

A Unified Nonlinear Controller for a Platoon of Car-Like Vehicles

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Abstract— This paper presents a dynamics-based nonlinear tracking control scheme for a platoon of two car-like mobile robots. A unified controller is designed for both look-ahead and look-behind tracking. Look-ahead and look-behind tracking maneuvers require the following vehicle to follow the leading vehicle in two opposite directions: forward or backward, respectively. Furthermore, both steering control and driving control are also integrated in the unified controller. Tracking stability is ensured by proper design of a stable performance target equation. Simulation results show the control scheme work properly in both tracking cases. Simulations also investigate the influence of two important design parameters: the desired distance l and the desired steering angle ratio p . The results suggest that these parameters affect the system performance and require careful selection.

I. INTRODUCTION

Platooning has been one keen research topic on autonomous vehicles in recent years. A vehicle platooning system consists of one leading vehicle, which leads the platoon, and one or more following vehicle that autonomously track and follow the leading vehicle. An autonomous tracking controller is required for each of the following vehicles. Based on the relative distance, orientation, velocity and even acceleration of the leading vehicle, the controller generates the corresponding control inputs for the following vehicle. Many controllers for vehicle tracking have been proposed. In a planar configuration of platoon, the relative position between two vehicles is basically composed of two parts: longitudinal relative distance and lateral deviation. Longitudinal control systems, such as those in [1]–[6], concentrate on the longitudinal relative distance and take the difference between the relative position and a pre-determined spacing l as the tracking error. Based on it, different control laws have been proposed, for example, a simple proportional integral differential (PID) controller or with an additional quadratic term (PIQ controller) as in [1], [2], a controller with a compensator as in [3], [4], or using adaptive control as in [1], [6]. The results were impressive, especially when no turning was involved, the speed of the platoon could be as high as 20m/s [1]. Lateral control, on the other hand, is concerned with the deviation between trajectories of two vehicles. Steering control plays the most important role. Since the deviation of the two trajectories is expectedly minimized towards zero, it is necessary for steering control

to require more than just the current position information of the leading vehicle. The trajectory of the leading vehicle can be partially interpolated from several recent measurements. The steering control to steer the following vehicle to stay on that trajectory can be developed. Path following algorithms might be applicable. For instance, in [7] and [8], the time history of the leading’s coordinates and the motion parameters of the following vehicle are recorded. Using this, the steering commands can be computed based on kinematic model of the vehicle. Likewise, three methods of interpolation and steering control were introduced in [9]: linear interpolation, circular interpolation and interpolation integrating the relative heading angle. Most of the above-mentioned controllers guarantee good tracking performances only when the leading vehicle moves forward in front of the following vehicle. Backward tracking is still a challenge due to difficulties in backward driving as pointed out in [10]. Some attempts to overcome the difficulties have also been carried out. The controller in [10] imitates the human driving practice of a boat with the rudder. Backward tracking for trailer system have also been extensively studied in [11]–[13]. Lately, a tracking control method that based on output feedback theory has been introduced in [15], [16] and [17], referred as full-state tracking. This nonlinear tracking method ensures exponential stability and convergence, as well as integrates both longitudinal control and lateral control into one controller. In [16], a look-ahead and look-behind platooning control has been proposed based on the kinematics. In this paper, we will extend this method to develop the controller for a platoon of two vehicles. The proposed controller will be able to handle both forward tracking and backward tracking with proper selection of design parameters.

II. VEHICLE AND PLATOON DYNAMICS MODELLING

A. Vehicle dynamics

Consider a car-like mobile robot with front wheels for steering and rear wheels for driving. Its dynamics model can be described by the following equation [14], [17]

$$\begin{cases} \dot{q} &= G(\theta, \gamma)\mu \\ \dot{\mu} &= u \end{cases} \quad (1)$$

