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Observer Design for a Towed Seismic Cable

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***Abstract*— In this paper, we consider observer design for a towed seismic cable, attached to a depth controller at one end and with a prescribed motion at the other. Based on a finite number of measurements, a globally asymptotically stable observer is proposed. Locally, the proposed observer is exponentially stable. The stability analysis of the observer is based on Lyapunov theory. The existence and uniqueness of the solutions of the observer are based on semigroup theory. The implementation of the observer requires at least two measurements: The position of the cable, and the slope of the cable at the location of the depth controller.**

I. Introduction

In surveying of hydrocarbon reservoirs under seabed, offshore towing of seismic sensor arrays is extensively used. These operations are accomplished by a towed cable configuration which consists of a negatively buoyant lead-in cable attached to a towing vessel at one end, and to a neutrally buoyant cable called streamer at the other end. To detect the reflected acoustic pulses from a towed acoustic source, hydrophones are embedded in the streamers. To obtain better stability and controllability of the motion of the streamers, a surface tail buoy is attached to the downstream end of the streamers. The length of the lead-in cable varies typically between 200 m to 400 m, and the length of the streamers is normally between 3000 m and 6000 m. In special cases, the length of the streamers can be as long as 10000 m. A typical towed cable configuration is shown in Figure 1.

The dynamics of towed cables have been studied by several authors [6],[9],[13],[14],[15],[19] and references therein. Most of these consider towing of neutrally buoyant cable with a free downstream end. Paidoussis [14],[15] derived the equations of motion for the transverse displacement of a towed neutrally buoyant element. Dowling [6] determined the form of the linear displacements of a neutrally buoyant cylinder. Triantafyllou and Chryssostomidis [19] developed a procedure for calculating the response of a towed array of seismic hydrophones when a harmonic excitation was applied at the upstream end.

Knowledge of the accurate position of the whole cable is of great importance, not only for precise maneuvering, monitoring and control-related concerns, but also because accurate knowledge of the configuration of the cable is the first step of other tasks, e.g. to prevent the streamers from getting tangled during surveying operations. More importantly, this knowledge can be used to depress the influence of the signal noise of the recorded seismic data, which leads to a better interpretation of recorded data and thus to a more accurate depiction of the sea floor [17]. However, the configuration of the cable can not be measured directly. Since this requires continuous distribution of sensors along the whole cable, which is not possible in practice. Typically, there is a finite number of sensors collocated with the depth controllers. So to get information on the position of unmeasured points on the cable, we need an observer. The main purpose of an observer is to estimate unmeasured physical quantities, e.g. position, velocity etc., based on available measurements. In this paper, we present one such observer.

Observer design based on Lyapunov theory is well known and widely used for both linear systems and nonlinear systems. Balas [1] considered observer design for linear flexible structures described by FEM. Demetriou [4] presented a method for construction of observer for linear second order lumped and distributed parameter systems using parameter-dependent Lyapunov functions. Kristiansen [10] applied *contraction* theory [11] in observer design for a class of linear distributed parameter systems. The structural damping forces were included in the last two cases. Hence, exponentially stable observers can easily be designed.

In this paper, we consider observer design for a part of a cable towing configuration shown in Figure 1. We consider a streamer, which is attached to a depth controller at one end, and has a prescribed motion at the other (Figure 2). The equation of motion for the seismic streamer is in the form of a nonlinear partial differential equation (PDE), adopted from [14]. The dynamics of the depth controller are described by an ordinary differential equation (ODE). Based on a finite number of measurements, a globally asymptotically stable observer is designed. Locally, the proposed observer is exponentially stable. The stability analysis of the observer is based on *Lyapunov theory*. The existence and uniqueness of the solutions are established using *semigroup theory*.

The paper is organized as follows. First, a model of the towed seismic cable is presented. Then, a globally asymptotically stable observer is designed. After that, we consider the existence and uniqueness (and stability) of the solutions of the observer.

II. System Model

Neglecting the bending stiffness and the material damping, the equation of motion for a neutral, flexible cylinder with small transverse excitations in the axial flow is given

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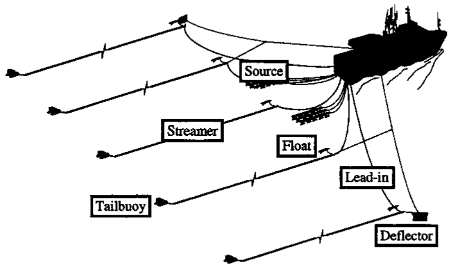

Fig. 1. A typical towed cable configuration.

