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Linear Estimator for Road Departure Warning Systems

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Abstract - **Most single vehicle road departure accidents in America occur due to either loss of control or road/lane departure caused by speeding or driver inattentiveness. Many active safety systems currently in use or under development are aimed at preventing accidents either by countering vehicle instability or by trying to prevent road departure. In either case these active safety systems need clean and reliable real-time vehicle dynamics variables to accurately assess the threat levels. Since it is not always feasible to measure the required information, estimation techniques are commonly used to fill in the gap. In this paper, we developed a Kalman filter to estimate two vehicle handling variables that are costly to measure—lateral velocity and relative heading angle. It is shown that it is critical to first obtain an accurate estimation of road super-elevation (bank angle) before those two states can be accurately estimated. By properly assigning the Kalman filter observer gains, we achieved robust estimation performance across a wide array of uncertain conditions. The work reported here will be used to support the data analysis for the Road Departure Crash Warning (RDCW) Field Operational Test, to be carried out at the University of Michigan Transportation Research Institute (UMTRI) .**

I. INTRODUCTION

Road departure crashes are one of the leading causes of highway fatalities in the U.S. USDOT defines an 'off-roadway crash' as one in which the first harmful event occurs off the roadway. An estimated 6.1 million police reported crashes involving light vehicles occurred in the U.S. in 2000, causing approximately 41,000 fatalities [1]. Of these crashes, the USDOT estimates that approx. 1.12 million are associated with a single vehicle departing the roadway due to inadvertent lateral drifting or loss of control and account for about 41% of all fatalities. Thus road departure accidents not only form a significant portion of all police reported accidents but are also involve more fatalities than other accidents.

The University of Michigan Transportation Research Institute (UMTRI) is conducting the Road Departure Crash Warning Field Operational Test as part of the Intelligent Vehicle Initiative [2]. This is a three year project to develop and operationally test a system to warn drivers when they may be drifting inadvertently from path of safe travel, or when they are driving too fast for an approaching curve. The purpose of this project is to anticipate the suitability and driver acceptance of widespread deployment of such a system on the U.S. passenger vehicle fleet.

Most active safety systems need to detect impending road departure and/or vehicle instability. Furthermore, to prevent accidents, they need measurements (or estimates) of various handling characteristics like lateral velocity and yaw rate.

A classic metric for determining impending road departure is Time to Lane Crossing (TLC). TLC may be understood as the time until the vehicle CG will cross either edge of the roadway under certain conditions [3]. This definition of TLC in its truest sense usually results in extremely large value of TLC for most driving conditions. Therefore, a modified TLC definition is used in this paper:

$$\dot{y} = u \cdot \Psi + v$$
$$TLC = \frac{y'}{\dot{y}} \quad \dot{y} \neq 0 \tag{1}$$

where y is the vehicle lateral displacement from the center of the road, u is the vehicle forward speed, Ψ is the relative heading angle and v is the vehicle lateral speed. y' is defined as $y \pm lanewidth/2$, and depending on the sign of $\dot{y}$, only the one that is reducing to zero is used in the calculations. Another approximation of TLC can also be obtained by integrating a vehicle dynamics model (e.g., a bicycle model) to find the future values of lateral displacement and heading angle and comparing those to the road profile ahead, assuming that steering remains constant.

No matter which version of TLC is used, in order to calculate TLC accurately, one needs to have both vehicle lateral speed and relative heading angle. This is the main reason why we focus on the estimation of these two parameters in this paper.

Lateral velocity is also the most important vehicle dynamics variable for active safety applications that are aimed at preventing or countering vehicle instability. Examples include Vehicle Stability Control [4], Steer by Wire [5], Four Wheel Steering [6], etc. Lateral velocity is difficult to measure and requires expensive sensors like the optical flow sensor [7]. However, even these expensive

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sensors are sometime vulnerable to environment lighting variation, dirt/mud, and vehicle vertical/roll/pitch motions. An alternative approach is to estimate lateral speed using other vehicle handling variables that are measured, like yaw rate and steering angle.

