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A Total Chattering-Free Sliding Mode Control for Sampled-Data Systems

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***Abstract*—The discrete-time equivalent control, obtained from $s_{k+1} = 0$, is a chattering-free sliding mode control in theory; however, it involves two practical problems in implementation, they are the numerical accessibility problem and the physical admissibility problem. The numerical accessibility problem is due to the unknown external disturbances, which appears in the equivalent control. The physical admissibility problem arises in the reaching phase, in which the control law is to drive the current state to the sliding surface in one sampling period. In this paper, a one-step delay estimator for the disturbance is employed to approximate the equivalent control with an $O(T^2)$ accuracy, where T is the sampling period. This leads to an $O(T^2)$ boundary layer in the vicinity of the sliding surface. On the other hand, the control admissibility issue is solved by extending the reaching phase, leading to $s_{k+h} = 0$, with a positive on-line tuning parameter h.**

I. INTRODUCTION

A sampled-data control system has its innate limitation on the switching frequency when implementing variable structure control laws. Continuous-time variable structure control with limited switching frequency leads to a serious chattering problem. Design of discrete-time sliding mode control becomes essential to sampled-data systems. It was pointed out in [1] that the system, when discrete variable structure control is employed, can at best achieve quasi-sliding motion, in which sliding mode is attained only at the sampling instants. In between consecutive sampling instants, the state trajectory will deviate from the sliding surface leading to minute errors with magnitude subject to the sampling period, T. In [2], a switching type of discrete-time sliding mode control law

$$u_k = \begin{cases} u_k^+; s_k > 0 \\ u_k^-; s_k < 0 \end{cases}$$

was proposed to yield a system trajectory converging to the discrete-time sliding surface, s_k=0 [2]. The upper and the lower limits of the switching control, u_k^+ and u_k^-, were to satisfy the reaching condition $| s_{k+1} |<| s_k |$. It was ascertained later in [3] that u_k^+ and u_k^- will converge to an identical value as discrete-time sliding mode is achieved and maintained. The ascertainment suggested that the switching behavior in $u(k)$ will degenerate into a continuous form. In other words, a continuous control is an ultimate solution to the problem of discrete-time sliding mode. Furuta's result gave a similar implication [4]. The proposed discrete-time control law took the form of state feedback with a continuous gain and a switching gain. The continuous gain was to maintain the state on the sliding surface, while the switching gain was a function of the sliding vector s_k to steer the state into a sliding region. It is seen that the magnitude of the switching gain dies away as discrete-time sliding mode occurs. The control law is then left with the continuous part, leading the system into a chattering-free sliding motion.

The nonswitching type of discrete-time sliding mode control was first proposed by Drakunov and Utkin in the context of discrete equivalent control, u_k^{eq}, which brings the state to the sliding surface in one step (s_{k+1}=0) [5]. Such a control law gives a definite solution to the discrete-time sliding mode problem. Although u_k^{eq} is usually not numerically accessible (due to unknown disturbances) and not physically admissible (due to a far apart current state from the sliding surface), it provides us with an implication that a chattering-free sliding mode control law does exist theoretically. The latter endeavor of much research yielded useful results in accordance with this standpoint. Bartolini *et. al.* proposed a smooth, self-adaptive mechanism to tune up the discrete control law to approach the equivalent control [6]. Corradini and Orlando made use of time-delay control to estimate the effects of unknown system perturbations and came up with a dwindling control activity, which became chattering-free asymptotically [7]. In [8], the system state is driven by Gao's method and finally crosses the sliding hyperplane in every successive sampling period. This results a zigzag motion about the sliding hyperplane [8]. A further improvement without switching control signal was proposed by Bartoszewicz [9]. In [9], the system state remains in a certain band around the sliding hyperplane, but not to pass through the hyperplane in each successive control step. It

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provides tremendous progress than [8] and will be used to evaluate the performance with our approach in this paper.