by the nonlinear model [14],

$$\begin{aligned} mw_{tt} &= (Tw_x)_x - (2aUw_t)_x \\ &\quad -F_n(\alpha)\ ,\ x \in]0, L[&& (1) \\ Mw_{tt} &= -Tw_x + 2aUw_t + \tau,\ x = L && (2) \\ w &= W_0 + A_0 \sin(\omega t)\ ,\ x = 0 && (3) \\ w(x,0) &= w_0(x), w_t(x,0) = v_0(x), x \in [0, L] && (4) \end{aligned}$$

where

$$\begin{aligned} m &= \rho_c + a = \frac{\rho_w \pi d^2}{4} \cdot (C_m + C_a) && (5) \\ T(x) &= T_0 + F_t \cdot (L - x) - Bh - aU^2 && (6) \\ B &= \rho_w g \pi \frac{d^2}{4} && (7) \\ F_t &= \frac{1}{2} \rho_w \pi d C_f U^2 && (8) \\ \beta &= \frac{1}{U} w_t + w_x && (9) \\ \alpha &= \arctan \beta && (10) \\ F_n(\alpha) &= F_{n1} \sin\alpha + F_{n2} \sin\alpha \left|\sin\alpha\right| && (11) \end{aligned}$$

where the nonlinearity is due to $F_n(\alpha)$. Here, L is the length of the cable, m is the sum of the structural and added mass per unit length, ρ_c is the density of the cable per unit length, a is added mass per unit length, C_m is the structural mass coefficient, C_a is the added mass coefficient, ρ_w is the density of the ambient water, d is the diameter of the cable, $T(x)$ is the effective tension of the cable at x, T_0 is the aft tension, B is the buoyancy force per unit length, h is the distance to the free surface, g is the gravitation constant, F_t is the tangential hydrodynamic force per unit length of the cable, C_f is the friction coefficient, α is the angle of attach, F_n is the normal hydrodynamic force per unit length of the cable, $U > 0$ is the tow speed of the towing vessel, W_0 is the initial position of the tow point, A_0 is the amplitude of the prescribed harmonic motion at the upstream end ($x = 0$), M is the total mass of the depth controller at the downstream end ($x = L$), $w(x, t)$ is the vertical displacement of the cable at point x and time t, $\tau : \mathbb{R}^+ \to \mathbb{R}$ is the control force generated by the depth controller at $x = L$ (see Figure 2). The subscripts $(\cdot)_t$ and $(\cdot)_x$ denote the partial derivative respect to t and x, respectively, and w_0 and v_0 are some given initial conditions of the cable.

For further discussion on this topic and the model (1)-(3), see [6],[9],[14],[13] and the references therein.

Remark 1: The model (1)-(3) has been considered by numerous authors, e.g. [9],[14],[13]. One important term has been excluded, namely, the instantaneous tangential dragforce. This was pointed out by e.g. [6],[15]. The complete model will therefore be considered in future work.

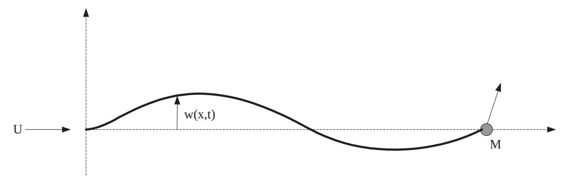

Fig. 2. Towed seismic cable.

A. Assumptions and Problem Statements

Without loss of generality, we assume that

Assumption 1: The tension $T(x)$ satisfies the inequality

$$0 < T_{\min} \leq T(x) < \infty,\ x \in [0, L]$$

and $T(x) \gg \{a, m, U\}$, $\forall x \in [0, L]$.

Assumption 1 does not cause any restriction in practice. Typically, the tow speed of the towing vessel U is between 0 m/s and 2.5 m/s, the added mass $a \leq 1$ and the structural mass $m \leq 1$, while the value of the tension $T(x)$ is of order 10^3. Assumption 1 is thus reasonable.

Assume that the system (1)-(3) is perfectly known. We consider the following two problems:

Problem 1: Design an observer for the system based on the measurements: $w(L, t)$ and $w_x(L, t)$, $t \geq 0$.

Problem 2: In addition to the measurements in Problem 1, we have $N - 1$ measurements of the position of the cable and $N - 1$ measurements of the slope of the cable at x_i, where $x_i \in]0, L[$, $i = 1, ..., N - 1$. Design an observer for the system based on these measurements.

