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Hybrid Model Identification for Fault Diagnosis of Non–linear Dynamic Processes

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***Abstract*— This work addresses an approach for fault diagnosis of industrial processes using hybrid models. A non–linear dynamic process can, in fact, be described as a composition of different affine submodels selected according to the process operating conditions. This paper concerns the identification of the hybrid model parameters through the input–output data acquired from the non–linear process. Therefore, the fault detection scheme adopted to generate residual signals exploits this estimated hybrid model. In order to show the effectiveness of the developed technique, the results obtained in the fault diagnosis of a real industrial plant are finally reported.**

I. INTRODUCTION

There is an increasing interest in the development of model-based fault detection and fault diagnosis methods, as can be seen in the many papers submitted to the IFAC (International Federation of Automatic Control) Congress and IFAC Symposium SAFEPROCESS [1], [2], [3]. The majority of real industrial processes are non–linear [4], [5] and cannot be modelled by using a single model for all operating conditions. Since a mathematical model is a description of system behaviour, accurate modelling for a complex non–linear system is very difficult to achieve in practice. Sometime for some non–linear systems, it can be impossible to describe them by analytical equations. Instead of exploiting complicated non–linear models obtained by modelling techniques, it is also possible to approximate the plant by a collection of local affine models obtained by identification procedures [6].

Residual are signals representing inconsistencies between the model and the actual system being monitored. Any inconsistency will indicate a fault in the system. Residual must, therefore, be different from zero when a fault occurs and zero otherwise. However, the deviation between the model and the plant is influenced not only by the presence of the fault but also the modelling error. Several techniques had been proposed for Fault Detection and Isolation (FDI) in dynamic systems using either unknown input observers, parity relations, sliding mode observers, gain–parametrised observers [3], [9].

In particular, in this work, hybrid model [8], [6] identification is combined with the model–based method to formulate a diagnosis technique using the estimated model itself for residual generation. Hybrid models can, in fact, be exploited to describe the behaviour of non–linear dynamic systems since these prototypes are described by a composition of affine models. Each submodel approximates the system locally around an operating point and a selection procedure determines which particular submodel has to be used. Such a multiple–model structure is called multiple–model approach. Under such an identification and diagnosis scheme, a number of local affine models are designed and the estimate of outputs is given by a composition of local outputs. The diagnostic signal (residual) is the difference between the estimated and actual system output [3], [9]. In this paper, the different operating points can be selected by means of clustering method [9]. On the basis of knowledge of the operating point regions, the identification of the structure and the parameters of each local model composing the hybrid system can be performed [10], [11], [9].

The remainder of this paper is organised as follows. Section II presents the structure of the hybrid model, while Section III illustrates how to integrate the Frisch scheme [12] method for the identification of linear systems within a general procedure for hybrid model identification. Section IV shows the design of the diagnostic scheme for FDI of dynamic systems. The application of such a fault detection and identification approach to a real industrial plant is described in Section V. The example demonstrates the effectiveness of the technique proposed. Finally, some concluding remarks are included in Section VI.

II. HYBRID PROTOTYPE MODELLING

The main idea underlying the mathematical description of non–linear dynamic systems is based on the interpretation of single input–single output, non–linear, time–invariant regression models in the form: [9], [6] such as:

$$y(t+n) = F\big(y(t+n-1), \cdots, y(t), u(t+n-1), \cdots, u(t)\big) \tag{1}$$

where $u(\cdot)$ and $y(\cdot)$ belong respectively to the bounded input $\mathcal{U}$ and output $\mathcal{Y}$ sets, n is the finite system memory (*i.e.* the model order) and $F(\cdot)$ is a continuous non–linear function defining a hypersurface from a $\mathcal{A}_n$ to $\mathcal{Y}$, being $\mathcal{A}_n$ the Cartesian product $\mathcal{U}^n \times \mathcal{Y}^n$. The identification of the non–linear system can be translated to the approximation of its mathematical model in the form of Eq. (1) using a parametric structure that exhibits arbitrary accuracy interpolation properties.