From Eq.(1), another important variable in calculating $\dot{y}$ is the relative heading angle, or heading angle "error". This variable is defined as the angle between the vehicle x-axis and the tangent to the road. This term represents the effect of vehicle misalignment and contributes to $\dot{y}$ significantly, especially at high forward speed.

In order to estimate both lateral speed and relative heading angle, vehicle lateral displacement needs to be measured. For the Road Departure Crash Warning Field operational test, the vehicles are equipped with an advanced camera system to measure lateral displacement (or lane position). The main purpose of this paper is thus to develop an algorithm to estimate lateral velocity and heading angle by using lateral displacement in addition to the commonly available measurements, including vehicle forward speed, yaw rate, front axle steering angle and vehicle lateral acceleration. More specifically, we use the Kalman filter technique to develop an estimation scheme that performs the following operations:

1. Filter the measured data.
2. Estimate lateral velocity and heading angle
3. Provide predicted vehicle measurements (like lateral displacement) when sensor measurements are temporarily unavailable.

The measured data available is lane position, yaw rate, lateral acceleration, steering angle and forward velocity. Also available is an estimate of road curvature. Measurements are assumed to be always available. Dropout in camera-based measurements is an important issue, and in the future, a Kalman filter would be useful in managing this.

The Kalman filter developed here will be used within a road departure warning system for online calculation of TLC as a part of the crash warning algorithm. Although the RDCW onboard system does not use the techniques shown here, it may be used off line as a tool to refine all the real time driving data that is collected throughout the field operational test.

The proposed Kalman filter is extensively tested using Carsim™ [8] vehicle dynamics simulation software that has a 27 degree of freedom (DOF), nonlinear vehicle dynamics model. The results obtained show that a Kalman filter can be used to filter the measured parameters and estimate lateral velocity & heading angle quite well.

II. Model Development

As stated earlier, the vehicle response used in this paper are obtained from CarSim, a commercial software. The classic two DOF bicycle model is used for the development of the Kalman filter, which is known to predict vehicle lateral behavior reasonably well when lateral acceleration is below 0.3g.

$$\begin{bmatrix} \dot{y} \\ \dot{v} \\ \dot{\Psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 & u & 0 \\ 0 & \frac{-(C_{\alpha f}+C_{\alpha r})}{mu} & 0 & \frac{bC_{\alpha r}-aC_{\alpha f}}{mu}-u \\ 0 & 0 & 0 & 1 \\ 0 & \frac{bC_{\alpha r}-aC_{\alpha f}}{I_z} & 0 & \frac{-(a^2C_{\alpha f}+b^2C_{\alpha r})}{I_z u} \end{bmatrix} \begin{bmatrix} y \\ v \\ \Psi \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{aC_{\alpha f}}{I_z} \end{bmatrix} \delta_f + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} r_d + \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix} \gamma \quad (2)$$

where: (values show nominal Carsim™ values)

a : Distance of front axle from CG (1.014 m)

b : Distance of rear axle from CG (1.676 m)

u : Longitudinal velocity (20 m/s)

I_z : Vehicle yaw moment of inertia (2741.9 kg/m^2)

m : Mass of vehicle (1707 kg)

y : Lateral displacement (from center of road) (m)

v : Lateral velocity (m/s)

Ψ : Relative heading angle (rad)

r : Yaw rate (rad/sec)

$C_{\alpha f}$: Front axle cornering stiffness (2450.9 N/rad)

$C_{\alpha r}$: Rear axle cornering stiffness (1820.6 N/rad)

δ_f : Front wheel steering wheel angle (rad)

g : Acceleration due to gravity (m/sec^2)

γ : Super elevation angle (rad)

r_d : Desired yaw rate (rad/sec), yaw rate needed to follow the centre of the lane

$r_d = \frac{u}{R}$, R : Road radius of curvature (m)

III. Kalman filter development

The various signals available are classified as measured variables to be filtered (outputs of the bicycle model), the measured inputs to the model & the unknown disturbances. The measured outputs are lateral displacement & yaw rate. The input is the steering angle measurement. The unknown disturbance is bank angle.