III. Observer Design

Let us start with Problem 1. We denote the measurements as follows: $y_1 = w|_L$ and $y_2 = w_x|_L$. Using the *coordinate* *error feedback* [11], we propose the observer

$$\bar{w}_t = \hat{w}_t - \frac{1}{M}[H_1 y_1 + H_2 T y_2] \cdot \delta(x-L), \quad x \in]0,L] \tag{12}$$

$$m\bar{w}_{tt} = (T\hat{w}_x)_x - (2aU\hat{w}_t)_x - F_n(\hat{\alpha}), \quad x \in]0,L[\tag{13}$$

$$M\bar{w}_{tt} = -T\hat{w}_x + 2aU\hat{w}_t + \tau - H_1\hat{w}_t - H_2 T\hat{w}_{xt}, \; x = L \tag{14}$$

$$\hat{w} = W_0 + A_0\sin(\omega t), \; x = 0 \tag{15}$$

$$\hat{w}(x,0) = \hat{w}_0(x), \hat{w}_t(x,0) = \hat{v}_0(x), \; x \in [0,L] \tag{16}$$

where $\hat{w}$ is the observed value of w, $\delta(\cdot)$ denotes the Dirac delta function, $F_n(\cdot)$ is given by (11), $\hat{\alpha}$ is similarly defined as α, H_1 and H_2 are positive observer gains, and $\hat{w}_0$ and $\hat{v}_0$ are initial conditions of the observer. The observer (12)-(15) is inspired from the work by [3],[12]. Combination of (12) and (13)-(14) gives the observer dynamics

$$m\hat{w}_{tt} = (T\hat{w}_x)_x - (2aU\hat{w}_t)_x - F_n(\hat{\alpha}), \quad x \in]0,L[\tag{17}$$

$$M\hat{w}_{tt}|_L = -T\hat{w}_x + 2aU\hat{w}_t - H_1\tilde{w}_t - H_2 T\tilde{w}_{xt} + \tau \tag{18}$$

$$\hat{w}|_0 = W_0 + A_0\sin(\omega t) \tag{19}$$

where $\tilde{w} = \hat{w} - w$ denotes the observer error. Subtracting (17)-(19) by (1)-(3) gives the observer error dynamics

$$m\tilde{w}_{tt} = (T\tilde{w}_x)_x - (2aU\tilde{w}_t)_x - \tilde{F}_n, \quad x \in]0,L[\tag{20}$$

$$M\tilde{w}_{tt}|_L = -T\tilde{w}_x - \tilde{H}_1\tilde{w}_t - H_2 T\tilde{w}_{xt} \tag{21}$$

$$\tilde{w}|_0 = 0 \tag{22}$$

where

$$\tilde{H}_1 = H_1 - 2aU \tag{23}$$

$$\tilde{F}_n = F_n(\hat{\alpha}) - F_n(\alpha) \tag{24}$$

To analyse (20)-(22), we define the Lyapunov function

$$E(t, \tilde{w}_t, \tilde{w}_x) = E_1 + E_2 + E_3 \tag{25}$$

where

$$E_1 = \frac{1}{2}\frac{(M\,\tilde{w}_t|_L + H_2\,T\tilde{w}_x|_L)^2}{M + \tilde{H}_1 H_2}$$

$$E_2 = \frac{1}{2}\int_0^L \left[T(x) + 2aU^2 - mU^2\right]\tilde{w}_x^2\,dx$$

$$E_3 = \frac{1}{2}mU^2\int_0^L \left(\frac{1}{U}\tilde{w}_t + \tilde{w}_x\right)^2 dx$$

Note that due to Assumption 1, $E_2 > 0$, $\forall\tilde{w}_x \neq 0$. Differentiation of E_1, E_2 and E_3 with respect to time along the solution trajectories of (20)-(22) gives

$$\dot{E}_1 = -\tilde{w}_t T\tilde{w}_x|_L - \frac{M\tilde{H}_1(\tilde{w}_t|_L)^2}{M + \tilde{H}_1 H_2} - \frac{H_2(T\tilde{w}_x|_L)^2}{M + \tilde{H}_1 H_2}$$

$$\dot{E}_2 = \int_0^L \left[T(x) + 2aU^2 - mU^2\right]\tilde{w}_x\tilde{w}_{xt}\,dx$$

$$\dot{E}_3 = -\int_0^L \left[T(x) + 2aU^2 - mU^2\right]\tilde{w}_x\tilde{w}_{xt}\,dx + \tilde{w}_t T\tilde{w}_x|_L + \left[\frac{1}{2}mU - aU\right](\tilde{w}_t|_L)^2 + \frac{U}{2}\,\tilde{w}_x T\tilde{w}_x|_L - \frac{U}{2}\,\tilde{w}_x T\tilde{w}_x|_0 + \frac{U}{2}\int_0^L \frac{dT}{dx}\tilde{w}_x^2\,dx - U\int_0^L \left(\frac{1}{U}\tilde{w}_t + \tilde{w}_x\right)\tilde{F}_n dx$$

where integration by parts has been successively applied. Note that $dT/dx = -F_t < 0$. Hence, we get