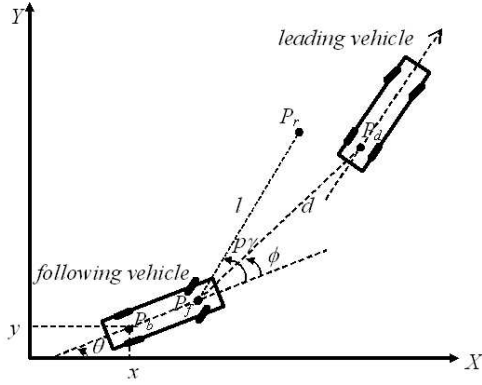


Fig. 1. Look-ahead tracking configuration

where $u = [u_m \ u_s]^T$ contains control inputs to the driving and steering wheels which are homogenous to the longitudinal acceleration u_m and the steering acceleration u_s , respectively; $\mu = [v \ \omega]^T$ consists of the longitudinal velocity v and the steering rate ω ; $q = [x \ y \ \theta \ \gamma]^T$ is the full-state configuration of the vehicle with (x, y) being the generalized coordinates, θ the heading angle and γ the steering angle; and

$$G(\theta, \gamma) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ \frac{1}{a} \tan \gamma & 0 \\ 0 & 1 \end{bmatrix} \quad (2)$$

with a being the length of the vehicle.

B. Platooning dynamics

In a platoon system considered in this paper, two car-like vehicles are moving in a horizontal plane. This platooning system can be executed in one of two modes: look-ahead tracking and look-behind tracking.

- **Look-Ahead Tracking:** The leading vehicle moves forward in front of the following vehicle as in Fig. 1. In this case, the tracked point is the rear point P_d of the leading vehicle. The relative distance between two vehicles is measured by the length $d > 0$ of P_fP_d and the relative orientation angle ϕ is formed by the longitudinal axis P_bP_f and P_fP_d ($-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$).
- **Look-Behind Tracking:** The leading vehicle moves backwards behind the following vehicle as in Fig. 2. In this case, the tracked point is the front point P_d of the leading vehicle. The relative distance between two vehicles is measured by the length $d > 0$ of P_bP_d and the relative orientation angle ϕ formed by the longitudinal axis P_bP_f and P_bP_d ($\frac{\pi}{2} \leq \phi \leq \frac{3\pi}{2}$).

For both cases, the point P_d of the leading vehicle is related to the point P_b of the following vehicle by the same formula as follows

$$P_d = z_d = \begin{bmatrix} x + \frac{1+f}{2}a \cos \theta + d \cos(\theta + \phi) \\ y + \frac{1+f}{2}a \sin \theta + d \sin(\theta + \phi) \end{bmatrix} \quad (3)$$

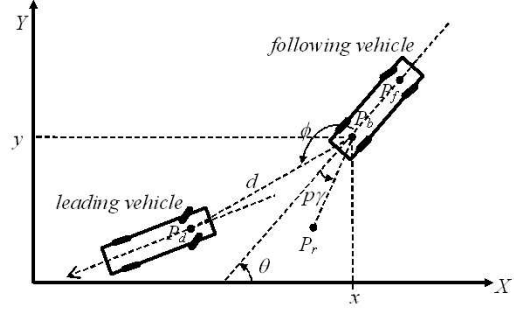


Fig. 2. Look-behind tracking configuration

where

$$f = \begin{cases} 1 & \text{for look-ahead tracking} \\ -1 & \text{for look-behind tracking} \end{cases}$$

The platoon dynamics is defined as the collective motions of both vehicles as well as the relative distance and orientation angle between two vehicles. A good performance of platoon maneuvers is ensured only if the following vehicle can follow the leading vehicle with a specified spacing and a tracking error bounded or going to zero.

III. PLATOONING TRACKING CONTROL DESIGN

The objective of tracking control is to drive the following vehicle automatically to follow and maintain a predetermined distance from the leading vehicle. In this section, a nonlinear output feedback controller is developed. The idea is motivated by the way a human driver does in platooning [16]. The driver keeps his eye focus on the leading vehicle with a comfortable distance and drives the vehicle so that his eye focus point is able to follow the leading vehicle with the same distance. Thus, the platoon system is formed and maintained.