Chattering control was the original method to acquire robustness in a continuous-time variable structure system. For discrete-time systems, however, many recent research results have indicated that a chattering-free control strategy can also render robustness with a priori knowledge of the unknown disturbances. To scrutinize the proposed control formats and the simulation results of the above references [2,4,6,7], one can observe that chattering can still occur in the reaching phase of the total control motion in spite of the chattering-free performance in the sliding phase. Such a provisional chattering phenomenon is still undesirable in many systems that require smoothness in the control actions.

In this paper, we will focus on the sampled-data systems to develop a total chattering-free discrete-time sliding mode control. We will carry on with Drakunov and Utkin's method to mend the deficiency of the incomplete chattering-free behavior. As was mentioned above, u_k^{eq} is usually not numerically accessible due to the unknown disturbances. Furthermore, if the current state is far from the sliding surface, the magnitude of u_k^{eq} can be too large to be physically admissible. These implementation problems will be studied judiciously throughout this paper. Morgan *et. al.* [10] pointed out that disturbances with certain smoothness can be estimated through the discrete-time system dynamic equation. Similar idea can also be seen in [7]. Thus, u_k^{eq} can be estimated with the accuracy of the disturbance estimation. Su *et. al.* [11] proposed a modified equivalent control law to maintain the state in an $O(T^2)$ boundary layer of the sliding surface ,denoted $\sum$. A chattering-free sliding motion in the boundary layer was observed. However, the control admissibility problem, which occurs in the reaching mode, is still unsolved yet. The attempt to drive the initial state to the sliding surface in one single sampling period requires a gigantic control magnitude. A natural conjecture to reduce the control magnitude is to prolong the reaching mode so as to render a 'soft landing' on $\sum$. Therefore, instead of using s_{k+1}=0 to yield u_k^{eq}, one can simply extend the reaching phase by using s_{k+h}=0, where h is a positive integer, to compute the equivalent control. In this paper, an on-line tuning parameter $h(k)$ is introduced. The tuning mechanism is operated according to the distance of $\sum$ from the current state. As the state is sufficiently close to $\sum$, the $h(k)$ parameter is set to one and the discrete-time sliding mode is attained.

The other parts of this paper are organized as follows. Section II is the problem formulation. Section III and IV represent the control concept, algorithm, and the simulation result. Final section is the conclusion.

II. Problem Formulation

In a sampled-data system, each of the control variables retains only one degree of freedom within the sampling period $kT \le t < (k+1)T$, leading to shrinkage in the control signal space from $(L^2_{[0,T]})^m$ to R^m, m being the number of control inputs. Therefore, a sampled-data controller will inherently be less capable than a continuous one [1]. In the context of sliding mode control, a sampled-data control law can at best achieve sliding mode in discrete time. Whereas in between consecutive sampling instants, the state will deviate from the sliding surface, forming a boundary layer in the vicinity of the sliding hyperplane [11].

A. An $O(T^2)$ boundary layer in sliding mode

Consider a linear time-invariant system with a prescribed sliding surface $\sum$

$$\dot{x} = Ax + Bu, \tag{1}$$

$$\textstyle\sum=\{\, x \mid s(x)=Cx=0\}, \tag{2}$$

where the state $x\in R^n$, the control $u\in R^m$, and the sliding vector $s\in R^m$; $A\in R^{nxn}$, $B\in R^{nxm}$, are constant matrices, and $C\in R^{m\times n}$ is chosen properly in order to achieve the desired sliding dynamics. The sampled-data system of (1) is

$$x_{k+1} = \Phi x_k + \Gamma u_k \tag{3}$$

where $\Phi = e^{AT}$, $\Gamma = [\int_0^T e^{A\lambda} d\lambda]B$, and T is the sampling period.

Assuming $C\Gamma$ is nonsingular, the discrete equivalent control can be obtained with s_{k+1}=0 and becomes

$$u_k^{eq} = -(C\Gamma)^{-1} C\Phi x_k, \tag{4}$$

which is a nonswitching type of control with chattering-free. With the equivalent control law (4) implemented through a zero-order-hold process in a sampled data system, sliding mode is attained only at each sampling instant, $s|_{t=kT} = s(x(kT)) = 0$, but not in continuous-time. During the sampling interval, the system state strays away from the sliding surface. The farthest deviation instant is about half of the sampling period, which results in an $O(T^2)$ boundary layer in the vicinity of the sliding surface, i.e. $s(kT+\tau)\in O(T^2)$, where $0\le\tau< T$. The proof is presented in section IV with a numerical example shown in Fig.1.