$$\begin{aligned}\dot{E} = &-\left[\frac{\tilde{H}_1 M}{M + \tilde{H}_1 H_2} - \frac{1}{2}mU\right](\tilde{w}_t|_L)^2 \\ &-\left[\frac{H_2}{M + \tilde{H}_1 H_2} - \frac{U}{2\,T|_L}\right](T\tilde{w}_x|_L)^2 \\ &-aU(\tilde{w}_t|_L)^2 - \frac{U}{2}\,\tilde{w}_x T\tilde{w}_x|_0 - \frac{F_t U}{2}\int_0^L \tilde{w}_x^2\,dx \\ &-F_{n2}U\int_0^L (\hat{\beta} - \beta)(\sin\hat{\alpha}\,|\sin\hat{\alpha}| - \sin\alpha\,|\sin\alpha|)\,dx \\ &-F_{n1}U\int_0^L (\hat{\beta} - \beta)(\sin\hat{\alpha} - \sin\alpha)\,dx\end{aligned} \tag{26}$$

where α and β are given by (10)-(9), respectively, and $\hat{\alpha}$ and $\hat{\beta}$ are the corresponding observed values. It is straightforward to verify that the last two terms of (26) are non-positive for $\forall\beta, \hat{\beta} \in \mathbb{R}$. Let the observer gains $\tilde{H}_1$ and H_2 be selected according to

$$\frac{\tilde{H}_1 M}{M + \tilde{H}_1 H_2} \geq \frac{1}{2}mU \tag{27}$$

$$\frac{H_2}{M + \tilde{H}_1 H_2} > \frac{U}{2\,T|_L} \tag{28}$$

Hence, the first two terms of (26) become negative. Thus, we get

$$\dot{E}(t, \tilde{w}_x, \tilde{w}_t) < 0, \quad t \geq 0, \forall\{\tilde{w}_x, \tilde{w}_t\} \neq 0$$

According to Lyapunov's stability theorem, the origin $(\tilde{w}_x, \tilde{w}_t) = (0,0)$ of (20)-(22) is globally asymptotically stable. Using the inequality

$$\tilde{w}(x,t)^2 \leq L\int_0^L \tilde{w}_x(x,t)^2 dx, \; x \in [0,L]$$

which a simple consequence of integration by parts and the boundary condition (22), it follows that $\tilde{w} = 0$ is asymptotically stable. Hence, the origin $(\tilde{w}, \tilde{w}_x, \tilde{w}_t) = (0,0,0)$ of (20)-(22) is globally asymptotically stable.

The angle of attach α can locally be approximated as

$$\sin\alpha \approx \beta = \frac{1}{U} w_t + w_x \tag{29}$$

Using (29) in (26) gives

$$\begin{aligned}\dot{E} &= -\left[\frac{\tilde{H}_1 M}{M+\tilde{H}_1 H_2} - \frac{1}{2} mU\right] \left(\tilde{w}_t|_L\right)^2 \\ &\quad -\left[\frac{H_2}{M+\tilde{H}_1 H_2} - \frac{U}{2\, T|_L}\right] \left(T\tilde{w}_x|_L\right)^2 \\ &\quad -aU\left(\tilde{w}_t|_L\right)^2 - \frac{U}{2}\ \tilde{w}_x T\tilde{w}_x|_0 - \frac{F_t U}{2}\int_0^L \tilde{w}_x^2 dx \\ &\quad -F_{n1}U\int_0^L \left(\frac{1}{U}\tilde{w}_t + \tilde{w}_x\right)^2 dx \\ &\quad -F_{n2}U\int_0^L \left(\hat{\beta}-\beta\right)\left(\hat{\beta}\left|\hat{\beta}\right| - \beta\left|\beta\right|\right) dx\end{aligned}$$

Thus, there exists a constant $C > 0$ such that

$$\dot{E}(t) \leq -C\cdot E(t), \quad t \geq 0$$

Hence, the origin $(\tilde{w}, \tilde{w}_x, \tilde{w}_t) = (0,0,0)$ of (20)-(22) is locally exponentially stable. Problem 1 is thus solved.

Theorem 1: Consider the observer error dynamics (20)-(22). Assume that τ is chosen such that the solutions of the closed loop system of (1)-(3) are well-posed. Let the observer gains H_1 and H_2 be chosen according to (27)-(28). Then the origin of (20)-(22) is globally asymptotically stable and locally exponentially stable.

