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Fault residuals for Compact Disc players based on redundant and non-linear sensors

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***Abstract*— In the process of achieving good performance of Compact Disc players it is important to handle surface defects as well as possible. The first prerequisite for handling these defects is to detect their beginnings and ends. Two servo loops are formed to keep the optical pick-up focused on, and radially tracked at the information track on the Compact Disc. The pick-up feeds the controllers with sensor signals, and some signals for defect detection. However, due to optical cross couplings detection based on these signals can at times give false or no detections. In this paper a method to estimate fault residuals is designed in such a way that the cross couplings are removed. This is done based on a model of the optical system, from the physical focus and radial distances to the four detector signals. Combining this model with a fault model, enable us to solve an inverse function to find the distances as it would have been if no faults had occurred. Two different approaches are pursued in this paper. Both are based on the Newton-Raphson method. Both methods solve the respective inverse problems.**

I. INTRODUCTION

Even though Compact Discs (CD) have been on the market in more than two decades, there are still performance issues to improve. Many people has experienced that their CD player, have problems playing discs with scratches, finger prints etc.

The Optical Pick-up Unit (OPU), used to read the data from the spiral shaped track, is controlled to be focused and radially tracked on the data track. The job of the two controllers is to keep the focus and radial distance equal zero. The focus and radial distances are illustrated in Fig. 1. In the OPU some optics are used for generating four detector signals, (two relating to each loop), the differences of these pairs are used to approximate focus and radial distances, the sum of these pairs give information of the amount of the reflected energy received at the detectors in the OPU. The actuators in both loops are linear electro-magnetic actuators, see [1] and [2]. Sometimes CD players have problems playing disc with surface defects. The reason is that the sensor is not reliable during a defect. A way to handle a defect is first to detect the defect as fast and reliable as possible and then detected, adapt the controller to handle the detected defect.

In many commercial CD players the sum signals are used for detection, since a defect typically will cause these sum signals to decrease remarkable. The sum signals are in principle not correlated with disturbances in the system, see [3], [4] and [5]. But due to the optical cross couplings in the system, this is not entirely true in practice. A radial distance changes the focus sum, and a focus distance changes the radial sum. In [6] a physical model of the optical system as with focus and radial distance as inputs, (e_{f} and e_{r}), was developed. This model also includes cross couplings from focus distance to radial detector signals, and from radial distance to focus detector signals. The output set of the model is a set in which the detector signals will be in the normal situation, where only disturbances occur.

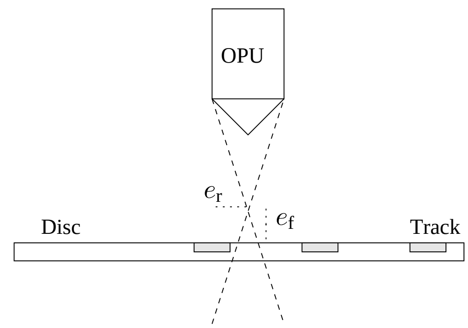

Fig. 1. The focus error e_{f} is the distance from the focus point of the laser beam to the reflection layer of the disc, the radial error is the distance from the centre of the laser beam to the centre of the track. The OPU emits the laser beam towards the disc surface and computes indirect measurements of e_{f} and e_{r} based on the received reflected light. In addition the OPU generates two residuals which can be used to detect surface defects as scratches.

Practical experiences have shown that it is preferable to distinguish between disturbances which the controller shall reject and faults which the controller shall not react to. Disturbances are phenomena like mechanical shocks, eccentricity of the disc etc. Faults are phenomena like scratches and finger prints on the disc surface, see [4]. The values of the four detector signals are caused by the two distances and the fault. I.e. it makes good sense to describe the fault by two parameters, i.e. focus and radial deviations of the sampled detector values from the model caused by the defect. These parameters are residuals which are well suited for the detection of the defects.

Unfortunately it is not a simple job to find these distances and residuals, since the only known information is the model and the sampled detector values. The model maps from $(e_\mathrm{f}, e_\mathrm{r}) \in \mathcal{R}^2$ to a 2 dimensional subset of the four detector signals in $\mathcal{R}^4$, The output set of the model is the set of values which the detector signal can be in as a response to e_f and e_r. This means that the output set of the model outputs is a surface in $\mathcal{R}^4$ with co-dimension 2. For the control purposes it is much more interesting to solve the inverse problem of this model since it would be helpful for control and fault detection and accommodation purposes to calculate e_f and e_r from the detector signals, and also use e_f and e_r to compute the fault parameters. Unfortunately, where is no global solution of inverse of the mapping given by model. The estimated distances and parameters found in this paper are in [7] and [8] used for detection of defects, and these show a clear potential of the residuals.