B. The external disturbances problem

Now consider the case of linear time-invariant system with external disturbances, which is described as

$$\dot{x} = Ax + Bu + Df, \tag{5}$$

where the unknown disturbance $f\in R^r$ is a bounded, smooth function of time with $|f(t)|\le f_{max}$, and $D\in R^{nxr}$ is a constant matrix satisfying the matching condition $rank[B, D]=rank[B]$ [12]. The sampled-data system of (5) becomes

$$x_{k+1} = \Phi x_k + \Gamma u_k + d_k, \tag{6}$$

where $d_k = \int_0^T e^{A\lambda} Df((k+1)T-\lambda)d\lambda$ is the lumped effect of the disturbance $f(t)$ within the sampling period $kT \le t < (k+1)T$. The discrete equivalent control

$$u_k^{eq} = -(C\Gamma)^{-1}C(\Phi x_k + d_k) \quad (7)$$

can be applied conceptually to (5). Unfortunately, it cannot be implemented physically without obeying the law of causality.

C. The control admissibility problem

An admissible control u_k in a practical system is usually bounded, i.e.

$$|u_k| \le M,\ M > 0. \quad (8)$$

To check for the magnitude of the equivalent control law of (7), rewrite u_k^{eq} in Taylor's series expansion

$$u_k^{eq} = -(C\Gamma)^{-1}C[(I + TA + \ldots)x_k + d_k]$$
$$= -(C\Gamma)^{-1}[s_k + O(T) + Cd_k], \quad (9)$$

where $d_k = \int_0^T e^{A\lambda} Df((k+1)T-\lambda)d\lambda \in O(T)$ and $C\Gamma = C[TI + (T^2A)/2 + \ldots]B \in O(T)$. If the state is located in the neighborhood of Σ, or $s_k \in O(T)$, then $u_k^{eq} \in O(1)$, and the control admissibility constraint (8) may hold. On the contrary, if the state is far away from Σ, or $s_k \in O(1)$, then the magnitude of u_k^{eq} is a reciprocal function of T, $u_k \in O(1/T)$, and u_k^{eq} becomes too large to be admissible.

III. The Chattering Free Controller

To deal with the causality problem in u_k^{eq} of (7), Su *et. al* proposed the modified control law [11]

$$u_k = -(C\Gamma)^{-1}C(\Phi x_k + d_{k-1}), \quad (10)$$

where d_{k-1} is the disturbance in the previous sampling instant and can be computed by the past information

$$d_{k-1} = x_k - \Phi x_{k-1} - \Gamma u_{k-1}. \quad (11)$$

Substituting (10) into system (6) yields

$$s_{k+1} = Cx_{k+1} = C(d_k - d_{k-1})$$
$$= C\int_0^T e^{A\lambda} D\{f[(k+1)T-\lambda] - f(kT-\lambda)\}d\lambda$$
$$= C\int_0^T e^{A\lambda} D\int_{kT-\lambda}^{(k+1)T-\lambda} \dot{f}(\sigma)d\sigma d\lambda \in O(T^2)\,. \quad (12)$$

If the external disturbance $f(t)$ is a smooth function of time, the resultant sliding motion is of $O(T^2)$ accuracy.

To deal with the control admissibility problem in (10), we propose to extend the reaching phase to decrease the magnitude of u_k. A boundary layer, Σ_T, containing Σ is defined as

$$\Sigma_T = \{x \,|\, \|(C\Gamma)^{-1}C(\Phi x + d_{k-1})\| \le M\}. \quad (13)$$

As the state is located inside of Σ_T, the control u_k of (10) is admissible and sliding mode is attained with $O(T^2)$ accuracy [11]. If the state is outside of Σ_T, a modified control law $u_{eq}^*(k)$ is obtained by solving s_{k+h}=0 (where h>1) instead of s_{k+1}=0. The integer h is an on-line tuning parameter, h_k. It is chosen such that $u_{eq}^*(k) \in O(1/hT)$ satisfies the admissibility condition in (8). As the state is driven toward the vicinity of Σ_T, the parameter h_k approaches to identity ($h_k \to 1$) and the $O(T^2)$ sliding mode is attained.