Consider now the problem 2. Let the measurements be denoted as follows: $y_1 = w|_L$, $y_2 = w_x|_L$, and $y_{i1} = w|_{x_i}$, $y_{i2} = w_x|_{x_i}$, $i = 1, ..., N-1$. Then, we replace equations (12)-(14) by

$$\begin{aligned}\bar{w}_t &= \hat{w}_t - \sum_{i=1}^{N-1} \frac{h_i y_{i1}}{m}\cdot\delta\left(x - x_i\right) \\ &\quad -\frac{H_1 y_1 + H_2 T y_2}{M}\cdot\delta\left(x-L\right), \quad x\in\left]0,L\right]\end{aligned} \tag{30}$$

$$\begin{aligned}m\bar{w}_{tt} &= -\sum_{i=1}^{N-1} h_i\left[\hat{w}_t + U\left(\hat{w}_x - y_{i2}\right)\right]\cdot\delta\left(x-x_i\right) \\ &\quad +\left(T\hat{w}_x\right)_x - \left(2aU\hat{w}_t\right)_x - F_n\left(\hat{\alpha}\right), \ x\in\left]0,L\right[\end{aligned} \tag{31}$$

$$M\bar{w}_{tt}|_L = -T\hat{w}_x + 2aU\hat{w}_t - H_1\hat{w}_t - H_2 T\hat{w}_{xt} \tag{32}$$

where $\delta(\cdot)$ again denotes the Dirac delta function, and H_1, H_2 and h_i are positive observer gains. Using (30) in (31) and (32), and subtracting the resulting equations by (1)-(3), we get the observer error dynamics

$$\begin{aligned}m\tilde{w}_{tt} &= -\sum_{i=1}^{N-1} h_i\cdot\left[\tilde{w}_t + U\tilde{w}_x\right]\cdot\delta\left(x-x_i\right) \\ &\quad +\left(T\tilde{w}_x\right)_x - \left(2aU\tilde{w}_t\right)_x - \tilde{F}_n\ , \quad x\in\left]0,L\right[\end{aligned} \tag{33}$$

$$M\tilde{w}_{tt}|_L = -T\tilde{w}_x - \tilde{H}_1\tilde{w}_t - H_2 T\tilde{w}_{xt} \tag{34}$$

$$\tilde{w}|_0 = 0 \tag{35}$$

where $\tilde{H}_1$ and $\tilde{F}_n(\cdot)$ are given by (23)-(24), respectively.

To analyse (33)-(35), we use the Lyapunov function (25). It can be verified that the time derivative of the Lyapunov function (25) along the solution trajectories of (33)-(35) is

$$\dot{E} = (*) - \sum_{i=1}^{N-1} h_i\cdot\left(\tilde{w}_t|_{x_i} + U\ \tilde{w}_x|_{x_i}\right)^2$$

where $(*)$ is the expression given in equation (26). Let $h_i > 0$, and choose H_1 and H_2 according to (27)-(28). Hence, we obtain

$$\dot{E} < 0, \quad t\geq 0, \forall\left\{\tilde{w}_x, \tilde{w}_t\right\}\neq 0$$

This shows that the origin $(\tilde{w}_x, \tilde{w}_t) = (0,0)$ of (33)-(35) is globally asymptotically stable. Moreover, by replacing the locally approximation (29), the locally exponentially stability of the origin $(\tilde{w}_x, \tilde{w}_t) = (0,0)$ is established. Problem 2 is thus solved.

Lemma 2: Consider the observer error dynamics (33)-(35). Assume that τ is chosen such that the solutions of the closed loop system of (1)-(3) are well-posed. Let $h_i > 0$, $i = 1,\ldots,N-1$, and choose H_1 and H_2 according to (27)-(28). Then the origin of (33)-(35) is globally asymptotically stable and locally exponentially stable.

IV. Existence and Uniqueness of Solutions

In this section, we consider existence and uniqueness of the solutions of the proposed observers. This will be done by using semigroup theory. First, we assume that the control law τ is chosen such that the solutions of the closed loop system of (1)-(3) are well-posed. Hence, by showing the existence and uniqueness of the solutions of the observer error dynamics (20)-(22) and (33)-(35), we get the existence and uniqueness of the solutions of the proposed observers.

Consider the observer error dynamics (20)-(22). Let $\tilde{w}(x,t)$ be a regular solution of (20)-(22) and define $\mathbf{w}(t) = (\tilde{w}(\cdot,t), \tilde{w}_t(\cdot,t), \tilde{w}_t(L,t))$. Equations (20)-(22) can be compactly written as

$$\dot{\mathbf{w}} = \mathbf{A}\mathbf{w} + \mathbf{F}(\mathbf{w}), \quad \mathbf{w}_0 \in H \tag{36}$$

where

$$\begin{aligned}\mathbf{A}\mathbf{w} &= \left[\tilde{w}_2,\ \frac{\left(T\tilde{w}_{1,x}\right)_x - 2aU\tilde{w}_{2,x}}{m}, -\frac{T\tilde{w}_{1,x}|_L}{M}\right. \\ &\qquad \left. - \frac{\tilde{H}_1\tilde{w}_3 + H_2\ T\tilde{w}_{2,x}|_L}{M}\right]^T, \ \mathbf{w}\in D(\mathbf{A}) \\ \mathbf{F}(\mathbf{w}) &= -\left[0, \frac{F_n(\hat{\alpha}) - F_n(\alpha)}{m}, 0\right]^T, \ \mathbf{w}\in H\end{aligned}$$

