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Multi-Model Control of Nonlinear Systems Using Closed-Loop Gap Metric

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ABSTRACT

A new multi-linear model approach is used to control a strongly nonlinear process. A global controller is built from a weighted combination of local controller outputs with the weights being functions of a defined closed-loop gap metric.

1. Introduction

For linear systems powerful controllers can be synthesized to meet certain performance and robustness requirements using well-known linear controller design methods. However, linear controllers can exhibit serious performance limitations when applied to nonlinear systems since nominal linear models used during design cannot represent the nonlinear plant in its whole operating range. On the other hand, several remedies for this problem can be devised. Gain scheduling (Rugh, 1991) and adaptive control (Aström and Wittenmark, 1984) are some of these more advanced approaches.

Another approach for the control of nonlinear plants is the multi-linear model approach (Foss et al., 1995; Murray-Smith and Johansen, 1997). In this approach the nonlinear plant is described by a combination of local linear models, each of which is valid in a particular operating region. First, local controllers for the local models are tuned. Next, the control actions of these local controllers are combined in the form of a global controller to be implemented on the nonlinear plant. Most of the time, the weights used to combine the multi-models or the local control actions are calculated from output estimation errors which are the differences between the outputs of local linear models and the actual output of the nonlinear plant (Banerjee et al., 1997).

In this paper, we define a closed-loop gap metric and use it to compute the weights of the global multi-linear controller. A different closed-loop gap metric was previously defined by (Lee et al., 2000) for the measure of performance difference between centralized and decentralized closed-loop systems. In the previous works gap metric is used as a criterion of robustness and it is used for the design of controllers such as decentralized control design (Lee et al., 2000) and control design for recycled multi unit process (Kadiman, 2003) .

2. Gap Metric

In the literature, the gap and other metrics were used to study how close different operators are (e.g. Newburgh, 1951; Berkson, 1963). In Zames and El-Sakkary (1980), the gap metric was used to establish a topology to quantify the tolerable uncertainties that preserve closed loop stability. El-Sakkary (1985) shows that the gap metric is better suited to measure the distance between two linear systems than a metric based on norms. Gap metric denoted by $\delta(P_1, P_2)$ introduces the notion of "distance" between two closed operators P_1 and P_2 as the "gap" between their graphs. The calculation of the gap metric begins with two finite dimensional linear systems with the same number of inputs and outputs whose normalized coprime factorizations are given by:

$$P_i(s) = N_i(s)\, D_i^{-1}(s) \quad \text{for } i=1 \text{ and } 2. \qquad (1)$$

It can be shown that the gap can be computed using the projection operators or the coprime factorizations (Georgiou, 1988):

$$\delta(P_1,P_2) = \max\left\{ \begin{array}{c} \inf_{Q\in H_\infty} \left\| \begin{pmatrix} D_1 \\ N_1 \end{pmatrix} - \begin{pmatrix} D_2 \\ N_2 \end{pmatrix} Q \right\|_\infty \\ \inf_{Q\in H_\infty} \left\| \begin{pmatrix} D_2 \\ N_2 \end{pmatrix} - \begin{pmatrix} D_1 \\ N_1 \end{pmatrix} Q \right\|_\infty \end{array} \right\} \quad (2)$$

Properties of the gap:

1. The gap defines a metric on the space of (possibly unstable) linear systems.
2. $0 \le \delta(P_1, P_2) \le 1$

Galan et al. used (2003) the gap metric concept for the first time within the context of multi-linear model-based control framework. The concept of distance between dynamic systems is used as a criterion to select a set of local linear models that can explain the nonlinear plant behavior. Gap metric is used to analyze the closeness among candidate *open-loop* models, so that a suitable set of local models can be chosen to design a multi-model controller (a robust H_∞ - controller in this case).

In this paper, the application of gap metric is quite different. While gap metric is used off-line for multi-model selection before (Galan et al., 2003), here it is used on-line to compute the weights of local controllers.

3. Controller Design

3a. Process Description and Local Controllers

Without any loss of generality, it is best to demonstrate the method through an example. The process is a highly nonlinear reactor taken from Uppal et al. (1976). The following set of nonlinear differential equations describe the system:

$$\dot{x}_1 = -x_1 + Da\, r(x_1, x_2) - (x_f - 1) \quad (3)$$

$$\dot{x}_2 = -x_2 + B\, Da\, r(x_1, x_2) + \beta\,(u - x_2) \quad (4)$$

$$y = x_2 \quad (5)$$

$$r(x_1, x_2) = (1 - x_1)\exp\left[\frac{x_2}{1 + x_2/\gamma}\right] \quad (6)$$

where x_1, x_2, u and x_f are reagent conversion, the dimensionless temperature (output), the dimensionless coolant temperature (input) and dimensionless feed concentration respectively. The nominal values for the constants in equations (3-6) are Da = 0.072, γ = 20, B = 8, β = 0.3 x_f = 1. System shows output multiplicity as shown by the S-shaped steady-state curve in Figure 1. There are three steady-state points corresponding to input u=0. A possible closed-loop trajectory from the lowest stable steady-state (model 1) to the middle unstable steady state point (model 2) is also shown. The linear transfer functions at the three steady-states are given in Table 1.