The focus point P_r is defined l meters away from the vehicle with l being the expected spacing between two vehicles ($P_fP_r = l$ in look-ahead tracking and $P_bP_r = l$ in look-behind tracking). The focus point P_r has a directional angle defined by the longitudinal axis of the vehicle (P_bP_f) and the focus line P_fP_r (in the look-ahead tracking as shown in Fig. 1) or P_bP_d (in the look-behind tracking as shown in Fig. 2). It is p times as much as the steering angle γ . This focus point P_r can be expressed with respect to P_b as follows

$$P_r = z = \begin{bmatrix} x + \frac{1+f}{2}a \cos \theta + l \cos(\theta + p\gamma) \\ y + \frac{1+f}{2}a \sin \theta + l \sin(\theta + p\gamma) \end{bmatrix} \quad (4)$$

where l and p are two system parameters that will affect the performance of the platoon system. The platooning is then measured by the output tracking error as follows

$$\tilde{z} = z - z_d = R^T(\theta) \begin{bmatrix} l \cos p\gamma - d \cos \phi \\ l \sin p\gamma - d \sin \phi \end{bmatrix} \quad (5)$$

where $R(\theta)$ defines a standard rotation matrix of θ as follows

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

The target performance of the platooning system can be specified by a second-order system for the closed-loop output tracking error

$$\ddot{z} + 2\xi\lambda\dot{z} + \lambda^2z = 0 \quad (6)$$

where the natural frequency $\lambda > 0$ and the damping ratio $0 < \xi \leq 1$ can be specified for a desired target performance.

Equation (6) can be rewritten equivalently as

$$\dot{z} = \dot{z}_d - 2\xi\lambda\dot{z} - \lambda^2z \quad (7)$$

Taking time derivative of (4) with using the dynamics model (1) produces

$$\dot{z} = \frac{\partial z}{\partial q} \dot{q} = \frac{\partial z}{\partial q} G\mu = E(\theta, \gamma)\mu = R^T(\theta)\bar{E}(\gamma)\mu \quad (8)$$

where the decoupling matrix

$$\bar{E}(\gamma) = \begin{bmatrix} 1 - \frac{l}{a} \tan \gamma \sin p\gamma & -lp \sin p\gamma \\ \tan \gamma \left(\frac{1+f}{2} + \frac{l}{a} \cos p\gamma \right) & lp \cos p\gamma \end{bmatrix} \quad (9)$$

Taking the differentiation of (8) again yields

$$\dot{z} = \frac{\partial(E\mu)}{\partial q} G\mu + E\dot{\mu} = R^T(\theta)\bar{H}(\gamma)\mu + R^T(\theta)\bar{E}(\gamma)u \quad (10)$$

where $\bar{H}(\gamma)$ is a nonlinear vector function of γ .

$$\bar{H}(\gamma) = \begin{bmatrix} \bar{H}_{11} & \bar{H}_{12} \\ \bar{H}_{21} & \bar{H}_{22} \end{bmatrix}$$

and

$$\begin{aligned} \bar{H}_{11} &= -\tan \gamma \left\{ \frac{1+f}{2} \dot{\theta} + \frac{l}{a} (\dot{\theta} + p\omega) \cos p\gamma \right\} \\ \bar{H}_{12} &= -\frac{l}{a} \frac{v}{\cos^2 \gamma} \sin p\gamma - lp(\dot{\theta} + p\omega) \cos p\gamma \\ \bar{H}_{21} &= \dot{\theta} - \frac{l}{a} (\dot{\theta} + p\omega) \tan \gamma \sin p\gamma \\ \bar{H}_{22} &= \left(\frac{1+f}{2} + \frac{l}{a} \cos p\gamma \right) \frac{v}{\cos^2 \gamma} - lp(\dot{\theta} + p\omega) \sin p\gamma \end{aligned}$$

Similarly, the second-order time differentiation of (3) is computed

$$\begin{aligned} \ddot{z}_d &= R^T(\theta) \left\{ \begin{bmatrix} \dot{v} - \frac{1+f}{2} a \dot{\theta}^2 \\ v \dot{\theta} + \frac{1+f}{2} a \ddot{\theta} \end{bmatrix} \right. \\ &\quad \left. + R^T(\phi) \begin{bmatrix} \left\{ \ddot{d} - d(\dot{\theta} + \dot{\phi})^2 \right\} \\ \left\{ 2\dot{d}(\dot{\theta} + \dot{\phi}) + d(\ddot{\theta} + \ddot{\phi}) \right\} \end{bmatrix} \right\} \quad (11) \end{aligned}$$

where

$$\begin{aligned} \dot{\theta} &= \frac{v}{a} \tan \gamma \\ \ddot{\theta} &= \frac{\dot{v}}{a} \tan \gamma + \frac{v\omega}{a \cos^2 \gamma} \end{aligned}$$