Let $t = kT$ be the current time with current state x_k. The sampled-data system with sampling period hT is

$$x(t+hT)\big|_{t=kT} = A_d x(t)\big|_{t=kT} + B_d u(t)\big|_{t=kT} + d_h(t)\big|_{t=kT}, \quad (14)$$

where $d_h(t)\big|_{t=kT} = \int_0^T e^{A\lambda} Df(t+hT-\lambda)d\lambda$, $B_d = [\int_0^{hT} e^{A\lambda}d\lambda]B$, and $A_d = e^{A(hT)}$. The proposed control is

$$u_{eq}^*(k) = -(CB_d)^{-1}CA_d x_k - (C\Gamma)^{-1}Cd_{k-1}. \quad (15)$$

As soon as an appropriate h_k is given, $A_d(h_k)$, $B_d(h_k)$ and d_{k-1}, can be computed subsequently to render $u_{eq}^*(k)$. The dynamics of the discrete-time sliding mode without the higher order terms is approximated by substituting (14) into s_{k+1} and becomes

$$s_{k+1} = [C\Phi - C\Gamma(CB_d)^{-1}CA_d]\, x_k + Cd_k - Cd_{k-1}$$
$$\cong [C(I + TA) - (1/h_k)C(I + h_kTA)]x_k + C(d_k - Cd_{k-1})$$
$$\cong [(h_k - 1)/h_k]\, s_k + O(T^2). \quad (16)$$

Since $(h_k - 1)/h_k < 1$, s_k is stable in discrete time.

The on-line tuning parameter h_k is estimated at each sampling interval such that $u_{eq}^*(k)$ satisfies the constraint (8) during the extended reaching phase. On the other hand, the reaching condition of sliding mode can also be assured at each sampling instant from a continuous-time perspective,

$$\dot{s}(kT) = CAx(kT) + CBu_{eq}^*(kT) + CDf(kT)$$
$$\cong CAx(kT) - [C(I + h_kTA)x(kT)]/(h_kT) - (Cd_{k-1})/T + CDf(kT)$$
$$\cong -[Cx(kT)]/(h_kT) + O(T) \approx -s(kT)/(h_kT), \quad (17)$$

where $d_{k-1} = \int_0^T e^{A\lambda} D[f(kT-\lambda)]d\lambda = Df(kT)T + O(T^2)\,.$

It yields

$$s\dot{s}\big|_{t=kT} \cong -\frac{1}{h_kT}s^2\big|_{t=kT} < 0\,. \quad (18)$$

With equations (16) and (17), the proposed control (15) is validated to drive the system state toward the sliding surface gradually. We come to the following Lemma.

Lemma 1. The control law (15) with the on-line tuning parameter h_k is able to drive the linear system (5) with a smooth exogenous disturbance $f(t)$ toward the $O(T^2)$ boundary layer Σ_T in (13).

A. The Simplified Chattering Free Controller

According to Euler's method

$$\dot{x}(t)\big|_{t=kT} = \lim_{T\to 0}[x(kT+T) - x(kT)]/T\,. \quad (19)$$

The approximated state equation of (6) can be described as

$$x_{k+1} = (I + TA)\, x_k + TB\, u_k + d_k. \quad (20)$$

The simplified control, based on (20), becomes

$$u^*(k) = -(Ch_kTB)^{-1}C(I + h_kTA)x_k - (C\Gamma)^{-1}Cd_{k-1}$$

$$= -[(CB)^{-1}Cx_k]/(h_kT) - (CB)^{-1}CAx_k - (C\Gamma)^{-1}Cd_{k-1}, \quad (21)$$

where $d_{k-1} = x_k - \Phi x_{k-1} - \Gamma u^*(k-1)$, and $d_{-1}=0$, for k=0, 1 ,…., n.