For the purpose of building the Kalman Filter, all measurements (lateral displacement, yaw rate & steering) are assumed to be corrupted by properly scaled white noise. Further to accurately model the system, the yaw rate measurement has a random walk error (r_{bias}), the derivative of which is white noise modeled as a unknown disturbance. To account for error in the bicycle model a unknown steering disturbance (δ_{dist}) is modeled as white noise. Thus the three unknown disturbances (yaw bias, steering disturbance and bank angle) are modeled as having first derivatives as white noise.

The Kalman filter model finally obtained is:

$$\begin{bmatrix}\dot{y}\\ \dot{v}\\ \dot{\Psi}\\ \dot{r}\\ \dot{r}_{bias}\\ \dot{\gamma}\end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & u & 0 & 0 & 0\\ 0 & \frac{-(C_{af}+C_{ar})}{mu} & 0 & \frac{bC_{ar}-aC_{af}}{mu}-u & 0 & g\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & \frac{bC_{ar}-aC_{af}}{I_z} & 0 & \frac{-(a^2C_{af}+b^2C_{ar})}{I_z u} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}}^{A}\begin{bmatrix}y\\ v\\ \Psi\\ r\\ r_{bias}\\ \gamma\end{bmatrix}$$

$$+\overbrace{\begin{bmatrix}0 & 0\\ \frac{C_{af}}{m} & 0\\ 0 & -1\\ \frac{aC_{af}}{I_z} & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}}^{B}\begin{bmatrix}\delta_{act}\\ r_d\end{bmatrix}+\overbrace{\begin{bmatrix}0 & 0 & 0\\ \frac{C_{af}}{m} & 0 & 0\\ 0 & 0 & 0\\ \frac{aC_{af}}{I_z} & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}}^{G}\begin{bmatrix}\delta_{dist}\\ \dot{\gamma}\\ \dot{r}_{bias}\end{bmatrix} \tag{3}$$

$$\begin{bmatrix}y_m\\ r_m\end{bmatrix}=\overbrace{\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 1 & 0\end{bmatrix}}^{C}\begin{bmatrix}y\\ v\\ \Psi\\ r\\ r_{bias}\\ \gamma\end{bmatrix}+\overbrace{\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}}^{D}\begin{bmatrix}\delta_{act}\\ r_d\end{bmatrix}+\overbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}^{H}\begin{bmatrix}\delta_{dist}\\ \dot{\gamma}\\ \dot{r}_{bias}\end{bmatrix}+[\text{noise}]$$

This is the format used for Kalman filter applications, along with the noise covariance matrices:

$Q = diag\left[\sigma^2_{\delta d},\sigma^2_{\dot{\gamma}},\sigma^2_{\dot{r}bias}\right]$: disturbance covariance martix

$\sigma^2_{\delta d}$:covariance of steering disturbace

$\sigma^2_{\dot{\gamma}}$: covariance of $\dot{\gamma}$

$\sigma^2_{\dot{r}bias}$: covariance of $\dot{r}_{bias}$

$R = diag\left[\sigma^2_{ym},\sigma^2_{rm}\right]$: measurement covariance matrix

σ^2_{ym} : covariance of lateral displacement measurent

σ^2_{rm} : covariance of yaw rate measurement

For the purpose of this paper, we assumed a measurement noise having standard deviation of 0.1m for lateral displacement and 1 deg/sec for yaw rate. Also a steering disturbance having standard deviation of 0.1 deg at the wheel is assumed.

The parameters for the 2 DOF bicycle model were chosen to match those for a ‘Big Car’ in Carsim™ shown above.