And the time differentiation of (5) is

$$\dot{\dot{z}} = R^T(\theta) \begin{bmatrix} -l(\dot{\theta} + p\omega) \sin p\gamma - \dot{d} \cos \phi + d(\dot{\theta} + \dot{\phi}) \sin \phi \\ l(\dot{\theta} + p\omega) \cos p\gamma - \dot{d} \sin \phi - d(\dot{\theta} + \dot{\phi}) \cos \phi \end{bmatrix} \quad (12)$$

By substituting (5), (10), (11) and (12) into (7) and rearranging the equation, (7) becomes

$$R^T(\theta)\bar{E}(\gamma)u = R^T(\theta)\bar{F}(v, \dot{v}, \gamma, \omega, d, \dot{d}, \ddot{d}, \phi, \dot{\phi}, \ddot{\phi}) \quad (13)$$

where \bar{F} is a nonlinear vector function

$$\begin{aligned} \bar{F} &= \begin{bmatrix} \dot{v} \\ \frac{1+f}{2} \dot{v} \tan \gamma \end{bmatrix} \\ &\quad + lR^T(p\gamma) \begin{bmatrix} (\dot{\theta} + p\omega)^2 - \lambda^2 \\ -\frac{v\omega}{a \cos^2 \gamma} - 2\xi\lambda(\dot{\theta} + p\omega) \end{bmatrix} \\ &\quad + R^T(\phi) \begin{bmatrix} \ddot{d} + 2\xi\lambda\dot{d} + \lambda^2d - d(\dot{\theta} + \dot{\phi})^2 \\ d(\ddot{\theta} + \ddot{\phi}) + 2(\dot{d} + \xi\lambda d)(\dot{\theta} + \dot{\phi}) \end{bmatrix} \quad (14) \end{aligned}$$

Multiplying the orthogonal matrix $R(\theta)$ to both sides of (13) produces

$$\bar{E}(\gamma)u = \bar{F}(v, \dot{v}, \gamma, \omega, d, \dot{d}, \ddot{d}, \phi, \dot{\phi}, \ddot{\phi}) \quad (15)$$

Note that (15) is independent of all the three global parameters (x, y, θ) .

Suppose $(v, \dot{v}, \gamma, \omega, d, \dot{d}, \ddot{d}, \phi, \dot{\phi}, \ddot{\phi})$ are measurable (by using Inertial Navigation System onboard and range sensors such as cameras and laser scanners), the control inputs u can be generated from (13) if and only if the matrix \bar{E} is non-singular:

$$\det(\bar{E}) = lp \cos p\gamma + \frac{1+f}{2} lp \tan \gamma \sin p\gamma \neq 0 \quad (16)$$

Since f only takes two values -1 or 1 , we have the following equality

$$\frac{1+f}{2} \tan \gamma = \tan \left(\frac{1+f}{2} \gamma \right) \quad (17)$$

then (16) becomes

$$\begin{aligned} \det(\bar{E}) &= lp \cos p\gamma + lp \tan \left(\frac{1+f}{2} \gamma \right) \sin p\gamma \\ &= lp \frac{\cos \left[\left(p - \frac{1+f}{2} \right) \gamma \right]}{\cos \left(\frac{1+f}{2} \gamma \right)} \neq 0 \quad (18) \end{aligned}$$

Condition (18) is applicable to both tracking cases. For practical wheeled mobile robots, the steering γ is restricted $|\gamma| \leq \gamma_{max} < \frac{\pi}{2}$. Thus, condition (16) is equivalent to

$$\begin{cases} lp \neq 0 & \text{(a)} \\ \left| p - \frac{1+f}{2} \right| < \frac{\pi}{2\gamma_{max}} & \text{(b)} \end{cases} \quad (19)$$