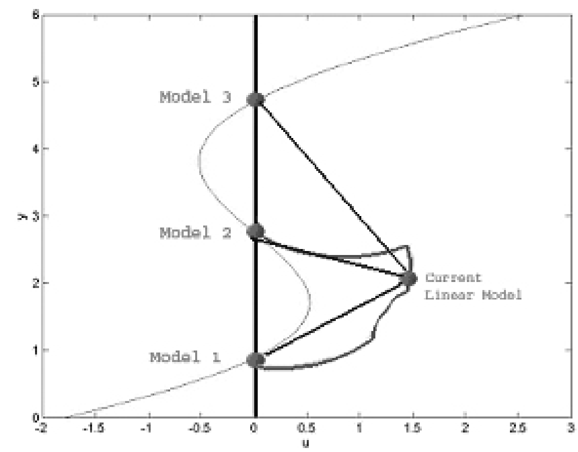

Figure 1. Steady –state input-output map and a closed loop trajectory for the reactor.

Table 1. Local models.

Model 1 →	$g_1(s) = \dfrac{0.3s + 0.35}{s^2 + 1.4s + 0.46}$
Model 2 →	$g_2(s) = \dfrac{0.3s + 0.53}{s^2 + 0.36\,s - 0.41}$
Model 3 →	$g_3(s) = \dfrac{0.3s + 1.26}{s^2 + 1.6\,s + 1.6}$

The models 1, 3 are stable and the model 2 is unstable. The gap metrics calculated for these open-loop models are given in Table 2. The diagonal entries are the comparison of the model with itself and therefore zero. The gap metric between a stable and an unstable model is essentially one indicating dissimilarity. On the other hand, the gap metric between the two stable models (1 and 3) is relatively low.

Table 2. The gap metrics between nominal models (Galan et al. 2003).

$\delta_{i,j}$	1	2	3
1	0. 0000	1. 0000	0.3656
2	1. 0000	0. 0000	0.9998
3	0.3656	0.9998	0. 0000

The global controller's task is to move the reactor operation between the above three steady states. The multi-models are chosen as the three local models given in Table 1. These nominal models are controlled using PI controllers. Each local controller is tuned so that the closed-loop servo response has a settling time of 5.

3b. Construction of Global Controller Using Gap Metric Weighting

The global controller is given by

$$u(t) = \sum_{i=1}^{3} w_i(t) u_i(t) \tag{7}$$

where u_i is the control output of the i^{th} local (PI) controller whose transfer function is denoted by c_i. The weights w_i are adjusted based on the closed-loop gap metrics that will be defined next.

Once the local controllers c_i are designed (see above) the local servo closed-loop transfer functions $\frac{c_i g_i}{1+c_i g_i}$ $i=1,2,3$ can be easily computed. Similarly we can compute servo closed loop transfer functions on the trajectory at any time t as follows:

i) First, linearize the nonlinear equations at the current state corresponding to time t (assuming the model and states are available). Denote the corresponding transfer function g_t.

ii) Then, form three fictitious servo closed-loop operators for g_t by using the three local controllers: $\frac{c_i g_t}{1+c_i g_t}$, $i=1,2,3$.

Now, the closed-loop gap metric is defined by:

$$\delta_i = \delta\left(\frac{c_i g_i}{1+c_i g_i}, \frac{c_i g_t}{1+c_i g_t}\right) \quad i=1,2,3 \tag{8}$$

The gap metric defined in this way indicates the distance between the closed-loop performance of the i^{th} local controller at the current simulation point to the desired i^{th} local closed loop performance.

At this point, weights of each controller are determined from equation 9 and the global process input is calculated from equation 7.

$$w_i = \frac{(1-\delta_i)}{\sum_{k=1}^{3}(1-\delta_k)} \tag{9}$$

Similarly, controllers can be designed for disturbance rejection as well in which case the closed loop gap is defined as,

$$\delta_i = \delta\left(\frac{g_{d\,i}}{1+g_i c_i}, \frac{g_{d\,t}}{1+g_t c_i}\right) \quad i=1,2,3$$

where g_d is the disturbance transfer function and g_{d_t} is the disturbance transfer function on the closed-loop trajectory at the current point.