IV. Simulation with Bicycle Model

This Kalman filter was initially tested using data obtained from simulations of the 2 DOF bicycle model itself. The data obtained from the bicycle model was made noisy by adding artificial white noise, disturbances, drift and bias as shown in Fig. 1

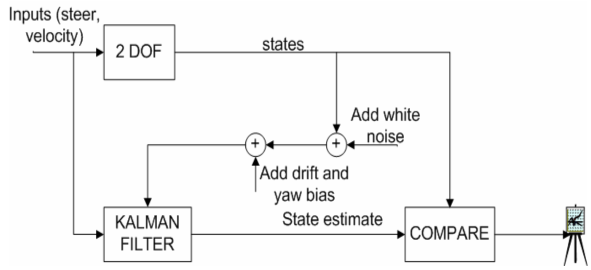


Fig. 1. Kalman filter Setup

The results (not shown for lack of space) show that the Kalman filter setup is correct and may now be tested in more realistic situations. One interesting observation is the correlation between the error in lateral velocity estimate and the road inclination angle estimate. Accurate estimation of lateral velocity depends on good estimate of super elevation.

V. Simulation with Carsim™

Next the Kalman filter was tested using data obtained from Carsim™. Carsim™ is a 27 DOF, non-linear vehicle dynamics simulation software package that produces results more reliable and closer to real life data than the 2 DOF linear bicycle model. All further testing of the Kalman filter is carried out on a ‘virtual test track’ designed for this paper in Carsim™. The ‘virtual test track’ consists of a road with two ninety degree turns at different radii of curvature and corresponding super elevation angles as shown in Fig. 2. It also consists of a straight section with a step change in superelevation angle and a maneuver with two double lane changes. Carsim™ incorporates the MacAdam driver model to generate the steering angle based on road profile. [9]

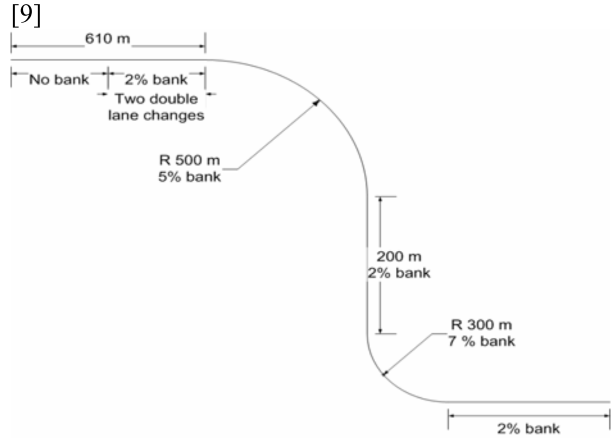


Fig. 2. ‘Virtual Test’ course

The bank angles were designed from the AASHTO ‘Green Book’ Highway Design manual so as to be close to real life conditions [10].