Condition (19-a) requires (i) ($l \neq 0$), i.e., the focus point P_r can not be fixed at the front center point P_f of the following vehicle in look-ahead tracking or at the back point P_b in look-behind tracking; and (ii) ($p \neq 0$), i.e., P_r can not be fixed on the longitudinal center axis. Condition (19-b) indicates that the selectable range of parameter p is bounded. Furthermore, [17] has shown that the parameters should be chosen ($l > 0, p > 0$) for look-ahead tracking, and ($l < 0, p < 0$) for look-behind tracking. Therefore, (19) leads to

- For look-ahead tracking:

$$l > 0 \text{ and } 0 < p < 1 + \frac{\pi}{2\gamma_{max}} \quad (20)$$

- For look-behind tracking:

$$l < 0 \text{ and } -\frac{\pi}{2\gamma_{max}} < p < 0 \quad (21)$$

With proper selection of parameters l and p to ensure matrix \bar{E} 's regularity, the resultant nonlinear controller can be obtained as follows

$$u = \bar{E}^{-1}(\gamma)\bar{F} \left(v, \dot{v}, \gamma, \omega, d, \dot{d}, \ddot{d}, \phi, \dot{\phi}, \ddot{\phi} \right) \quad (22)$$

This control input u is highly nonlinear and depends solely on parameters in vehicular coordinates $(v, \dot{v}, \gamma, \omega, d, \dot{d}, \ddot{d}, \phi, \dot{\phi}, \ddot{\phi})$. That is, no global measurements such as generalized position and orientation (x, y, θ) are required. It is a unified controller for both tracking cases: *look-ahead* and *look-behind*, with only one switching parameter f being of value 1 or -1, respectively. It also combines the action for driving and steering into the same controller.

IV. SIMULATION STUDIES

Simulations are carried out to verify the effectiveness of the proposed control scheme. Different sets of design parameters (l, p) are tested for both look-ahead and look-behind tracking control.

A. Look-ahead tracking control

In this situation, the leading vehicle is l meters in front of the following vehicle. The leading vehicle is maneuvered with the desired steering angle and acceleration profiles as shown in Fig. 3 and Fig. 4, respectively.

Parameters λ and ξ of the target performance are fixed at 1 and 0.5, respectively. We focus on the effects of system parameters l and p .

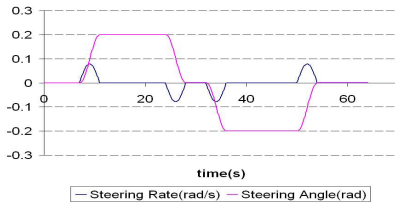


Fig. 3. Desired steering angle γ_d and steering rate ω_d for the leading vehicle

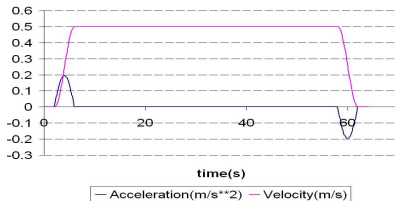


Fig. 4. Desired acceleration and velocity for the leading vehicle

Suppose the steering angle of the vehicle is limited $|\gamma| \leq \gamma_{max} = \frac{\pi}{9}$. Then, the condition (20) becomes

$$l > 0 \text{ and } 0 < p < 5.5 \quad (23)$$

1) Influence of parameter p :

Parameter l is fixed at a desired space of $2.5m$, while different values of p are tested in the range of $(0, 5.5)$. Simulation results, in Figs. 7-9, show that the following vehicle successfully follows the leading vehicle. The tracking errors are small along the straight path. However, during turns, the value of parameter p clearly has an effect on the tracking performance. With smaller values of p , the following vehicle tries to cut corner to catch up with the leading vehicle. In contrast, larger values of p result in overshooting before turning. The best of the tested values is $p = 2$. Obviously, p value should not be near its upper and lower limits in (23) to avoid matrix \bar{E} being singular.

2) Influence of parameter l :

Parameter p is fixed at 2 and parameter l is now changed from $1m$ to $7m$. The tracking results are shown in Figs. 10-12. It is clear that the controller can drive the vehicle to follow the leading vehicle. It also shows that the tracking errors are the same along the straight path. When turning, the tracking error is bigger with large values of l . And the following vehicle's trajectory is closer to that of the leading vehicle when l is small.

