in \{\eta_1, \eta_2\}$, $\eta_1 = 0.1$, $\eta_2 = 0.3$. Assuming the rate parameters are $\lambda_{21} = 0.4 \cdot 30$, $\lambda_{12} = 0.6 \cdot 30$, and that $P_{ij}(0, t)$'s are in steady-state, the probabilities of $\eta(t) = \eta_1$ and $\eta(t) = \eta_2$ are, respectively,

$$\begin{aligned} Pr\{\eta(t) = \eta_1\} &= \frac{\lambda_{21}}{\lambda_{12} + \lambda_{21}} = 0.4, \\ Pr\{\eta(t) = \eta_2\} &= \frac{\lambda_{12}}{\lambda_{12} + \lambda_{21}} = 0.6. \end{aligned}$$

For the monitoring purposes we assume that a non-delayed measurement of $y(t)$ is available. We design an observer in the form (7) with

$$L = \begin{bmatrix} -1 \\ -10 \end{bmatrix}, \quad \rho_1 = 0.4, \quad \rho_2 = 0.6.$$

The condition of Theorem 1 is satisfied since $-a_0 = 3 > 2 + 0.25 \cdot a_1^2 = 2.5625$. The condition (15) of Theorem 2 with $m = 2$ is verified by first showing that there is a positive definite solution, P, to the algebraic Riccati equation,

$$\bar{A}_0^{\mathrm{T}} P + P\bar{A}_0 + P\bar{A}_1 R^{-1} \bar{A}_1^{\mathrm{T}} P + P\bar{A}_2 R^{-1} \bar{A}_2^{\mathrm{T}} P + (2 + 0.5)R = 0,$$

where $R = diag([1, \frac{1}{10}])$. Then, (15) must hold for a sufficiently large Q. In fact, we confirm that (15) holds for $Q = diag([30, 30])$.

For the simulation example, we inject a fault at time $t = 4$ sec and remove it at a later time of $t = 7$ sec. We consider the ability of the observer to identify the fault condition. Figure 1 shows an overlay of twenty engine speed trajectories generated for different realizations of $\eta(t)$. These engine speed trajectories show a significant amount of variability due to the stochastic nature of $\eta(t)$. The observer estimates of the fault generated from one of these trajectories are shown in Figure 2, confirming that the observer can identify the fault quite well, and differentiate it from transients caused by initial conditions changes (such as for $0 \leq t \leq 4$). From the discussion in Remark 3 we obtain that the observer can function even if there is a stochastic uncertainty in the coefficient C_f of the model so that $C_f = C_{f0} + b\dot{\zeta}$ provided $b < 0.9354$.

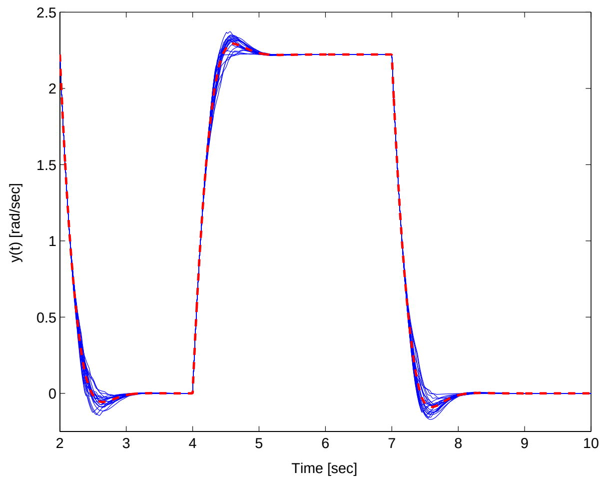

Fig. 1. The twenty trajectories of the engine speed deviation from the set-point, $y(t) = \omega(t) - \omega_d$, overlaid to show the variability caused by the random delay. The fault is injected at time $t = 4$ sec and is removed at time $t = 7$ sec. The dashed line shows the expected value of y from the twenty trajectories.

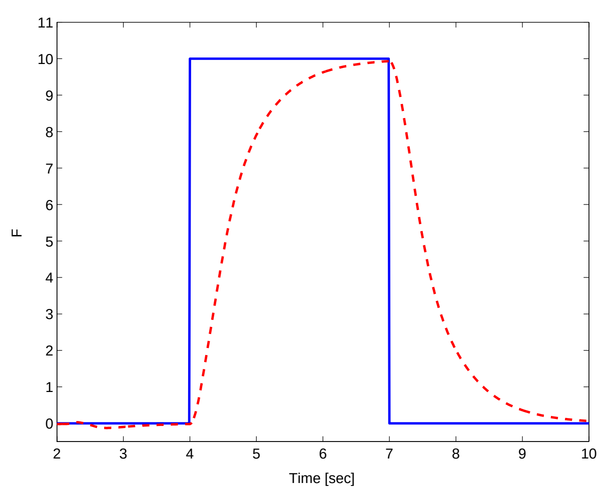

Fig. 2. The fault, F (solid line), and its observer-based estimate, $\hat{x}_2$ (dashed line).

V. Concluding Remarks

In the paper we considered a fault detection problem for a class of linear systems with a random delay which is relevant to applications in the area of control systems operating over networks. The delay value was assumed to be unknown and an observer was designed to estimate the fault.

The observer structure (7) was intuitively motivated by probabilistically averaging the model with respect to the random delay value, and by adding an output injection. At the same time, the optimality of choosing ρ_i in (7) equal to the probability of $\eta(t)$ taking the value η_i has not been theoretically justified. The relevant theoretical investigations on this topic, and the results of the numerical studies will be reported in the future publications.

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