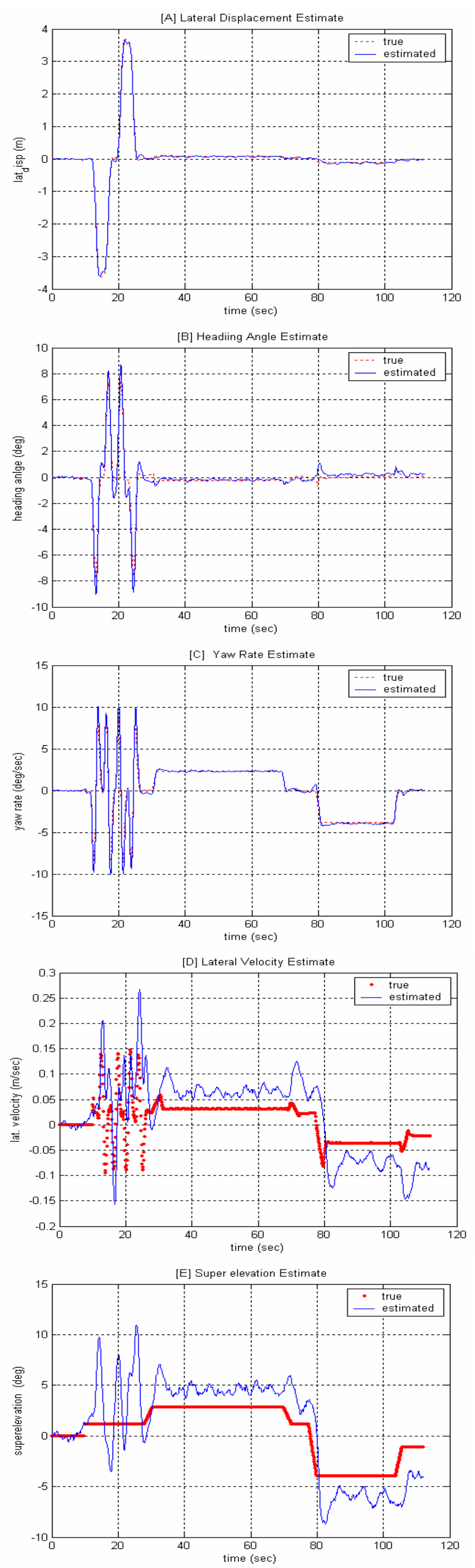


Fig. 3.: Carsim™ Simulation

The plots in Fig. 3 show that the estimation accuracy obtained from Carsim™ is not as good as the results obtained from the 2 DOF simulations. As seen from the plots, the estimation of lateral velocity is very poor when we use data from Carsim™. As mentioned earlier there is a close correlation between lateral velocity estimate and inclination angle estimate. From Fig. 3(e) it is clear that the estimate of super elevation angle is quite inaccurate. This also causes inaccuracy in lateral velocity estimation. The reasons for this are the disparity in the 2DOF model and the 27DOF non-linear model in Carsim™, which can be understood as model uncertainty (or disturbance). Also in this filter, super elevation angle is modeled as a disturbance whose first derivative is white noise; this is not really the case in real conditions.

To get accurate estimation of lateral velocity it is quite clear that a good estimate of super elevation angle is required. For this purpose we adopted a super elevation estimation algorithm proposed by H.E. Tseng [11].

VI. Bank Angle Estimation Algorithm

To improve lateral velocity estimation, we have adopted a superelevation estimation algorithm proposed in [11]. The bank angle estimation algorithm uses the 2DOF bicycle model and measurements of yaw rate, steering and lateral acceleration to dynamically estimate the super elevation angle

Step 1: Derive transfer functions:

$H_{\delta \to r}$: Transfer function from steering to yaw rate

$H_{\delta \to ay}$: Transfer function from steering to lareral accelration

$H_{\varphi \to r}$: Transfer function from super elevation to yaw rate

$H_{\varphi \to ay}$: Transfer function from super elevation to lateral accelration.

Step 2: Find three different estimates of super elevation angle based on yaw rate, lateral acceleration, and lateral velocity:

$$\sin\hat{\varphi}_r = H_{\varphi \to r}^{-1}(r - H_{\delta \to r}\,\delta)$$

$$\sin\hat{\varphi}_a = H_{\varphi \to ay}^{-1}(a_y - H_{\delta \to ay}\,\delta) \qquad (4)$$

$$\sin\hat{\varphi}_v = (u \cdot r - a_y)/g$$

where δ, a_y, and r are measured steer, lateral acceleration, and yaw rate, respectively.

Step 3: Create a combined estimate to reduce sensitivity to errors in the individual estimates:

$$\sin\hat{\varphi} = \sin\hat{\varphi}_v \cdot \max[0, 1 - |DFC| - |d\sin\hat{\varphi}_v / dta| \qquad (5)$$

where

$$DFC = H_{\varphi \to ay}(\sin\hat{\varphi}_a - \sin\hat{\varphi}_v) + u \cdot H_{\varphi \to r}(\sin\hat{\varphi}_r - \sin\hat{\varphi}_v)$$

VII. Kalman Filter with Bank Angle Estimation

The Kalman filter can be modified such that the estimated bank angle (γ) becomes a known input, we still assume some unknown error γ_{dist} in the inclination estimate, that becomes part of disturbance. And the co-variance matrices become:

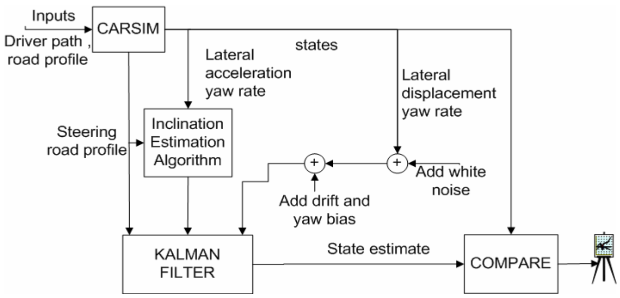


Fig. 4. Kalman filter with bank angle estimation

$R = diag\left[\sigma^2_{ym}, \sigma^2_{rm}\right]$: measurement covariance matrix

σ^2_{ym} :covarinace of lateral displacement measurent

σ^2_{rm} : covarinace of yaw rate measurement

$Q = diag\left[\sigma^2_{\delta d}, \sigma^2_{\gamma\ dist}, \sigma^2_{\dot{r}bias}\right]$: disturbance covariance martix

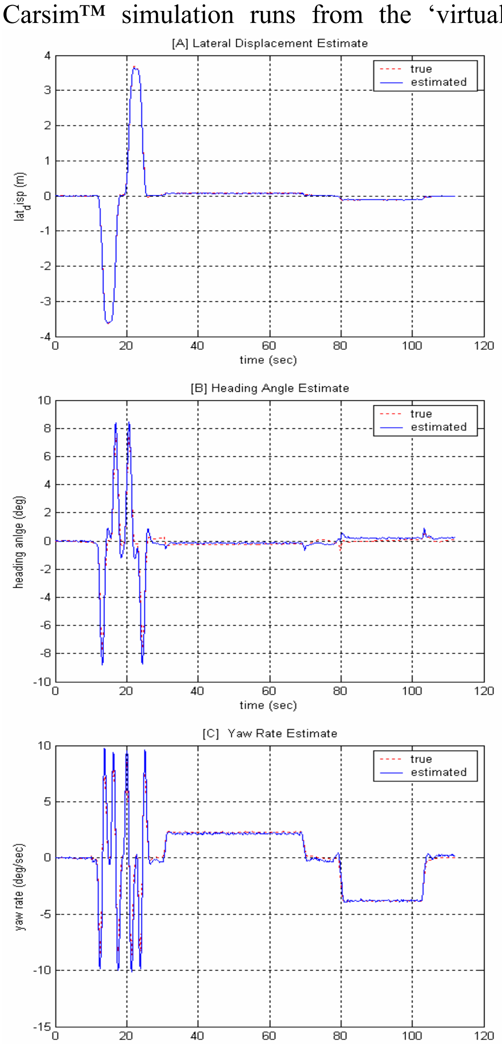
$\sigma^2_{\delta d}$: covarinace of steering disturbace

$\sigma^2_{\gamma\ dist}$:estimate of covarinace of γ estimate

$\sigma^2_{\dot{r}bias}$: covarinace of $\dot{r}_{bias}$

VIII. Performance Evaluation of Kalman Filter:

The Kalman filter is now tested with data obtained from Carsim™ simulation runs from the 'virtual test track'.