B. Simulation study for look-behind tracking

In this situation, the leading vehicle is placed l meters behind the following vehicle. Similarly to the look-ahead tracking situation, the trajectory of the leading vehicle is generated with the desired steering angle and acceleration as shown in Fig. 5 and Fig. 6, respectively.

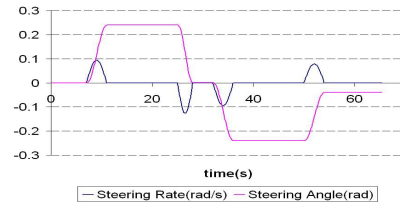


Fig. 5. Desired steering angle γ_d and steering rate ω_d of the leading vehicle

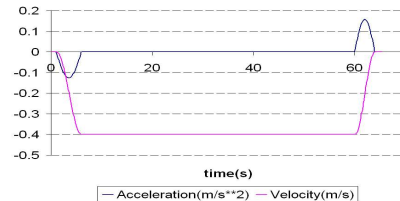


Fig. 6. Desired acceleration and velocity of the leading vehicle

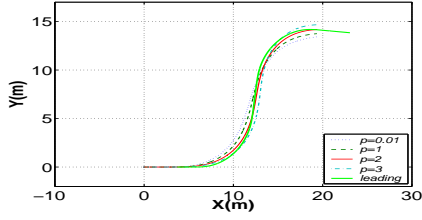


Fig. 7. Vehicle trajectories (x, y) with different values of p

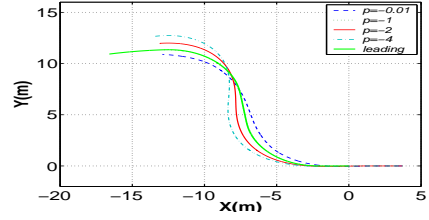


Fig. 13. Vehicle trajectories (x, y) with different values of p

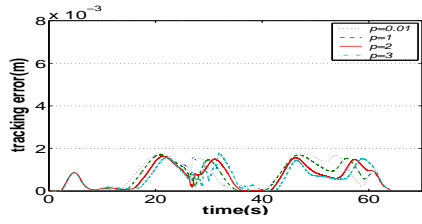


Fig. 8. Tracking errors $\|\tilde{z}\|$ for different values of p

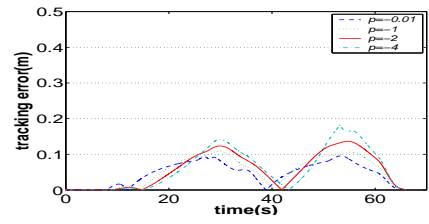


Fig. 14. Tracking errors $\|\tilde{z}\|$ for different values of p

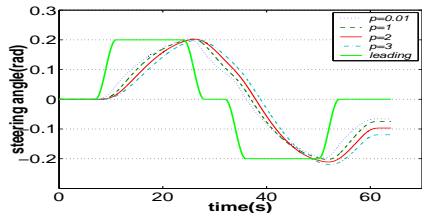


Fig. 9. Steering angle γ for different values of p

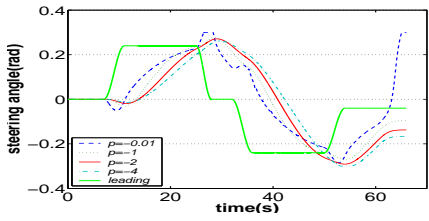


Fig. 15. Steering angle γ for different values of p

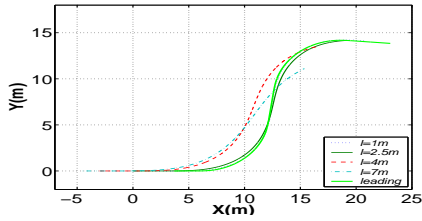


Fig. 10. Vehicle trajectories (x, y) with different values of l

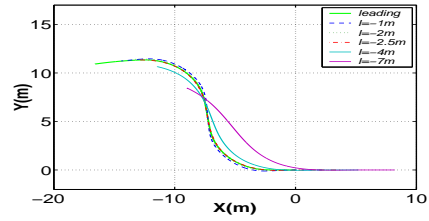


Fig. 16. Vehicle trajectories (x, y) with different values of l

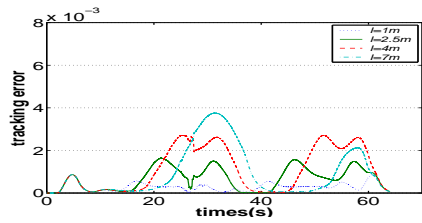


Fig. 11. Tracking errors $\|\tilde{z}\|$ for different values of l

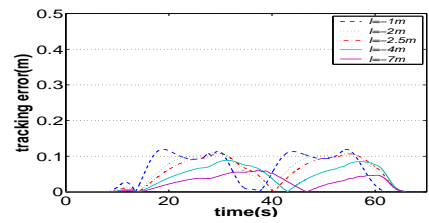


Fig. 17. Tracking errors $\|\tilde{z}\|$ for different values of l

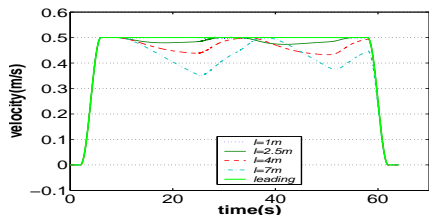


Fig. 12. Vehicle velocity v for different values of l

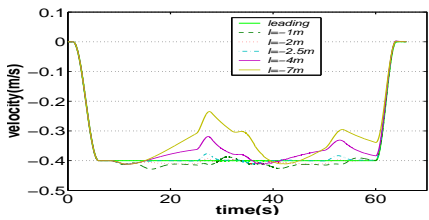


Fig. 18. Vehicle velocity v for different values of l

For the controller (22), parameters λ and ξ are both fixed at 1.

Suppose the steering angle of the vehicle is limited $|\gamma| \leq \gamma_{max} = \frac{\pi}{9}$. Then, the condition (21) becomes

$$l < 0 \text{ and } -4.5 < p < 0 \quad (24)$$

1) *Influence of parameter p:*

Parameter l is fixed at $l = -2.5m$. Several values of p in range $(-4.5, 0)$ are tested with results shown in Figs. 13-15.