Substituting (20) into $s_{k+1} = Cx_{k+1}$, we obtain

$$s_{k+1} = C\Phi x_k - (C\Gamma)(h_kT)^{-1}(CB)^{-1}C(I+h_kTA)x_k + C(d_k - d_{k-1})$$

$$= \frac{2(h_k-1) - T(CAB)(CB)^{-1}}{2h_k} s_k + C(d_k - d_{k-1}) + G(T^2), \quad (22)$$

where $G(T^2) = C[(T^2A^2)/2 + (T^3A^3)/3! + \ldots]x_k$

$$-(1/h_k)C[(T^2A^2)/3! + (T^3A^3)/4! + \ldots]B(CB)^{-1}Cx_k$$

$$-C[(T^2A)/2 + (T^3A^2)/3! + (T^4A^3)/4! + \ldots]B(CB)^{-1}CAx_k . \quad (23)$$

During the reaching phase, h_k is estimated at each sampling interval such that $2(h_k-1) >> -TCAB(CB)^{-1}$ and the control constraint (8) is satisfied. Then

$$s_{k+1} \cong [(h_k-1)/h_k]s_k + C(d_k - d_{k-1}) + G(T^2). \quad (24)$$

and

$$\dot{s}(kT) = CAx(kT) - [1/(h_kT)]C(I + h_kTA)x(kT)$$

$$-CB(C\Gamma)^{-1}Cd_{k-1} + CDf(kT) \approx -s(kT)/(h_kT). \quad (25)$$

Hence, the continuous-time reaching condition holds as (18).

IV. Numerical Example

Given the linear time-invariant system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f \quad (26)$$

where $f(t)=0.5+5\sin(0.1t)-\cos(t)$ is the unknown external disturbance. Let $s(x) = Cx = [1 \ \ 1]x$.

A. An $O(T^2)$ boundary layer in sliding mode

In order to verify the $O(T^2)$ dynamics during the sliding surface purely, the equivalent control (4) is applied to (26) by neglecting $f(t)$. The initial condition is given as $x_1(0)=2$, $x_2(0)=-1$ and the sampled-data system of (26) is obtained with the sampling period $T = 0.1$second. Let

$$x(t)=C_1(t)\underline{v}_1 + C_2(t)\underline{v}_2 \quad (27)$$

be the solution of $\dot{x} = Ax$ in (26). To differentiate (27) yields

$$\dot{x}(t) = \dot{C}_1(t)\underline{v}_1 + \dot{C}_2(t)\underline{v}_2 = Ax(t). \quad (28)$$

Substituting (27) into (28) and comparing the coefficient parts of both equations, we obtain

$$\dot{C}_1(t) = C_1(t)\lambda_1 + C_2(t), \quad (29)$$

and

$$\dot{C}_2(t) = C_2(t)\lambda_1. \quad (30)$$

The given numerical example has repeated eigenvalues $\lambda_1=\lambda_2=1$ with multiplicity 2. We can find two linearly independent eigenvectors $\underline{v}_1 = [1,1]^T$ and $\underline{v}_2 = [0,1]^T$ with these eigenvalues. Let $C_2(t)=\beta e^{\lambda_1 t}=\beta e^t$ and $C_1(t)=\alpha e^t+\beta te^t$ be the solution of (30) and (29) respectively. Substituting them into (27), the solution of $\dot{x} = Ax$ in (26) is

$$x(t) = (\alpha e^t + \beta te^t)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta te^t \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (31)$$

The coefficients α and β are decided by the initial condition of the system. Let the solution of (26) without disturbance be the form as following

$$x(t) = \alpha e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta\{te^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^t \begin{bmatrix} 0 \\ 1 \end{bmatrix}\} + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \quad (32)$$

where K_1 and K_2 are arbitrary constants. Hence,

$$\dot{x}(t) = \alpha e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta\{te^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^t \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}. \quad (33)$$

Substituting (32) into (26), we compare the coefficient parts with (33) and obtain

$$\begin{bmatrix} K_2 \\ 2K_2 - K_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) = 0. \quad (34)$$

It implies $K_2=0$ and $K_1=u(k)$. The solution of (26) between the successive sampling instants can be described as