By using redundancy of detector signals and a simple model of the possible faults in this paper, a local solution based on Newton-Raphson's method is described. This local solution of the inverse problem can also be used for solving inverse problems for other applications like this. This description is followed by a proposal of two different algorithms to locally solve the given inverse problems given by the fault model. The first algorithm is based on a fault model which is an orthogonal projection on the model's output set. The second one is based on a fault model which is a scaling projection on the same output set. The algorithms are subsequently tested by simulations, and the results of these simulations are that the algorithm based on the scaling projection is clearly the best handling the inverse problem with a type of faults like the ones in the CD player case.

II. The disturbance set

The nominal controllers are designed to handle changes in the sensor signals due to the disturbances inside the Disturbances set. Surface defects such as scratches and finger print can be viewed as deviations from this set. The disturbance set in $\mathcal{R}^4$, can be defined in the following way:

1 Definition (The disturbance set) *The disturbance set $\mathcal{D} \in \mathcal{R}^4$ is defined as the set in which any sample s_m in $\mathcal{R}^4$ of the detector signals, will be if only disturbances occur.*

$\mathcal{D}$ can be modelled by some functions mapping from e_f and e_r to the four detector signals

$$\begin{bmatrix} D_1 \\ D_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} f_1(e_\mathrm{f}, e_\mathrm{r}) \\ f_2(e_\mathrm{f}, e_\mathrm{r}) \\ f_3(e_\mathrm{f}, e_\mathrm{r}) \\ f_4(e_\mathrm{f}, e_\mathrm{r}) \end{bmatrix}. \tag{1}$$

This model is found in [6]. These functions are based on a first principle model of the optical system. Each of the four function $f_i(e_\mathrm{f}, e_\mathrm{r}),\ i \in \{1, 2, 3, 4\}$, can be simplified to the following structure.

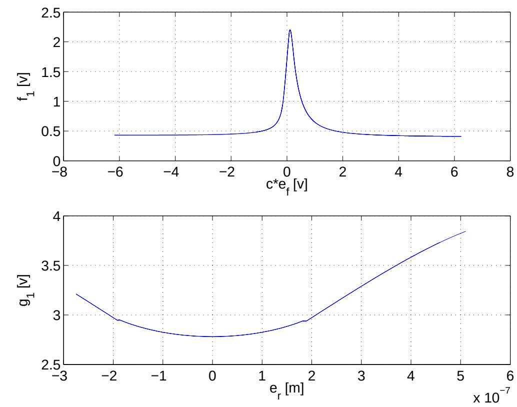

Fig. 2. The two factorising functions of $f_1(\cdot)$, $h_1(e_\mathrm{f})$ in the upper part, and $g_1(e_\mathrm{f})$ in the lower part.

$$f_i(e_\mathrm{f}, e_\mathrm{r}) = h_i(e_\mathrm{f}) \cdot g_i(e_\mathrm{r}), \tag{2}$$

$h_i(e_\mathrm{f}) \in \mathcal{R}$ and $g_i(e_\mathrm{r}) \in \mathcal{R}$. D_1 and D_2 are symmetric and related to the focus loop and S_1 and S_2 are symmetric as well and related to the radial loop. Due to the symmetry not all the functions are plotted. $h_1(\cdot)$ and $g_1(\cdot)$ are plotted in Fig. 2, and $h_3(\cdot)$ and $g_3(\cdot)$ are plotted in Fig. 3. From these functions it is clear that the optical model does not have a global inverse. Instead does the scope is to find a method to compute the local inverse at given points based on a given fault model.

The implementation of this model has a drawback. The functions are not differentiable at all points, even though the real system is. In the description of the distance function it can be seen that differentiability is an important requirement to the disturbance set model, see Section III.

As a consequence, a differentiable model is needed. This, not at all point differentiable, model is instead approximated by splines, and due to the properties of splines, this splined-model is differentiable at all points. The spline approximations are done by using cubic splines.