The tracking is successful. It is shown that along the straight path, the tracking is extremely good. When the leading vehicle turns, the following vehicle's steering turns to the opposite site for a while before turns back to the same direction. It is because the tracked point is the front point of the leading vehicle, not the rear point which is considered as the reference point of the leading vehicle. Thus, when the leading vehicle is about to turn left, for instance, its front point will move to the right side.

The range of p is divided into two parts. With values of p in $(-1 < p < 0)$, the controller will drive the following vehicle to cut short the corner to catch up with the leading vehicle. On the other hand, the values of p in $(-4.5 < p < -1)$ will give overshoots when the leading vehicle turns. The tracking performance is the best when $p = -1$, as shown in Fig. 13.

2) *Influence of parameter l:*

Now parameter p is fixed at $p = -1$. Several values of l in range $(-1, -7)m$ are tested with results shown in Figs. 16-18. The tracking performance in the look-behind case is also influenced by parameter l the same way as that in look-ahead tracking. With bigger value of l , the tracking vehicle will take more direct "shortcuts" to follow the leading vehicle. The tracking error, however, is slightly smaller when a larger value is chosen for parameter l . That is because in this case the error is caused by the swing of the desired point P_d , that is defined at the steering wheel side instead of the fixed side. In other words, the desired point P_d looks more dynamic than the reference point of the leading vehicle from the following vehicle's side.

V. CONCLUSIONS

The proposed unified platoon tracking controller is suitable for both look-ahead and look-behind maneuvers. It is an integrated action for both velocity control and steering control. The design and implementation of the controller depend on the selection of 4 parameters. The target performance is determined by the natural frequency λ and the damping ratio ξ . The non-singularity condition also introduces two platoon parameters (l, p) . Extensive simulation results show that proper selections of these two system parameters are important to the performance of the platooning tracking system.