4. Results and Discussion

Figure 2 shows a sequence of set point responses of the process output obtained using the local controllers and the closed-loop gap metric weights. Here, linearization on the closed-loop trajectory is performed every 1-time unit. In Figure 3, it is seen that when the closed-loop trajectory is in the neighborhood of a particular local steady state, the corresponding closed-loop gap metric between the current point and that steady-state is close to zero indicating that the achieved performance is similar to the local controller's performance. It also follows that when a particular gap is small, the corresponding weight is high (Figure 4), indicating that the control action is dominated by the corresponding local controller. For example, in the time interval 10-20, where the set-point change is made to go to the second steady-state, the second gap (gap 2) approaches zero after the initial transient due to the overshoot. As a result, the second weight takes a relatively high value in the same interval.

Figure 5 shows the contributions of the local controllers to the global control action. It is noted that the controller designed around the unstable point has the largest influence on the closed-loop transient behavior.

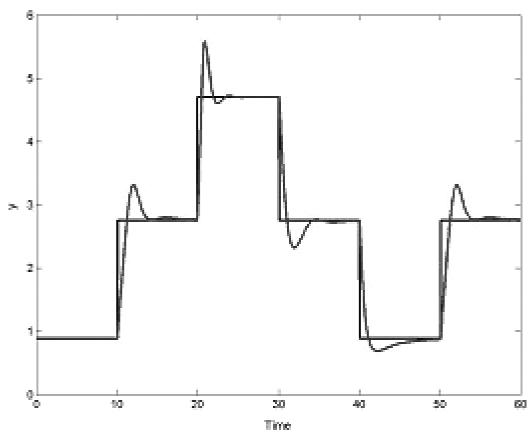

Figure 2. Servo response of the output using global controller when gap metric weights are used.

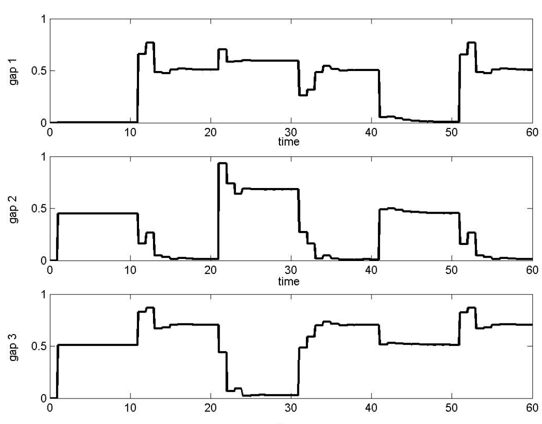

Figure 3. Calculated gap metrics.

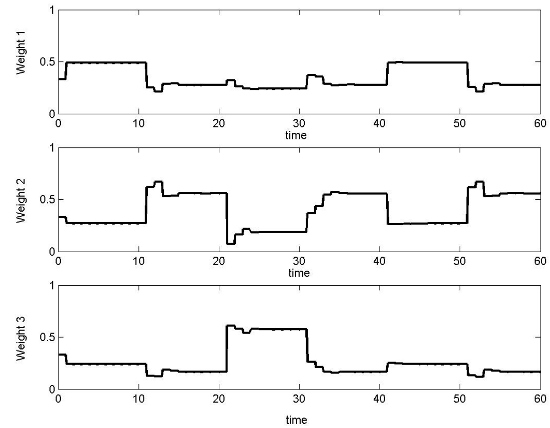

Figure 4. Weights of the local controller outputs.

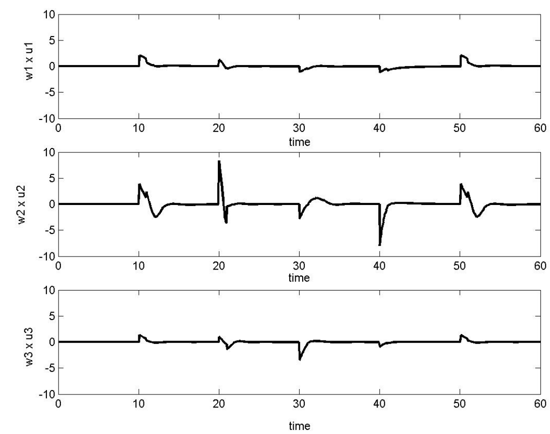

Figure 5. Contribution of the local controllers ($w_i u_i$) to the global control action.

To study the behavior of the control system under disturbance rejection, the dimensionless feed concentration was changed from 1 to 0.9 at time 20 when the reactor was operating at the middle unstable steady-state point 2. The closed loop response is shown in Figure 6 and the gaps are displayed in Figure 7. Gaps 1 and 3 (corresponding to the stable points) are equal to 1 indicating the dissimilarity of the current closed-loop performance around the unstable steady state to the local controllers' performances at stable points 1 and 2 Before the disturbance enters most weight on the controllers is on model 2 (gap almost zero). However after the disturbance is rejected the weight of the model 1 is increased. This is because the disturbed system approaches to the model 1 in the steady-state input-output map. This in turn causes the disturbed model to appear in between the first and the second model.