This disturbance set is particularly interesting in the case of fault detections, since faults move the detector signals outside this set, see [5]. A measure of the deviation from the model to a given sample, s_m caused by the fault would be a signal well suited for detection of a fault. An example of D_1 and D_2 where s_m, due to a defect is outside $\mathcal{D}$, represented by $\mathbf{f}$, is illustrated in Fig. 4. The controllers controlling focus and radial distances are only trying to reject the movement of the detector signals inside the disturbance set. So even though that a sample's deviation from $\mathcal{D}$ is quite interesting in case of detecting a fault, it does not explain everything a fault causes. A fault does cause movements of the detector signals which can be assumed as being inside $\mathcal{D}$ but is not, [9]. It is as a consequence quite interesting

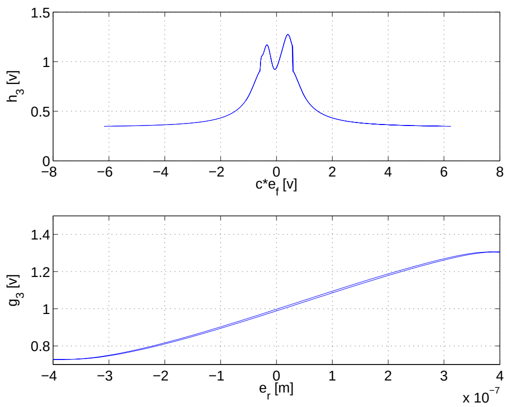

Fig. 3. The two factorising functions of $f_3(\cdot)$, $h_3(e_{\mathrm{f}})$ in the upper part, and $g_3(e_{\mathrm{f}})$ in the lower part.

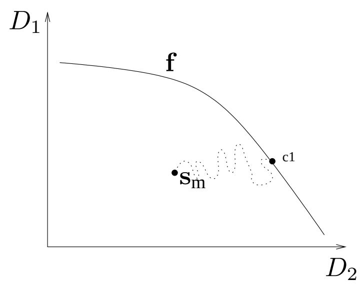

Fig. 4. The measured detector signal, $\mathbf{s}_{\mathrm{m}}$, is due to a fault outside the output set of the optical model, $\mathbf{f}(\cdot)$, this is illustrated with the detector signals D_1 and D_2, which are the two focus detectors measured in [V].

to know an estimate of this fault caused movement inside $\mathcal{D}$. This means it is interesting to calculate $\mathbf{x} = \begin{bmatrix} e_{\mathrm{f}} \\ e_{\mathrm{r}} \end{bmatrix}$ as it would have been if no faults had occurred. It is of equal interest to compute some parameters describing the fault. All these can be found by solving the inverse problem. In the following two methods are described to solve the inverse problems.

III. Two Methods for Solving the Inverse Problem

The solution of the inverse problem can be found based on the use of the redundancy of the detector signals. These signals can be modelled as:

$$\mathbf{s}_{\mathrm{m}} = \mathbf{g}(\mathbf{f}(e_{\mathrm{f}}, e_{\mathrm{r}}), f_{\mathrm{f}}, f_{\mathrm{r}}). \tag{3}$$

Where $\mathbf{g}(\ldots, f_{\mathrm{f}}, f_{\mathrm{r}})$ is a model of the fault. This means by solving the inverse problem for finding e_{f} and e_{r}, gives the fault parameters f_{f} and f_{r} as well.

Remark 1 In the given application there are some additional requirements to the solution of the inverse problem. Due to low pass filtering nature of the OPU the following limits are relevant: $|e_{\mathrm{f}}[n] - e_{\mathrm{f}}[n+1]| < \gamma_{\mathrm{f}}$ and $|e_{\mathrm{r}}[n] - e_{\mathrm{r}}[n+1]| < \gamma_{\mathrm{r}}$, where n is any sample, and γ_{f} and γ_{r} are the maximum variations in e_{f} and e_{r}. This luckily solves the non-uniqueness problem and makes it possible to find the local solution to the inverse problem.