A hybrid prototype defined through the composition of simple models having local validity is the natural candidate to perform this task, as it combines function interpolation properties with mathematical tractability. In the following the proposed hybrid structure is defined and its properties in terms of interpolation characteristics of arbitrary non–linear

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functions are recalled [6]. The hybrid prototype is formed by a collection of parametric submodels described by the model:

$$y(t+n)=\sum_{j=0}^{n-1}\alpha_j^{(i)}y(t+j)+\sum_{j=0}^{n-1}\beta_j^{(i)}u(t+j)+b^{(i)} \quad (2)$$

in which the system operating point is described by the input and output samples $y(t+n-1),\cdots,y(t)$ and $u(t+n-1),\cdots,u(t)$, that can be collected with a vector $\mathbf{x}_n(t)=[y(t),\cdots,y(t+n-1),u(t),\cdots,u(t+n-1)]^T$. The switching function $\chi_i\big(\mathbf{x}_n(t)\big), i=1,\ldots,M$ is:

$$\chi_i\big(\mathbf{x}_n(t)\big)=\begin{cases}\chi_i\big(\mathbf{x}_n(t)\big)=1 \text{ if } \mathbf{x}_n(t)\in\mathcal{A}_n^{(i)}\\ \chi_i\big(\mathbf{x}_n(t)\big)=0 \text{ otherwise}\end{cases} \quad (3)$$

where $\{\mathcal{A}_n^{(1)},\ldots,\mathcal{A}_n^{(M)}\}$ is a partition of $\mathcal{A}_n$, whose structure will be characterised in the following.

Thus output $y(t+n)$ of the non–linear dynamic system of Eq. (1) can be approximated by the hybrid piecewise affine model $f(\cdot)$ in the form:

$$y(t+n)=f\big(\mathbf{x}_n(t)\big)=\sum_{i=1}^{M}\chi_i\big(\mathbf{x}_n(t)\big)[\mathbf{x}_n(t),1]^T\mathbf{a}_n^{(i)} \quad (4)$$

where the model parameters are collected in the vector $\mathbf{a}_n^{(i)}=[\alpha_0^{(i)},\ldots,\alpha_{n-1}^{(i)},\beta_0^{(i)},\ldots,\beta_{n-1}^{(i)},b^{(i)}]^T$. It is worth noting that the model is affine in each $\mathcal{A}_n^{(i)}$, $\mathbf{a}_n^{(i)}$ being the affine submodel parameters.

Since the model of Eq. (1) is assumed to be continuous, $f(\cdot)$ is forced to be continuous over the whole $\mathcal{A}_n$. In such a case the parameter vectors are constrained to satisfy the following relation:

$$\lim_{\substack{\mathbf{x}_n(t)\to\bar{\mathbf{x}}_n\\ \mathbf{x}_n(t)\in\mathcal{A}_n^{(i')}}} f\big(\mathbf{x}_n(t)\big)=\lim_{\substack{\mathbf{x}_n(t)\to\bar{\mathbf{x}}_n\\ \mathbf{x}_n(t)\in\mathcal{A}_n^{(i'')}}} f\big(\mathbf{x}_n(t)\big) \quad (5)$$

$\bar{\mathbf{x}}_n$ being an accumulation point for both $\mathcal{A}_n^{(i')}$ and $\mathcal{A}_n^{(i'')}$, *i.e.* if

$$\bar{\mathbf{x}}_n(t)^T\ \mathbf{a}_n^{(i')}=\bar{\mathbf{x}}_n(t)^T\ \mathbf{a}_n^{(i'')}. \quad (6)$$

The straightforward application of Eq. (6) to all the accumulation points common to neighbouring regions leads to an infinite number of constraints. Yet, in [6] it is shown that the adoption of regions with straight borders guarantees that only a finite number of them is linearly independent. In fact, if regions whose boundaries are convex polyhedra are considered, continuity can be ensured simply by setting the value of the local models only on the vertices of the boundaries. In particular, it is undoubtedly convenient to "triangulate" the domain $\mathcal{A}_n$, i.e. to partition it into $2n$–dimensional simplexes. Moreover, we will assume that the triangulation is such that two simplexes are either disjoint, or have in common a whole k–dimensional boundary, with $k=0,1,\ldots,2n-1$. In this way, the local affine model of Eq. (4) can be forced to assume given values at most in $2n+1$ vertices of each simplex, which are affinely independent points.