where $\mathbf{w} = (\tilde{w}_1, \tilde{w}_2, \tilde{w}_3)$, $\mathbf{w}_0$ denotes the initial condition of the problem, and $\tilde{w}_{j,x}$ denotes $\partial\tilde{w}_j/\partial x$, $j = 1, 2$. For notational simplicity, the symbol ˜ will be left out in the following, i.e. $\tilde{w}$ will be replaced by w. The space H and the domain of operator $\mathbf{A}$ are defined as

$$\begin{aligned}H &= H_0^1(\Omega)\times L_2(\Omega)\times\mathbb{R} \\ D(\mathbf{A}) &= \left\{\mathbf{w}\in H_0^2(\Omega)\times H_0^1(\Omega)\times\mathbb{R}\right|\ w_2|_L = w_3\right\}\end{aligned}$$

where $\Omega =]0, L[$ and

$$L_2(\Omega) = \left\{ f \mid \int_\Omega |f(x)|^2 \ dx < \infty \right\}$$
$$H_0^k(\Omega) = \left\{ f \mid f, \ f', \ldots, \ f^{(k)} \in L_2(\Omega); f(0) = 0 \right\}$$

In H, we define the inner-product

$$\langle \mathbf{w}, \mathbf{v} \rangle_H = \int_0^L \left(T + 2aU^2 - mU^2 \right) w_{1,x} \cdot v_{1,x} \ dx$$
$$+ mU^2 \int_0^L \left(\frac{w_2}{U} + w_{1,x} \right) \left(\frac{v_2}{U} + v_{1,x} \right) dx$$
$$+ \frac{\left(Mw_3 + H_2 \ Tw_{1,x}|_L \right) \left(Mv_3 + H_2 \ Tv_{1,x}|_L \right)}{M + \tilde{H}_1 H_2}$$

where $\mathbf{v} = (v_1, v_2, v_3) \in H$ and $\mathbf{w} = (w_1, w_2, w_3) \in H$. It can be verified that $(H, \langle \cdot, \cdot \rangle_H)$ is a Hilbert space. Note that the Lyapunov function (25) can be expressed as

$$E(t) = \frac{1}{2} \langle \mathbf{w}(t), \mathbf{w}(t) \rangle_H = \frac{1}{2} \|\mathbf{w}(t)\|_H^2 \qquad (37)$$

Theorem 3: Let H_1 and H_2 be given by (27)-(28). Then, $\mathbf{A}$ generates a C_0-semigroup $\left\{ e^{\mathbf{A}t} \right\}_{t \geq 0}$ of contractions on H, and this semigroup is exponentially stable.

Proof: With *Lumer-Phillips theorem* (see e.g.[16]), it is straightforward to show that $\mathbf{A}$ generates a C_0-semigroup of contractions on H. Note that $\mathbf{A}$ is dissipative since,

$$\dot{E} = \langle \mathbf{w}, \mathbf{A}\mathbf{w} \rangle_H \leq 0, \quad t \geq 0 \qquad (38)$$

The last part of the theorem follows from the energy multipliers method and (Th. 4.1, p. 116, [16]). Consider the following functional

$$W(t) = t \cdot E(t) + W_1(t) \qquad (39)$$

where

$$W_1(t) = 2m \int_0^L x w_t w_x \ dx + 2aU \int_0^L x w_x w_x \ dx$$

and $E(t)$ is given by (37). It is straightforward to verify that there is a constant $C > 0$ such that the following holds

$$(t - C) \cdot E(t) \leq W(t) \leq (t + C) \cdot E(t), \ t \geq 0 \qquad (40)$$

Moreover, it can be shown that the time derivative of (39) along the solution trajectories of (36) (with $\mathbf{F} = 0$) satisfies

$$\dot{W}(t) \leq 0, \ t \geq t_1 \qquad (41)$$

for sufficiently large time $t_1 > 0$. By combining (40)-(41) and (38), we get

$$E(t) \leq \frac{t_1 + C}{t - C} E(0), \ t_{\max} > \max(C, t_1)$$

i.e. $E(t)$ decays as $O(1/t)$, $t > t_{\max}$. Since, $\|\mathbf{w}(t)\|_H^2 = \left\| e^{\mathbf{A}t} \mathbf{w}_0 \right\|_H^2 = 2E(t)$, it follows that the solution of (36) is bounded for $\forall t \geq 0$, and decays as $O(1/\sqrt{t})$, $t > t_{\max}$. Hence, there exists an integer $p > 1$ such that

$$\int_0^\infty \left\| e^{\mathbf{A}t} \mathbf{w}(0) \right\|_H^p \ dt < \infty, \ \forall \mathbf{w}_0 \in D(\mathbf{A})$$