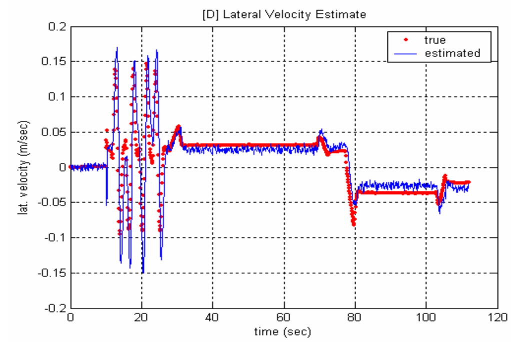


Fig. 5. Carsim™ Simulation with bank angle estimate

The plots in Fig. 5 show that the performance of the filter for the nominal case is quite acceptable. The measurements of yaw rate and lateral displacement are filtered very well and we get accurate estimates of heading angle and lateral velocity. Further simulations with practical disturbances and/or uncertainties are conducted to assess the performance of the Kalman Filter. Perturbed cases include common plant variations such as increase in vehicle mass and reduction in road/tire friction, or offset in steering angle measurement, and time delay in road curvature information. A 300kg mass increase is used to simulate the maximum variation in loading condition of the vehicle. Similarly, a friction coefficient of 0.2 is used to simulate driving under icy or wet conditions. A 0.5 second delay in road curvature measurement is used to account for the processing time required by the image processing module to calculate road curvature from camera images and a 5 degree offset (a the wheel) is used to check the robustness of the algorithm under such constant errors. For all these simulations, the vehicle is assumed to be driven on a 'virtual test track' with combined turning, lane-change and straight-line driving with part of the track super-elevated. The steering angle is generated by the built-in driver model, which is based on the preview driver model [9].

The estimation results are summarized in Table 1. It can be seen from the results that the Kalman filter performs satisfactorily for all the disturbances. In fact for certain practical disturbances, like delay in road curvature measurement that is caused by the time required by the vision processing module, the Kalman Filter performance is better than the nominal case in most respects!

To further test the accuracy of the Kalman Filter and to access its utility in actual Road Departure prevention applications, the data obtained from the Kalman Filter was used to perform TLC analysis and the results were compared the actual TLC obtained from Carsim™ data. Two simulations were performed for this purpose. The first run is a simple curve with the driver model following an offset path close to the inner edge, for this case the simple definition of TLC was used, as mentioned in equation [1]. The second run is the on the 'virtual test track' in Carsim™ with a short preview driver aimed at trying to simulate a inexperienced or distracted driver, so that more instances of low TLC are achieved . In this more complex case a more sophisticated definition of TLC is used where future course of the car is predicted using data obtained from the Kalman filter as initial condition and steering assumed to be constant. Results are summarized in Table 2.

IX. CONCLUSIONS

We have presented the development of a Kalman filter to filter the measured signals and estimate lateral velocity and heading angle. These signals can now be used to dynamically observe the Time to Lane Crossing (TLC) during the analysis of the operational test of the RDCW system. This filter may also apply to most other active safety systems that prevent road departure and vehicle instability. Since this filter filters or estimates the whole suite of vehicle handling parameters (yaw rate, heading angle, lateral velocity and lateral displacement) it can be used for any vehicle control system that needs all or any combination of the above parameters to perform its task.

The Kalman filter development has been extensively tested for various operating conditions using realistic, non-linear and highly advanced simulations using Carsim™.
We also tested and verified a useful algorithm for getting an approximate estimate of road bank angle presented by Tseng. This may be applicable to many active safety applications.

Table 1: Simulation Results with various disturbances

Scenario	Standard Deviations of error			
	Lateral Displacement (m)	**Heading Angle (deg)**	**Yaw Rate (deg/sec)**	**Lateral Velocity (m/s)**
Nominal w/o bank angle	0.037	0.37	0.46	0.056
Nominal with bank angle	0.014	0.30	0.39	0.021
With 300 kg mass increase	0.018	0.39	0.63	0.022
Delay in road curvature measurement	0.008	0.23	0.41	0.017
Offset in Steering Measurement	0.021	0.67	1.06	0.044
Change in Tire properties (low μ)	0.022	0.64	1.92	0.044
Higher Speed (30 m/s)	0.009	0.33	1.30	0.106

Table 2: Results for TLC analysis

Scenario	Standard Deviations of error				
	Lateral Displacement (m)	**Heading Angle (deg)**	**Yaw Rate (deg/sec)**	**Lateral Velocity (m/s)**	**TLC (sec)**
Offset on Simple Curve	0.011	0.05	0.18	0.01	0.41
DLC with Short Preview Driver	0.016	0.30	0.51	0.035	0.87

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