Under these assumptions, the continuity constraints (one for each simplex vertex) can be collected in a finite matrix C_n such that:

$$C_nA_n=\mathbf{0}, \quad (7)$$

with

$$A_n=\begin{bmatrix}\mathbf{a}_n^{(1)} & \ldots & \mathbf{a}_n^{(M)}\end{bmatrix}^T.$$

III. DYNAMIC SYSTEM IDENTIFICATION

It is assumed that the input–output data $u(t)$ and $y(t)$, $(t=0,1,\ldots,L_i)$ generated by a system of the type (2) are available.

Restricting our investigation to find order n and parameters $\mathbf{a}_n^{(i)}$ for local model (2) in region $\mathcal{A}_n^{(i)}$, the following matrix should be defined:

$$X_k^{(i)}=\begin{bmatrix} y(k) & \mathbf{x}_k^T(0) & 1\\ y(k+1) & \mathbf{x}_k^T(1) & 1\\ \vdots & \vdots & \\ y(k+N_i-1) & \mathbf{x}_k^T(N_i-1) & 1\end{bmatrix} \quad (8)$$

$$\Sigma_k^{(i)}=\left(X_k^{(i)}\right)^T X_k^{(i)} \quad (9)$$

with $k+N_i-1\le L_i$ and N_i is chosen so that $k+N_i-1$ is large enough to avoid unwanted linear dependence relationships due to limitations in the dimension of the vector spaces involved.

To determine the model order n in region $\mathcal{A}_n^{(i)}$, it is possible to consider the sequence of increasing–dimension positive definite or positive semidefinite $\big((2k+2)\times(2k+2)\big)$ matrices:

$$\Sigma_2^{(i)},\quad \Sigma_3^{(i)},\quad \ldots\quad \Sigma_k^{(i)},\quad \ldots \quad (10)$$

testing their singularity as k increases. As soon as a semidefinite positive matrix $\Sigma_k^{(i)}$ is found then $n=k$, and the parameters $\mathbf{a}_n^{(i)}$ describe the dependence relationship of the first vector of $\Sigma_n^{(i)}$ on the remaining ones as:

$$\Sigma_n^{(i)}\,[-1\,,\,\mathbf{a}_n^{(i)T}]^T=0 \quad (11)$$

It is worth noting that the vectors $\mathbf{x}_n(0),\mathbf{x}_n(1),\ldots,\mathbf{x}_n(N_i-1)$ in (8) must belong to the region $\mathcal{A}_n^{(i)}$ according to the partition defined in (3).

Note also that in the presence of noise and disturbance the above procedure described to determine order and model parameters would obviously be useless since matrices Σ_k would always be non–singular (positive definite).

In order to solve the problem in a mathematical framework, it is necessary to characterise the noise and disturbance affecting the input-output data. Following common assumptions [12], [10], [11], the noise signals $\tilde{u}(t)$ and $\tilde{y}(t)$ are assumed additive on input-output data $u^*(t)$ and $y^*(t)$ and region independent, so that:

$$\begin{cases} u(t) &=& u^*(t)+\tilde{u}(t)\\ y(t) &=& y^*(t)+\tilde{y}(t).\end{cases} \quad (12)$$

Obviously, only $u(t)$ and $y(t)$ are available for the identification procedure, and moreover every noise term $\tilde{u}(t)$

and $\tilde{y}(t)$ is modelled with a zero–mean white process and is supposed to be independent of every other term. These structures are also commonly known as Error-In-Variables (EIV) models.

Under these assumptions, and $\bar{\bar{\sigma}}_u$ and $\bar{\bar{\sigma}}_y$ being the input and output noise variances respectively, the generic positive definite matrix $\Sigma_k^{(i)}$ associated with the input-output noise-corrupted sequences can always be expressed as the sum of two terms $\Sigma_k^{(i)} = \Sigma_k^{*(i)} + \bar{\bar{\Sigma}}_k$ where:

$$\bar{\bar{\Sigma}}_k = \text{diag}[\bar{\bar{\sigma}}_y I_{k+1}, \bar{\bar{\sigma}}_u I_k, 0] \geq 0. \tag{13}$$

Thus, it is again possible to determine the order and parameters of the model in region $\mathcal{A}_n^{(i)}$ from the analysis of the sequence of increasing–dimension $\big((2k+2)\times(2k+2)\big)$ symmetric positive definite matrices:

$$\Sigma_2^{(i)}, \quad \Sigma_3^{(i)}, \quad \ldots \quad \Sigma_k^{(i)}, \quad \ldots \tag{14}$$