According to (Th. 4.1, p. 116, [16]), there exist $M \geq 1$ and $\mu > 0$ such that

$$\left\| e^{\mathbf{A}t} \right\|_H \leq M e^{-\mu t}, \ t \geq 0, \forall \mathbf{w}_0 \in D(\mathbf{A}) \qquad (42)$$

Since $\overline{D(\mathbf{A})} = H$, the inequality (42) holds for $\forall \mathbf{w}_0 \in H$. ■

To show the existence, uniqueness and stability of the solutions of the abstract problem (36), we need the following lemma:

Lemma 4: The operator $\mathbf{A}^{-1}$ exists and is compact; and $(\lambda \mathbf{I} - \mathbf{A})^{-1}$ is compact for $\forall \lambda > 0$. Moreover, the non-linear operator $\mathbf{F}(\cdot)$ is monotone, dissipative, and globally Lipschitz on H.

Theorem 5: The abstract problem (36) has a unique mild solution,

$$\mathbf{w}(t; \mathbf{w}_0) = Z(t)\mathbf{w}_0 = e^{\mathbf{A}t}\mathbf{w}_0 + \int_0^t e^{\mathbf{A}(t-\xi)} \mathbf{F}(Z(\xi)\mathbf{w}_0) d\xi, \ t \geq 0 \qquad (43)$$

for $\forall \mathbf{w}_0 \in H$, where $\left\{ e^{\mathbf{A}t} \right\}_{t \geq 0}$ is the linear C_0-semigroup generated by $\mathbf{A}$ and $\{Z(t)\}_{t \geq 0}$ is the nonlinear semigroup generated by the sum operator $\mathbf{A} + \mathbf{F}(\cdot)$. The unique mild solution (43) tends asymptotically to zero as $t \to \infty$ for $\forall \mathbf{w}_0 \in H$.

Proof: Since $\mathbf{F}$ is dissipative and globally Lipschitz, it follows from (Th. 4.2, [18]) that (36) has a unique weak solution $\mathbf{w}(t; \mathbf{w}_0)$ defined on $\mathbb{R}^+$ for every $\mathbf{w}_0 \in H$.

To prove that the solution (43) tends asymptotically to zero as $t \to \infty$ for $\forall \mathbf{w}_0 \in H$, we apply (Th. 4, [5]). This theorem requires that: i) $\mathcal{A} \triangleq -(\mathbf{A} + \mathbf{F})$ is maximal monotone and densely defined on H; ii) $0 \in R(\mathcal{A})$, where $R(\mathcal{A})$ is the range of the operator $\mathcal{A}$; iii) the operator $(\lambda \mathbf{I} + \mathcal{A})^{-1}$ is compact for some $\lambda > 0$, i.e. the solution of (36) is precompact for some $\lambda > 0$.

Since $\mathbf{A}$ is an infinitesimal generator of a C_0-semigroup of contractions, it follows from *Hiller-Yosida* theorem (see e.g. [16]) that $-\mathbf{A}$ is maximal monotone. From Lemma 4, it follows that $\mathbf{F}$ is continuous monotone. Thus, by (Th. 1 and Prop. 3.15, [20]) $\mathcal{A}$ defined on H is also maximal monotone on $D(\mathcal{A}) = D(\mathbf{A})$. Moreover, since $\overline{D(\mathbf{A})} = H$ (since $\mathbf{A}$ generates a C_0-semigroup of contractions on H), it follows that $\overline{D(\mathcal{A})} = H$. The condition i) is thus satisfied.

It can be verified that $0 \in R(\mathcal{A})$, which can simplest be shown by using a contradiction argument, i.e. assume that $0 \notin R(\mathcal{A})$, and show that this assumption is not true.

Using the fact that $\mathbf{A}$ and $\mathbf{F}$ are dissipative, it can be shown that $(\lambda \mathbf{I} + \mathcal{A})^{-1}$ is a compact operator for all $\lambda > 0$.

By (Th. 4, [5]), $-\mathcal{A}$ generates a nonlinear semigroup $\{Z(t)\}_{t \geq 0}$ on H. The unique mild solution of the problem (36) is given by (43). According to (Th. 4, [5]), the mild solution (43) tends to a compact subset $\Omega(\mathbf{w}_0) \subset \{\mathbf{w} \in H | \ \|\mathbf{w}\|_H \leq \|\mathbf{w}_0\|_H \}$, as $t \to \infty$. This subset is $Z(t)$-invariant. So to prove that every solution of (36) converges to zero as $t \to \infty$, we have to show that $\Omega(\mathbf{w}_0) = \{0\}$.