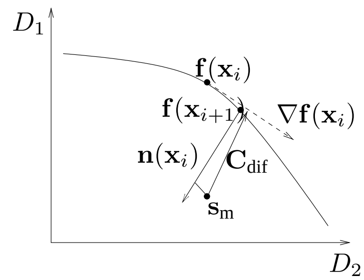

Fig. 5. Illustration of the principles of the orthogonal projection method. The illustration shows an example of one iteration in $\mathcal{R}^2$, D_1 and D_2 the two focus detectors measured in [V]. The subscript i represents the iteration number. $\mathbf{f}(\mathbf{x}_i)$ is the starting point, $\nabla\mathbf{f}(\mathbf{x}_i)$ is the gradient at the starting point, $\mathbf{n}(\mathbf{x}_i)$ is the normal vector to the gradient, $\mathbf{s}_{\mathrm{m}}$ is the sampled detector signals, $\mathbf{f}(\mathbf{x}_{i+1})$ is the function value of iteration $i+1$, $\mathbf{C}_{\mathrm{dif}}$ is the error at iteration $i+1$.

Based on the standard requirements to the CD player servos, see [2], the maximum deviation of focus and radial positions from sample to sample can be calculated to: $\gamma_{\mathrm{f}} = \gamma_{\mathrm{r}} \approx 0.014\mu$m.

In the following, descriptions are given of the two different approaches to solve the inverse problem and thereby find the right candidate point, and use this to compute the fault parameters.

The first approach is to model the faults as being orthogonal to $\mathcal{D}$, meaning that a normal distance function can be used, this approach has its strong side in solving the inverse problem in case of no defects. The second approach is based on a fault model, which changes the sensor signals in a direction towards the origin, which is stronger in solving the inverse problem if a defect occurs.

A. The orthogonal projection method

The subject is to find the point, $\mathbf{f}(\mathbf{x})$ in $\mathcal{D}$,where $\mathbf{x} = \begin{bmatrix} e_{\mathrm{f}} & e_{\mathrm{r}} \end{bmatrix}^T$. The following is known, if $\mathbf{n}(\mathbf{x})$ is defined as the normal vector to $\mathbf{f}(\mathbf{x})$:

$$\mathbf{s}_{\mathrm{m}} = k \cdot \mathbf{n}(\mathbf{x}) + \mathbf{f}(\mathbf{x}), \;\; k \in \mathcal{R}. \tag{4}$$

Since $\mathbf{f}(\cdot)$ is only local invertible, there are a number of points in $\mathcal{D}$ for which (4) is true. Due to Remark 1, the value of $\mathbf{x}[n]$ would be close to the value of $\mathbf{x}[n-1]$, this means that if an iterative algorithm is used for finding $\mathbf{x}[n]$, $\mathbf{x}[n-1]$ can be used as a starting value of the algorithm. In the following an algorithm, based on Newton-Raphson's method, is described. One iteration of the algorithm used on a problem in $\mathcal{R}^2$ is illustrated in Fig. 5.

1) The algorithm: In the following the iteration numbers are indicated in variable subscripts with i, meaning the ith iteration. The most important part of this algorithm is the projection of the vector $\mathbf{s} - \mathbf{f}(\tilde{\mathbf{x}}_i)$ on the tangent plane at the point $(\tilde{\mathbf{x}}_i)$, where $\tilde{\mathbf{x}}$ is estimated value of $\mathbf{x}$. The tangent plane is defined as:

$$\mathbf{y}(\mathbf{x}_{i+1}) = \mathbf{f}(\mathbf{x}_i) + \nabla\mathbf{f}(\mathbf{x}_i)\cdot(\mathbf{x}_{i+1} - \mathbf{x}_i). \tag{5}$$

Now find an orthonormal basis of the gradient:

$$\mathcal{U} = \text{orth}(\nabla\mathbf{f}(\mathbf{x}_i)). \tag{6}$$

Use this to form a transformation matrix P

$$\mathbf{P} = \mathcal{U}\cdot\mathcal{U}^*. \tag{7}$$

This means

$$\mathbf{y}_{i+1} = \mathbf{f}(\mathbf{x}_i) + \mathbf{P}\cdot(\mathbf{s} - \mathbf{f}(\mathbf{x}_i)), \tag{8}$$

$$\mathbf{P}\cdot(\mathbf{s} - \mathbf{f}(\mathbf{x}_i)) = \mathcal{U}\xi, \tag{9}$$

where

$$\xi = \mathcal{U}^*\cdot(\mathbf{s} - \mathbf{f}(\mathbf{x}_i)). \tag{10}$$

ξ is the Δy in $\mathcal{U}$ coordinates. This gives, the iteration increment Δx as

$$\Delta x = \mathbf{V}\Sigma^{-1}\mathbf{U}^*\cdot(\mathbf{s} - \mathbf{f}(\mathbf{x}_i)), \tag{11}$$

$$\nabla\mathbf{f}(\mathbf{x}_i) = \tilde{\mathbf{U}}\Sigma\tilde{\mathbf{V}}^+ \tag{12}$$

$$= \tilde{\mathbf{U}}\begin{bmatrix}\Sigma & 0\end{bmatrix}\tilde{\mathbf{V}}^*. \tag{13}$$