The solution of the above identification problem requires the computation of the unknown noise covariances $\bar{\bar{\sigma}}_u$ and $\bar{\bar{\sigma}}_y$, that can be achieved solving the following relation:

$$\Sigma_k^{*(i)} = \Sigma_k^{(i)} - \tilde{\Sigma}_k \geq 0 \tag{15}$$

in the variables $\tilde{\sigma}_u, \tilde{\sigma}_y$, where $\tilde{\Sigma}_k = \text{diag}[\tilde{\sigma}_y I_{k+1}, \tilde{\sigma}_u I_k, 0]$

Unfortunately the relation (15) admits for any k an infinite solution set describing a curve $\Gamma_k^{(i)}(\tilde{\sigma}_y, \tilde{\sigma}_u) = 0$ in the first orthant of the noise plane whose concavity faces the origin. In [11] a constructive methodology to numerically compute this curve is given. Since determination of the system order requires the increasing values of k to be tested, it is relevant to analyse the behaviour of the associated curves when k varies. As proven in [11], the solution sets of condition (15) for different values of k are non–crossing curves in the noise plane.

It is also important to observe that, since we assume that a system of type (2) has generated the noiseless data, for $k \geq n$ all the curves of type (15) have necessarily at least one common point, i.e. point $(\bar{\bar{\sigma}}_u, \bar{\bar{\sigma}}_y)$ corresponding to the true variances of the noise affecting the input and the output data. The search for a solution for the identification problem can thus start from the determination of this point in the noise space. This task can be achieved on the basis of the following properties.

With reference to the diagonal non–negative definite matrices $\tilde{\Sigma}_k$, the following properties hold:

Property 1:

1) If $k < n$ the matrices $\Sigma_k^{*(i)}$ are positive definite.
2) If $k > n$ the dimension of the null space of $\Sigma_k^{*(i)}$ and consequently, the number of eigenvalues equal to zero is $(k - n + 1)$.
3) For $k = n$, matrix $\Sigma_k^{*(i)}$ is characterised by a linear dependence relation among its $2k+2$ vectors, and the coefficients which link the first vector of $\Sigma_k^{*(i)}$ to the remaining ones are the parameters $\mathbf{a}_n^{(i)}$, of the system (2) which has generated the noiseless sequences.
4) For $k \geq (n+1)$, all the $k-n+1$ linear dependence relations among the vectors of the matrix Σ_k^* are characterised by the same $2n+2$ coefficients $\mathbf{a}_n^{(i)}$.

When the order n has been determined, the parameters $\mathbf{a}_n^{(i)}, i = 1, \ldots, M$ can be identified solving the following equation:

$$(\Sigma_n^{(i)} - \bar{\bar{\Sigma}}_n)\,[-1\,,\, \mathbf{a}_n^{(i)T}]^T = \mathbf{0} \qquad \text{for } i = 1, \ldots, M. \tag{16}$$

The previous result can be fully applied when the assumptions behind the Frisch scheme are satisfied (independence between input–output sequences, additive noise, noise whiteness).

In real applications, we are forced to relax these assumptions, thus no common point can be determined among curves $\Gamma_n^{(i)} = 0$ in the noise plane and a unique solution to the identification problem can be obtained only by introducing a criterion to select a different noisy point for each region as best approximation of the ideal case. With reference to the identification of the system order n in the i–th region $\mathcal{A}_n^{(i)}$, it must be noted that the $\Gamma_{n+1}^{(i)} = 0$ curve has a single point in common with the $\Gamma_n^{(i)} = 0$ curve in ideal conditions, which corresponds to a double singularity of the matrix $\Sigma_{n+1}^{*(i)}$. In real cases, the order n can be computed finding the point $(\tilde{\sigma}_u, \tilde{\sigma}_y) \in \Gamma_{n+1}^{(i)} = 0$ that makes $\Sigma_{n+1}^{*(i)}$ closer to the double singular condition (i.e. minimal eigenvalue equal to zero and the second minimum eigenvalue near to zero). As n is unknown, increasing system orders k must be tested, and the value of k associated to the minimum of the second eigenvalue of the matrix $\Sigma_{k+1}^{*(i)}$ corresponds to the order n. This criterion is consistent as it leads to the common point of the curves when the assumptions of the Frisch scheme are not violated.