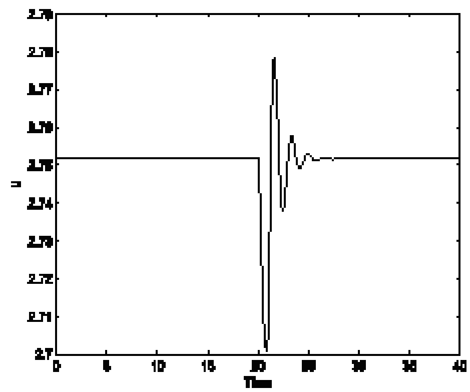

Figure 6. Disturbance rejection response.

This can also be seen in the contributions of the local controllers shown in Figure 9.

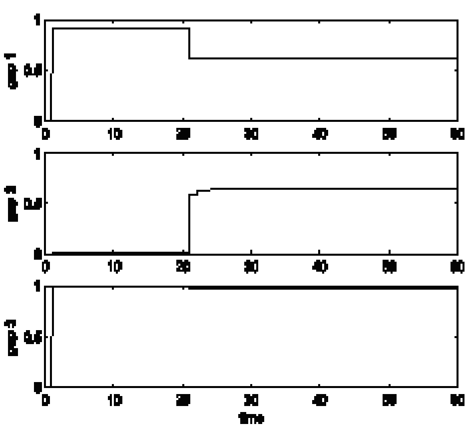

Figure 7. Calculated gap metrics for disturbance rejection.

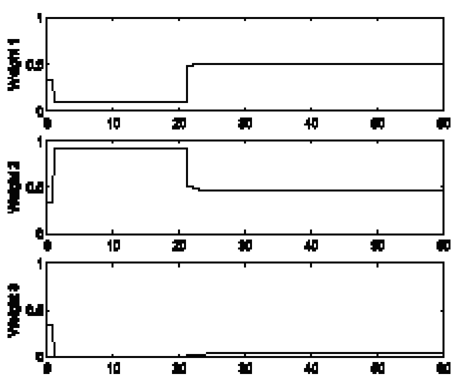

Figure 8. Weights of the local controller outputs.

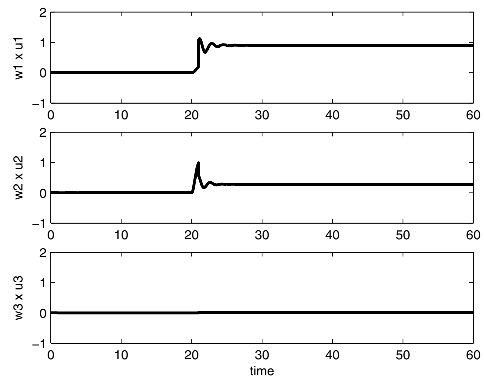

Figure 9. Contributions of local controllers for disturbance rejection.

5. Conclusions

A new multi-model control method is presented for nonlinear systems. A convex function of closed-loop gap metrics is used as weights to obtain the global controller output among different local controllers. An example of a highly nonlinear process is given to demonstrate the response characteristics of this approach.

References

Aström, K.J. and Wittenmark, B., 1984, *Computer Controlled Systems*, Prentice-Hall, NJ.

Banerjee, A, Arkun, Y., Ogunnaike, B., Pearson, R., 1997, *AIChE J.*, **43**, pp.1204-1226.

Berkson, E.,1963, *Pacific J. Math*, vol.13, pp.7-22.

El-Sakkary A., 1985, *IEEE Trans. On Automatic Control*, **AC-30**, 240-247.

Foss, B.A., Johansen, T.A. and Sorensen, A.V., 1995, *Control Eng. Practice*, **3**, pp. 389-396.

Galán, O., Romagnoli, J. A., Palazoglu, A. and Arkun, Y., 2003, *I&EC Research*, (in print).

Georgiou, T.T., 1988, *Systems & Control Letters*, **11**, pp. 253-257.

Murray-Smith, R., and Johansen, T.A. (eds.), 1997, *Multiple Model Approaches to Modeling and Control*, Taylor & Francis, London, England.

Newburgh, J. D., 1951, *Ann. Math.* vol. 53, pp. 250-255.

P.L. Lee, H. Li, I.T. Cameron, 2000 *Chem. Eng. Sci.,* vol. **55**,3743-3758.

Rugh, W., 1991, *IEEE Control Syst. Magazine*, **11**, pp. 79-84.

Y. Samyudia and K. Kadiman, (2003), *J. of Process Control*, vol. 13, 311-324

Zames, G. and El-Sakkary, A.K., 1980, *Proceedings of the Allerton Conf.,* pp.380-385.