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \alpha e^{KT} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \beta\{KTe^{KT} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{KT} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\} + \begin{bmatrix} u(k) \\ 0 \end{bmatrix}. \quad (35)$$

Subtracting $x_1(k)$ from $x_2(k)$ in (35), we obtain

$$\alpha = e^{-KT}[x_1(k)(1+KT) - x_2(k)KT - u(k)(1+KT)] \quad (36)$$

and

$$\beta = e^{-KT}[u(k) + x_2(k) - x_1(k)]. \quad (37)$$

During the sampling interval, the discrete-time equivalent control u_k^{eq} is solved with $s_{k+1} = 0$ and becomes

$$u_k^{eq} = -\frac{(1-2T)e^T x_1(k) + (1+2T)e^T x_2(k)}{(1 - e^T + 2Te^T)}. \quad (38)$$

The discrete-time sliding mode during the sampling instant is

$$s(\tau) = x_1(k)(1-2\tau)e^\tau + x_2(k)(1+2\tau)e^\tau$$

$$+u_k^{eq}(1 - e^\tau + 2\tau e^\tau). \quad (39)$$

The ideal discrete-time sliding mode is zero at each sampling instant theoretically which implies $s(k) = x_1(k)+x_2(k) = 0$. The deviated state trajectory during the sampling interval can be obtained by substituting (38) into (39) and becomes

$$s(\tau) = 2[x_2(k) - x_1(k)][\tau e^\tau - \frac{(1 - e^\tau + 2\tau e^\tau)}{(1 - e^T + 2Te^T)} Te^T]. \quad (40)$$

The maximum deviation point during each sampling period is obtained by differentiating (40) with respect to time; that is

$$\frac{ds(\tau)}{d\tau}\Big|_{\tau=\tau_{\max}} = 2[x_2(k) - x_1(k)]e^\tau[\tau + 1 - \frac{(Te^T)}{(1 - e^T + 2Te^T)}(2\tau+1)] = 0.$$

(41)

Because the state strays away during the sampling interval, i.e. $2[x_2(k)-x_1(k)]e^{\tau}\neq 0$ during the discrete-time sliding phase, the farthest deviation instant is estimated to be

$$\tau_{\max} = [-(1-e^T) - Te^T]/(1-e^T) \cong 0.5T\,. \qquad (42)$$

Substituting (42) into (40), the estimated trend of the deviated dynamics is

$$s(\tau)\big|_{\tau=\tau_{\max}} = [x_2(k)-x_1(k)]\Big[\frac{(1-2e^{T/2}+e^T)}{(1-e^T+2Te^T)}Te^{T/2}\Big]$$

$$= [x_2(k)-x_1(k)]\Big\{\frac{[T^2/(2\cdot 2!)+3T^3/4!+\ldots]}{[1+3T/2!+5T^2/3!+\ldots]}\Big[1+\frac{T}{2}+\frac{(T/2)^2}{2!}+\ldots\Big]\Big\}$$

$$\cong [x_2(k)-x_1(k)](T^2)\{[1/(2\cdot 2!)]+(3T/4!)+\ldots\}\in O(T^2)\,. \qquad (43)$$

Figure 1 shows that the farthest deviation instant is about half of T which results the $O(T^2)$ dynamic motion in the vicinity of Σ, i.e. $s(kT+\tau)\in O(T^2)$, where $0\leq\tau<T$.

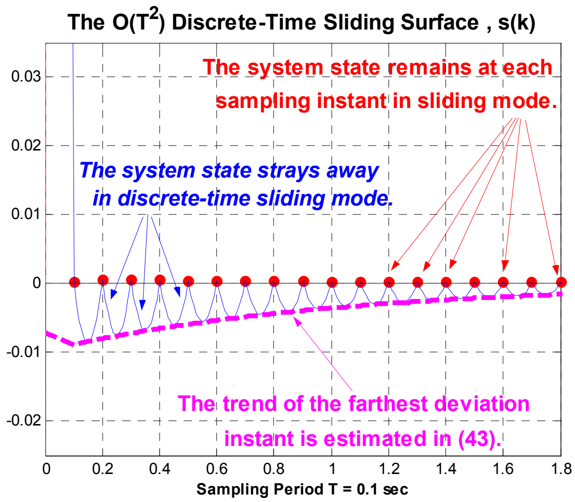

Fig. 1. An $O(T^2)$ boundary layer in the discrete-time sliding mode

B. The control admissibility problem

The same example with disturbance in (26) is used to compare the performance of both control methods (10) and (21). Given the control constraint M=15 and the sampling period T=0.01second, two simulations are shown in Fig. 2 and Fig. 3 with the initial conditions $x_1(0)$=2, $x_2(0)$=−1, and $x_1(0)$=−3, $x_2(0)$=−1, respectively.