Note that since the order n of the hybrid model (4) is region independent, it can be determined by choosing k that fulfil the following inequality:

$$\max_{i=1,\ldots,M} \lambda_k^{(i)} < \epsilon \tag{17}$$

when ϵ is an arbitrary positive constant and $\lambda_k^{(i)}$ is the minimal eigenvalue different from zero of matrix $\Sigma_{k+1}^{*(i)}$.

Once the model order n is selected, the parameters $\mathbf{a}_n^{(i)}, i = 1, \ldots, M$ cannot be computed from (16), because the curves $\Gamma_n^{(i)} = 0$ do not share the common point $(\bar{\bar{\sigma}}_u, \bar{\bar{\sigma}}_y)$. In this case, for each region a different noise $(\bar{\bar{\sigma}}_u^{(i)},\ \bar{\bar{\sigma}}_y^{(i)})$ must be considered and relation (15) should be rewritten as:

$$\Sigma_n^{*(i)} = \Sigma_n^{(i)} - \tilde{\Sigma}_n^{(i)} \geq 0 \tag{18}$$

where $\tilde{\Sigma}_n^{(i)} = \text{diag}[\bar{\bar{\sigma}}_u^{(i)} I_{n+1}, \bar{\bar{\sigma}}_y^{(i)} I_n, 0]$. The values $(\bar{\bar{\sigma}}_u^{(i)},\ \bar{\bar{\sigma}}_y^{(i)})$ can be computed by solving an optimisation problem which minimises both the distances between $(\bar{\bar{\sigma}}_u^{(i)},\ \bar{\bar{\sigma}}_y^{(i)})$ and $(\bar{\bar{\sigma}}_u^{(j)},\ \bar{\bar{\sigma}}_y^{(j)})$ with $i \neq j$ and the continuity constraints described by Eq. (7):

$$J((\bar{\bar{\sigma}}_u^{(1)}, \bar{\bar{\sigma}}_y^{(1)}), \ldots, (\bar{\bar{\sigma}}_u^{(M)}, \bar{\bar{\sigma}}_y^{(M)})) = \\ = d\left((\bar{\bar{\sigma}}_u^{(1)}, \bar{\bar{\sigma}}_y^{(1)}), \ldots, (\bar{\bar{\sigma}}_u^{(M)}, \bar{\bar{\sigma}}_y^{(M)})\right) + \\ + (C_n A_n)^T H C_n A_n \tag{19}$$

H being a definite positive weighting matrix and d a distance defined as:

$$d\left((\bar{\bar{\sigma}}_u^{(1)}, \bar{\bar{\sigma}}_y^{(1)}), \ldots, (\bar{\bar{\sigma}}_u^{(M)}, \bar{\bar{\sigma}}_y^{(M)})\right) = \\ = \sum_{i=1}^{M} \sum_{j=i+1}^{M} \sqrt{(\bar{\bar{\sigma}}_u^{(i)} - \bar{\bar{\sigma}}_u^{(j)})^2 + (\bar{\bar{\sigma}}_y^{(i)} - \bar{\bar{\sigma}}_y^{(j)})^2}. \tag{20}$$

It is worth observing that the matrix A_n collects the parameters $\mathbf{a}_n^{(i)}, i = 1, \ldots, M$ which depend on $(\bar{\bar{\sigma}}_u^{(i)}, \bar{\bar{\sigma}}_y^{(i)})$. Now, let us take into account the problem of determining the model order n. In the real case the item 2 of Property 1 is only approximately fulfilled (i.e. for $k > n$ null eigenvalue has multiplicity one, whereas the *second* minimum eigenvalue is very close to zero). Minimisation of cost function (19) can be computationally difficult, as it depends on $2M$ independent variables. Therefore, in order to decrease the complexity of the problem, a common parametrisation can be defined for all the curves $\Gamma_n^{(i)}(\bar{\bar{\sigma}}_u^{(i)}, \bar{\bar{\sigma}}_y^{(i)}) = 0$ by introducing polar coordinates:

$$\begin{cases} \bar{\bar{\sigma}}_u^{(i)} = \rho^{(i)} \cos \frac{\pi}{2} q \\ \bar{\bar{\sigma}}_y^{(i)} = \rho^{(i)} \sin \frac{\pi}{2} q \end{cases} \tag{21}$$

where $\rho^{(i)}$ is determined so that $\Gamma_n^{(i)}(\rho^{(i)} \cos \frac{\pi}{2} q, \rho^{(i)} \sin \frac{\pi}{2} q) = 0$ and $q \in [0, 1]$.