Where

$$\mathbf{U} = \tilde{\mathbf{U}}(1:2,:). \tag{14}$$

These equations result in

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \tilde{\mathbf{V}}\Sigma^{-1}\mathbf{U}^*\cdot(\mathbf{s} - \mathbf{f}(\mathbf{x}_i)) \Rightarrow \tag{15}$$

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \nabla\mathbf{f}(\mathbf{x}_i)^+\cdot(\mathbf{s} - \mathbf{f}(\mathbf{x}_i)). \tag{16}$$

The $\nabla\mathbf{f}(\mathbf{x}_i^+$ denotes the pseudo inverse of $\nabla\mathbf{f}(\mathbf{x}_i$. The algorithm for estimating $\tilde{\mathbf{x}}$ is the following:

1) Find the gradient, $\nabla(\tilde{\mathbf{x}}_i)$ to the point $(\tilde{\mathbf{x}}_i, \mathbf{f}(\tilde{\mathbf{x}}_i))$.
2) Project $\mathbf{s}$ to this tangent plane, from this projection $\tilde{\mathbf{x}}_{i+1}$ can be found. $\tilde{\mathbf{x}}_{i+1} = \tilde{\mathbf{x}}_i + \nabla\mathbf{f}(\tilde{\mathbf{x}}_i)^+\cdot(\mathbf{s} - \mathbf{f}(\tilde{\mathbf{x}}_i))$.
3) Compute a normal vector $\mathbf{n}(\mathbf{x})$ to the point$(\tilde{\mathbf{x}}_{i+1}, \mathbf{f}(\tilde{\mathbf{x}}_{i+1}))$.
4) Compute: $\mathbf{f}_\text{f} = \begin{bmatrix} r_\text{f} \\ r_\text{r} \end{bmatrix} = \begin{bmatrix} \left\| \begin{matrix} D_1 - \tilde{D}_1 \\ D_2 - \tilde{D}_2 \end{matrix} \right\| \\ \left\| \begin{matrix} S_1 - \tilde{S}_1 \\ S_2 - \tilde{S}_2 \end{matrix} \right\| \end{bmatrix}$.
5) If $|\mathbf{f}(\mathbf{x}) - \mathbf{s}| < \epsilon$ stop, else go to 1. ϵ is a small real constant.

r_f and r_r are residuals, since they are 0 in case of no faults and increase as the fault develops.

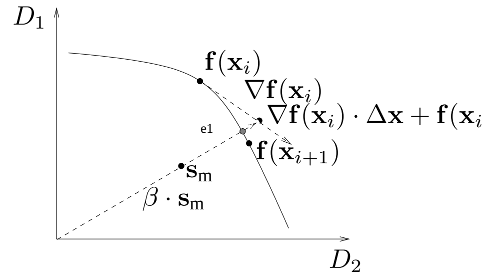

Fig. 6. Illustration of the principles of the scaling projection method. The Illustration shows an example in $\mathcal{R}^2$, D_1 and D_2 the two focus detectors measured in [V]. $\mathbf{f}(\mathbf{x}_i)$ is the starting point for the iteration, $\nabla\mathbf{f}(\mathbf{x}_i)$ is the gradient at the starting point, $\mathbf{s}_\text{m}$, $\beta\cdot\mathbf{s}$ is vector through origin and $\mathbf{s}_\text{m}$, $\nabla\mathbf{f}(\mathbf{x}_i)\cdot\Delta\mathbf{x}$ is the crossing between the gradient and the vector through $\mathbf{s}_\text{m}$, this can be used to find $\Delta\mathbf{x}$ and $\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta\mathbf{x}$, and $\mathbf{f}(\mathbf{x}_{i+1})$ is the new function value. The algorithm stops with iterations then norm$(\mathbf{f}(\mathbf{x}_{i+1}) - \beta\cdot\mathbf{s}) < \epsilon$, where ϵ is the stop parameter.