Assume that $\mathbf{w}_0 \in D(\mathbf{A})$. In this case $\Omega(\mathbf{w}_0) \subset D(\mathbf{A})$, and the solution $\mathbf{w}(t)$ is a strong one, i.e. $\mathbf{w}(t; \mathbf{w}_0)$ is differentiable and $\mathbf{w}(t; \mathbf{w}_0)$ lies in the set $D(\mathbf{A})$ for $\forall t \geq 0$. Let $\mathbf{v} \in \Omega(\mathbf{w}_0)$. Since $\Omega(\mathbf{w}_0)$ is $Z(t)$-invariant, the following holds

$$v(t) = \frac{1}{2} \|Z(t)\mathbf{v}\|_H^2 = \frac{1}{2} \|\mathbf{v}\|_H^2, \quad \forall t \in \mathbb{R}^+$$

Differentiation of $v(t)$ along the solutions of (36) gives

$$\begin{aligned} \frac{d}{dt} v(t) &= \langle Z(t)\mathbf{v}, \mathbf{A}Z(t)\mathbf{v} \rangle_H + \langle Z(t)\mathbf{v}, \mathbf{F}(Z(t)\mathbf{v}) \rangle_H \\ &= 0 \end{aligned}$$

Using the dissipativity of $\mathbf{A}$, we get

$$\mathbf{F}(Z(t)\mathbf{v}) = 0, \quad \forall t \in \mathbb{R}^+$$

From (43), we get $Z(t)\mathbf{v} = e^{\mathbf{A}t}\mathbf{v}$ (i.e. $\Omega(\mathbf{w}_0)$ is $e^{\mathbf{A}t}$-invariant). Now, using the fact that $Z(t)\mathbf{v} = e^{\mathbf{A}t}\mathbf{v}$ converges exponentially to zero (from Theorem 3), we get $\mathbf{v} = 0$ and $\Omega(\mathbf{w}_0) = \{0\}$ for $\forall \mathbf{w}_0 \in D(\mathbf{A})$. Further, since $\overline{D(\mathbf{A})} = H$ and $\{Z(t)\}_{t \geq 0}$ is a contraction semigroup on H, it follows that $\Omega(\mathbf{w}_0) = \{0\}$ for $\forall \mathbf{w}_0 \in H$. ■

Finally, let us consider the existence, uniqueness and stability of the solutions of the observer error dynamics (33)-(35). Again, let $\tilde{w}(x,t)$ be a regular solution of (33)-(35) and define $\mathbf{w}(t) = (\tilde{w}(\cdot,t), \tilde{w}_t(\cdot,t), \tilde{w}_t(L,t))$. Equations (33)-(35) can be compactly written as

$$\dot{\mathbf{w}} = \mathbf{A}\mathbf{w} + \mathbf{B}\mathbf{w} + \mathbf{F}(\mathbf{w}), \;\; \mathbf{w}_0 \in H \tag{44}$$

where

$$\mathbf{B}\mathbf{w} = \left[0, -\sum_{i=1}^{n-1} \frac{h_i}{m} \left(\tilde{w}_2 + U\tilde{w}_{1,x} \right) \cdot \delta(x - x_i), 0 \right]^T$$

for $\forall \mathbf{w} \in H$. It is straightforward to show that $\mathbf{B}$ is monotone, dissipative and globally Lipschitz. Hence, we have the following result.

Lemma 6: The abstract problem (44) has a unique mild solution,

$$\begin{aligned} \mathbf{w}(t; \mathbf{w}_0) &= Z(t)\mathbf{w}_0 = e^{\mathbf{A}t}\mathbf{w}_0 \\ &+ \int_0^t e^{\mathbf{A}(t-\xi)} \mathbf{B} Z(\xi)\mathbf{w}_0 \; d\xi \\ &+ \int_0^t e^{\mathbf{A}(t-\xi)} \mathbf{F}(Z(\xi)\mathbf{w}_0) \; d\xi, \;\; t \geq 0 \end{aligned} \tag{45}$$

for $\forall \mathbf{w}_0 \in H$. The unique mild solution (45) tends asymptotically to zero for $\forall \mathbf{w}_0 \in H$.

V. CONCLUSION

In this paper, we studied observer design for a seismic streamer, attached to a depth controller at the downstream end and with a prescribed motion at the upstream end. Based on a finite number of measurements, a globally asymptotically stable observer is designed. Locally, the proposed observer is exponentially stable. The stability analysis of the observer is based on Lyapunov theory. The existence and uniqueness of the solutions of the observer are based on semigroup theory. The stability analysis of the observer based on semigroup theory has also been discussed. There is agreement between the results obtained by Lyapunov theory and semigroup theory.

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