In such a way, the cost function has the form:

$$J(q) = d\left((\bar{\bar{\sigma}}_u^{(1)}(q), \bar{\bar{\sigma}}_y^{(1)}(q)), \ldots, (\bar{\bar{\sigma}}_u^{(M)}(q), \bar{\bar{\sigma}}_y^{(M)}(q))\right) + \\ + (C_n A_n)^T H C_n A_n. \tag{22}$$

The parametrisation chosen to simplify the minimisation problem leads to consistent results. In fact, when the data are generated by a continuous piecewise-affine dynamic system, all assumptions regarding the Frisch scheme being fulfilled and noise being region-independent, the curves $\Gamma_n^{(i)} = 0$ share a common point in the noise plane. In these conditions, cost function $J(q) = 0$ and the variances $(\tilde{\bar{\sigma}}_u, \tilde{\bar{\sigma}}_y)$ are identified exactly.

Finally, one should note how once the parameter q minimising the cost function (22) is computed, the matrices $\tilde{\Sigma}_n^{(i)}$ can be built and the model parameter $\mathbf{a}_n^{(i)}, i = 1, \ldots, M$ determined by means of relation:

$$(\Sigma_n^{(i)} - \tilde{\Sigma}_n^{(i)})\,[-1\,,\,\mathbf{a}_n^{(i)T}]^T = \mathbf{0} \quad \text{for } i = 1, \ldots, M. \tag{23}$$

This completes the multiple model identification procedure.

IV. FAULT DIAGNOSIS OF DYNAMIC SYSTEMS

The problem treated in this section regards the detection and isolation of the output sensor faults on the basis of the knowledge of the measured sequences $u(t)$ and $y(t)$. In the following it is assumed that the monitored system, depicted in Figure (1), can be described by a model of the type (4). $y(t) \in \Re^m$ is the system output vector and $u(t) \in \Re^r$ the input vector.

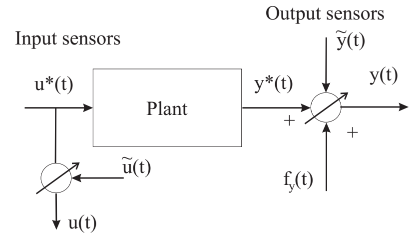


Fig. 1. The monitored system.

In real applications variables $u^*(t)$ and $y^*(t)$ are measured by means of sensors whose outputs are affected by noise (see relations (12)) and faults.

In this work, neglecting sensor dynamics, faults on the measured input and output signals $u(t)$ and $y(t)$ are considered and they can be modelled as:

$$\begin{array}{rcl} u(t) & = & u^*(t) + \tilde{u}(t) + f_u(t) \\ y(t) & = & y^*(t) + \tilde{y}(t) + f_y(t) \end{array} \tag{24}$$

in which, the vectors $f_u(t) = [f_{u_1}(t) \ldots f_{u_r}(t)]^T$ and $f_y(t) = [f_{y_1}(t) \ldots f_{y_m}(t)]^T$ are composed of additive signals assuming values different from zero only in the presence of faults. Usually these signals are described by step and ramp functions representing, respectively, abrupt and incipient faults (bias or drift).

There are different approaches to generate the diagnostic signals, residuals or symptoms, from which it will be possible to diagnose faults associated to sensors. In this work, a model–based approach is used to estimate the outputs $y(t)$ of the system from the input-output measurements [3]. As an example, residuals can be generated by the comparison of measured $y(t)$ and estimated $\hat{y}(t)$ outputs:

$$r(t) = \hat{y}(t) - y(t) \tag{25}$$

where $\hat{y}(t)$ is the estimate generated by the identified model of the type (4) of the process under investigation.

The symptom evaluation refers to a logic device which processes the redundant signals generated by the first block in order to estimate when a fault occurs and to univocally identify the unreliable sensor. Faults can be detected, for example, by using a simple thresholding logic:

$$|r(t)| \begin{cases} \le \text{Threshold} & \text{Fault-free conditions,} \\ > \text{Threshold} & \text{Faulty conditions.} \end{cases} \tag{26}$$

V. IDENTIFICATION AND FAULT DIAGNOSIS OF THE PLANT

The technique for input–output sensor fault diagnosis presented has been applied to the model of a real single–shaft industrial gas turbine with variable IGV angle working in parallel with electrical mains in a cogeneration plant. The non–linear turbine model was developed as explained in [9].