REFERENCES

- [1] D. Yanakiev and I. Kanellakopoulos, "Nonlinear spacing policies for automated heavy-duty vehicles," *IEEE Trans. Veh. Technol.*, vol. 47, no. 4, pp. 1365–1377, Nov 1998.
- [2] T. S. No, K.-T. Chong, and D.-H. Roh, "A Lyapunov function approach to longitudinal control of vehicles in a platoon," *IEEE Trans. Veh. Technol.*, vol. 50, no. 3, pp. 116–124, Jan 2001.
- [3] P. Daviet and M. Parent, "Longitudinal and lateral servoing of vehicles in a platoon," in *Proc. IEEE Intelligent Vehicles Symposium (IV'96)*, Tokyo, Japan, Sep 1996, pp. 41–46.
- [4] S. C. Warnick and A. A. Rodriguez, "A systematic antiwindup strategy and the longitudinal control of a platoon of vehicles with control saturations," *IEEE Trans. Veh. Technol.*, vol. 49, no. 3, pp. 1006–1016, May 2000.
- [5] J. K. Hedrick, M. Tomizuka, and P. Varaiya, "Control issues in automated highway systems," *IEEE Control Syst. Mag.*, vol. 14, no. 6, pp. 21–32, Dec 1994.
- [6] D. Swaroop, J. K. Hedrick, and S. Choi, "Direct adaptive longitudinal control of vehicle platoons," *IEEE Trans. Veh. Technol.*, vol. 50, no. 1, pp. 150–161, Jan 2001.
- [7] S. K. Gehrig and F. J. Stein, "A trajectory-based approach for the lateral control of a car following system," in *Proc. IEEE International Conference on Systems, Man, and Cybernetics (SMC'98)*, vol. 4, San Diego, CA, Oct 1998, pp. 3596–3601.
- [8] S. Kato, S. Tsugawa, K. Toduka, T. Matsui, and H. Fujii, "Vehicle control algorithms for cooperative driving with automated vehicles and intervehicle communications," *IEEE Trans. Intell. Transport. Syst.*, vol. 3, no. 3, p. 155, 2002.
- [9] R. White and M. Tomizuka, "Autonomous following lateral control of heavy vehicles using laser scanning radar," in *Proc. American Control Conference (ACC2001)*, vol. 3, Virginia, USA, Jun 2001, pp. 2333–2338.
- [10] S. Patwardhan, H.-S. Tan, J. Guldner, and M. Tomizuka, "Lane following during backward driving for front wheel steered," in *Proc. American Control Conference (ACC'97)*, vol. 5, New Mexico, USA, Jun 1997, pp. 3348–3353.
- [11] D.-H. Kim and J.-H. Oh, "Experiments of backward tracking control for trailer system," in *Proc. IEEE International Conference on Robotics and Automation (ICRA'99)*, vol. 1, May 1999, pp. 19–22.
- [12] C. Altafini, A. Speranzon, and B. Wahlberg, "A feedback control scheme for reversing a truck and trailer vehicle," *IEEE Trans. Robot. Automat.*, vol. 17, no. 6, pp. 915–922, Dec 2001.
- [13] M. Saeki, "Path following control of articulated vehicle by backward driving," in *Proc. IEEE International Conference on Control Applications (CCA 2002)*, vol. 1, Sep 2002, pp. 421–426.
- [14] G. Campion, G. Bastin, and B. D'Andréa-Novell, "Structural properties and classification of kinematic and dynamic models of wheeled mobile robots," *IEEE Trans. Robot. Automat.*, vol. 12, no. 1, pp. 47–62, Feb 1996.
- [15] D. Wang and G. Xu, "Full state tracking and internal dynamics of nonholonomic wheeled mobile robots," in *Proc. American Control Conference (ACC2000)*, Jun 2000, pp. 3274–3277.
- [16] D. Wang, G. Xu, and M. Pham, "Look-ahead/behind control of a car-like vehicle," in *Proc. Robotics and Mechatronics Congress*, Singapore, Jun 2001, pp. 101–106.
- [17] D. Wang and G. Xu, "Full state tracking and internal dynamics of nonholonomic wheeled mobile robots," *IEEE/ASME Trans. Mechatron., Focused Section on Advances in Robot Dynamics and Control*, vol. 8, no. 2, pp. 203–214, 2003.