The flexible parameter h_k is simply estimated as

$$h_k = ceil\{|u(k)|/M\}\,. \qquad (44)$$

The 'ceil' function rounds the elements of $|u(k)|/M$ to the nearest integers greater than or equal to $|u(k)|/M$, i.e. h_k is a positive integer.

We observed that the total chattering free dynamics is assured conformably during the reaching phase and the sliding mode. However, the control admissibility problem is solved by the proposed control scheme (21) only.

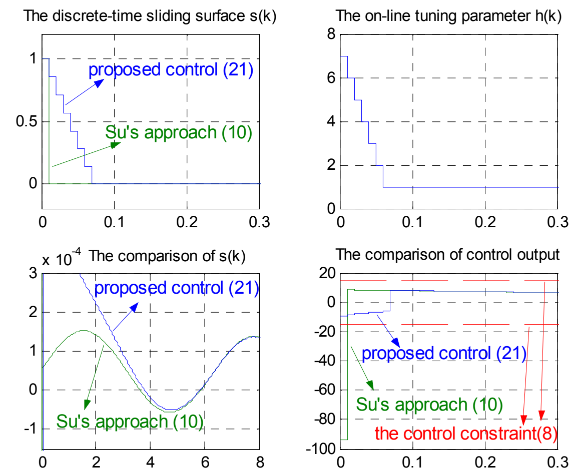

Fig. 2. The comparison with initial condition $x_1(0)$=2, $x_2(0)$= −1

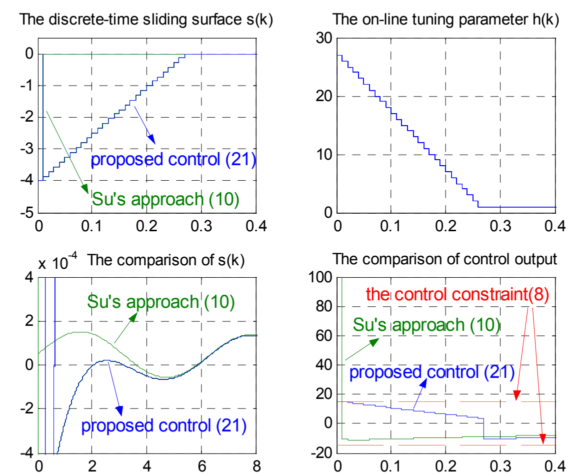

Fig. 3. The comparison with initial condition $x_1(0)$= −3, $x_2(0)$= −1

C. The comparison with Bartoszewicz's approach

According to [9], if the bound of the discrete-time lumped effect of $f(t)$ is known, i.e.

$$d_{low} \leq d_k \leq d_{upper} \qquad (45)$$

where d_{low} and d_{upper} are known constants. Let

$$d_{mean} = (d_{low}+d_{upper})/2. \qquad (46)$$

The control law proposed by Bartoszewicz is

$$u^{A\_Bart}(k) = -(C\Gamma)^{-1}[C\Phi x_k + Cd_{mean} - S_d(k+1)], \qquad (47)$$

where $S_d(k)$ is a known function defined in [9]. Let $S_d(k)$ be as same as in [9] and

$$S_d(k)=[(k^*-k)/k^*]s(0) \qquad (48)$$

where k=0, 1, …, k^*. The positive constant k^* is chosen by the designer in order to achieve good tradeoff between the necessary arrival time during the reaching phase and the

control magnitude required for the convergence rate.
The discrete-time sliding mode is obtained by applying (47) into the sampled-data system