The process consists of three major components: *the combustor*, *turbine*, and *condenser*. Furthermore, there are pumps and valves (not highlighted). The combustor boils the water and the steam generated drives the turbine. After the turbine, the condenser cools the steam. In turn, external cooling water cools the condenser. Pumps transport the water from the condenser tank back to the combustor tank.

Concerning the machine layout shown in Figure (2), the input control sensors are used for the measurement of:

$u_1(t)$, Inlet Guide Vane (IGV) angular position (α);
$u_2(t)$, fuel mass flow rate (M_f).

The output sensors are those used for the measurement of the following variables:

$y_1(t)$, pressure at the compressor inlet (p_{ic});
$y_2(t)$, pressure at the compressor outlet (p_{oc});
$y_3(t)$, pressure at the turbine outlet (p_{ot});
$y_4(t)$, temperature at the compressor outlet (T_{oc});
$y_5(t)$, temperature at the turbine outlet (T_{ot});
$y_6(t)$, electrical power at the generator terminal (P_e).

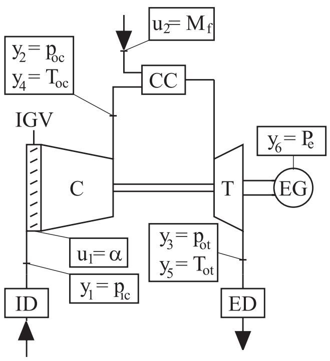


Fig. 2. Layout of the single–shaft industrial gas turbine with the monitored sensors highlighted.

The rotational speed sensors are not considered since the operation of the machine in parallel with electrical mains is at constant rotational speed. The measurements of ambient temperature T_a and relative humidity were also not considered, since they are not directly used by the gas turbine control system. The ambient temperature in particular, which is an important parameter for gas turbine performance, is taken into account by the machine control system by means of the measurements of compressor outlet pressure. This pressure P_a indeed depends on the compressor mass flow rate which, in turn, depends on ambient temperature [9].

According to Section (II) and (III), the non–linear dynamic process can be described as a composition of several affine submodels selected according to process operating conditions.

It is assumed that the monitored system, depicted in Figure (2), can be described by a model of the type given by Eq. (4). Moreover, as presented in Section (IV), the diagnostic scheme exploits the hybrid model to generate residuals.

The problem considered here thus regards the hybrid prototype identification and the sensor fault diagnosis on the basis of the knowledge of the measured sequences $u(t)$ and $y(t)$ acquired from the input–output sensors of the industrial gas turbine.

The process operates mainly at steady state conditions and the 8 process measurements, including temperatures, flow rates, pressures, control signals, turbine speed and torque can be acquired with a sampling rate of 0.1 s. The number of acquired samples is about $N = 5000$. Because of the presence of the input and output sensors, actual measurements are affected by faults and noise.

A pressure sensor bias (abrupt fault on the p_{ot} pressure sensor signal) and an input sensor fault (abrupt faults on the $\alpha(t)$ sensor signal) were simulated to experiment with both the identification and the fault diagnosis methods.

Because of the underlying physical mechanisms and because of the modes of the control signals, the process has non–linear steady state as well as dynamic characteristics.

A clustering algorithm [13] was exploited in connection with the identification method described in Section (III) in order to extract a number of $M = 3$ clusters (operating conditions) and $n = 2$ the number of sample delays of the inputs and outputs. After clustering, the system parameters $\mathbf{a}_n^{(i)}$, with $i = 1, \cdots, M$ for each output, were estimated using the Frisch scheme. The model was then validated on a separate data set.

In fault–free conditions, Table (I) reports the mean–square values of the output estimation errors $r(t)$ given by classical observers using a single model (*i.e.*, with $M = 1$ and $n = 2$) for all operating conditions [14]. These values are very large and cannot be used to detect faults reliability.