$$s_{k+1} = C\Phi x_k + C\Gamma\, u^{A\_Bart}(k) + Cd_k$$
$$= Cd_k - Cd_{mean} + S_d(k+1), \qquad (49)$$

which is the reaching law proposed by Bartoszewicz [9]. Finally,

$$s_{k+1} = Cd_k - Cd_{mean} \qquad (50)$$

for any $k \geq k^*$. Let d_{mean}=0. The constant k^* is chosen as same as the approximated convergence characteristic in (16), i.e. k^*=7 for the initial condition $x_1(0)$=2, $x_2(0)$= −1 and k^*=27 for the initial condition $x_1(0)$= −3, $x_2(0)$= −1, respectively. The same conditions are used here to compare the performance of both control methods (21) and (47).

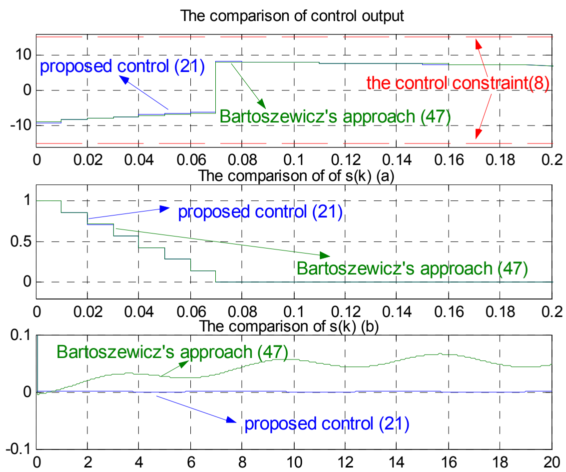

Fig. 4. The comparison with initial condition $x_1(0)$=2, $x_2(0)$= −1

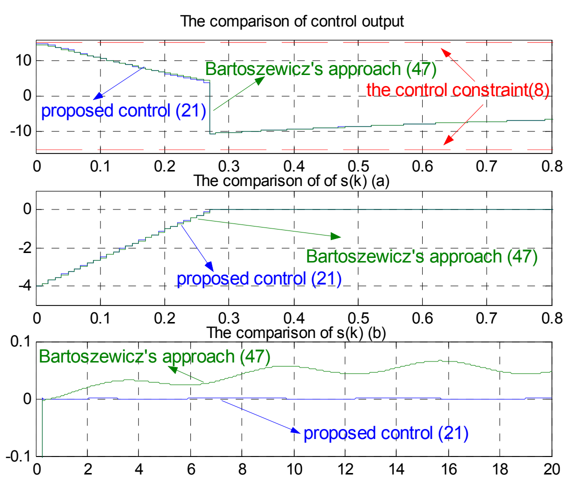

Fig. 5. The comparison with initial condition $x_1(0)$= −3, $x_2(0)$= −1

We observed that the characteristic of the control output and the discrete-time sliding mode dynamics behaves similarly in Fig. 4 and Fig. 5. However, the certain band around the sliding hyperplane (49) by using (47) is less accuracy than s_{k+1}=$C(d_k - d_{k-1}) \in O(T^2)$ by using proposed approach (21).

If the change rate of the external disturbance is already known, a modified reaching law proposed by Bartoszewicz is

$$s_{k+1} = Cd_k - Cd_{mean} + S_d(k+1) - \sum_{i=0}^{k} [s(i) - S_d(i)]$$
$$= Cd_k - Cd_{k-1} + S_d(k+1). \qquad (51)$$

Finally, $$s_{k+1} \cong Cd_k - Cd_{k-1}. \qquad (52)$$

This enhances the dynamic boundary layer around the sliding hyperplane. However, the prior knowledge of the disturbance change rate is unnecessary in our approach.

V. Conclusion

Based on the former contribution of related research, a total chattering-free sliding mode control for sampled-data systems is proposed. The phenomenon of the deviated state during the sampling period, which results in the dynamic motion in $O(T^2)$ vicinity of Σ, is described. The farthest deviation instant is shown to be about half of T. The deficiencies related with the discrete-time equivalent control, obtained from s_{k+1}=0,are focused, summarized, and solved.

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