A meaningful improvement has been obtained by using this identification technique where the process is described as an hybrid model identified using Frisch scheme method. The i-th output $y(t)$ of the plant ($i = 1, \cdots, m$ and $m = 6$) can be characterised as a hybrid multiple–input single–output (MISO) model with $r = 2$ inputs. The mean–square errors of the output estimation errors $r(t)$, under no–fault conditions, are collected in Table (I). The hybrid multiple–model approximates the real process very accurately. The results indicate that the composite model can serve as a reliable predictor for the real process. Using this model, a model–based approach for fault diagnosis can be exploited and applied to the actual power plant. The fault occurring

TABLE I

OUTPUT ESTIMATION ERRORS.

Output	p_{ic}	p_{oc}	p_{ot}
Classical observer	13.29	7.56	15.34
Hybrid model	2.04	3.22	1.67
Output	T_{oc}	T_{ot}	P_e
Classical observer	20.22	21.57	19.70
Hybrid model	2.55	2.58	1.70

on the single sensor $\alpha(t)$ or $p_{ot}(t)$ causes alteration of the sensor signals $\mathbf{u}(t)$, $\mathbf{y}(t)$ and of the residuals $\mathbf{r}(t)$

given by the predictive model (4) using $\mathbf{u}(t)$ and $\mathbf{y}(t)$ as inputs. Residuals indicate the fault occurrence according to Equation (26) whether their values are lower or higher than the thresholds fixed under fault–free conditions.

To summarise the performance of the FDI technique, the minimal detectable faults on the various sensors, expressed as percentages of the mean values of the relative signals, are collected in Table (II). The minimum values shown in Table (II) are relative to the case in which the fault must be detected as soon as it occurs. The results were obtained by using a single model for all operating conditions.

An improvement in the FDI performance has been obtained by using the hybrid multiple–model. Model parameters were identified under the assumptions of the Frisch scheme.

Table (II) summarises the performance of the enhanced FDI technique and shows the minimal detectable fault size for the various sensors. The fault sizes are expressed as percentages of the signal mean values.

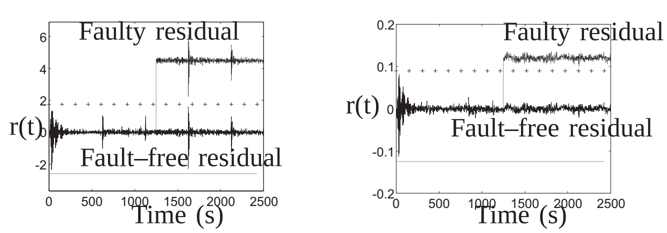
TABLE II

MINIMAL DETECTABLE STEP FAULTS.

Sensor	α	M_f	p_{ic}	p_{oc}
Classical observer	4%	4%	5%	7%
Hybrid model	1.8%	2.3%	0.60%	0.8%
Sensor	p_{ot}	T_{oc}	T_{ot}	P_e
Classical observer	5%	5%	2.5%	1.7%
Hybrid model	0.65%	1.7%	0.65%	1.2%

The residuals obtained by using the multiple–model approach are more sensitive to a fault occurring on the sensors, since the corresponding output estimation errors are smaller. Noise rejection is, in fact, achieved by means of the dynamic Frisch identification scheme. Moreover, smaller thresholds can be placed on the residual signals to declare the occurrence of faults.

As an example, fault–free and faulty residuals regarding the $\alpha(t)$ sensor signal are reported in Figures (3(a)) and (3(b)). These were generated by using a classical observer designed and the identified hybrid system, respectively. Fault-free thresholds were marked by using "$-$" and "$+$".



(a) (b)

Fig. 3. (a) single model and (b) hybrid model residuals $r(t)$ for the signal $\alpha(t)$.

The consequence is that the values of the faults, reported in Table II, obtained by using the hybrid multiple–model approach are lower than the ones corresponding to classical observers. Moreover, the minimal detectable faults on the various sensors seem to be adequate for the industrial diagnostic applications. However, these improvements are not free of charge: they have been obtained with a procedure of greater complexity and, consequently, with a growing computational cost.

VI. CONCLUSION

In this paper an off-line procedure was proposed for the identification and fault diagnosis of a dynamic system using an hybrid model identified from noisy input-output measurements. An hybrid model consists of several local affine models each for different operating point of the process. The identification algorithm requires the determination of the regions in which measured data can be approximated by affine dynamic submodels. Parameters and structure of submodels were estimated using a technique based on the rules of the Frisch scheme, traditionally exploited to analyse economic systems. The effectiveness of these procedures were tested on real data acquired from a dynamic non–linear process.

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