B. The scaling projection method

The orthogonal projection method is well suited for solving the inverse problem in cases where the detector signals are inside $\mathcal{D}$, or the faults are orthogonal to $\mathcal{D}$. The orthogonal distance function is not well suited for finding the right distance, since real faults for a CD player are not modelled well as orthogonal deviations. The study of the two focus detector signals of a compact disc surface defects presented in [10] indicates that a scaling model of the defects/faults is a much better model. This model can also be argued based on some physical arguments. Given a sample of detector signals in $\mathcal{D}$, $\mathbf{f}(\mathbf{x}_{i+1})$. The defect can be modelled by a decrease of the received energy by a factor $0 < k < 1$ at all four detectors. This means that the new defective sample, $\mathbf{s}_\text{m}$ is:

$$\mathbf{s}_\text{m} \approx \beta\cdot\mathbf{f}(\mathbf{x}_{i+1}). \tag{17}$$

Thus the point $\mathbf{s}_\text{m}$ will be on the line going through $\mathbf{0}$ to $\mathbf{f}(\mathbf{x}_{i+1})$. The general structure of the algorithm to calculate scaling distance function is the same as for the orthogonal distance function, and is also based on the fact that focus and radial distances cannot change much from sample to sample. Only two of the steps in the algorithm are changed. These are: the step calculating $\mathbf{x}_{i+1}$ and the step containing the stop criteria.

$$\mathbf{f}(\mathbf{x}_i) + \nabla\mathbf{f}\cdot\Delta\mathbf{x}_{i+1} \approx \beta\cdot\mathbf{s}_\text{m} \Rightarrow \tag{18}$$

$$\nabla\mathbf{f}\cdot\Delta\mathbf{x}_{i+1} - \beta\cdot\mathbf{s}_\text{m} \approx -\mathbf{f}(\mathbf{x}_i) \Rightarrow \tag{19}$$

$$\begin{bmatrix}\nabla\mathbf{f} & -\mathbf{s}_\text{m}\end{bmatrix}\cdot\begin{bmatrix}\Delta\mathbf{x}_{i+1} \\ \beta\end{bmatrix} \approx -\mathbf{f}(\mathbf{x}_i) \Rightarrow \tag{20}$$

$$\begin{bmatrix}\Delta\mathbf{x}_{i+1} \\ \beta\end{bmatrix} \approx -\left(\begin{bmatrix}\nabla\mathbf{f} & -\mathbf{s}_\text{m}\end{bmatrix}\right)^+\cdot\mathbf{f}(\mathbf{x}_i). \tag{21}$$

The new stop criteria is the derivative of the model error:

$$\left\|2\cdot(\mathbf{f}(\mathbf{x}) - \beta\cdot\mathbf{s}_\text{m})^T\begin{bmatrix}\nabla\mathbf{f} \\ -\mathbf{s}_\text{m}\end{bmatrix}\right\| < \epsilon. \tag{22}$$

β is not a residual since it is equal 1 in case of no faults and goes towards 0 as the fault develops, instead a related residual, α, is defined as:

$$\alpha = 1 - \frac{1}{\beta}. \tag{23}$$

Previously an algorithm, based on the scaling projection and Newton-Raphson's method, is described. One iteration of algorithm used on a problem in $\mathcal{R}^2$ is illustrated in Fig. 6.

1) The algorithm:

1) Find the gradient, $\nabla(\tilde{\mathbf{x}}_i)$ to the point $(\tilde{\mathbf{x}}_i, \mathbf{f}(\tilde{\mathbf{x}}_i))$.
2) Compute:
$$\begin{bmatrix} \tilde{\mathbf{x}}_{i+1} \\ \beta \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{x}}_i \\ 0 \end{bmatrix} + \left(\begin{bmatrix} \nabla \mathbf{f} & -\mathbf{s}_{\mathrm{m}} \end{bmatrix} \right)^+ \cdot \mathbf{s}_{\mathrm{m}}.$$
3) Compute: $\gamma = ||\mathbf{s}_{\mathrm{m}} - \mathbf{f}(\mathbf{x}_{i+1})||$, $r_{\mathbf{f}}$ and $r_{\mathbf{r}}$.
4) Jump to step 1 if:
$$\epsilon < \left\| 2 \cdot (\mathbf{f}(\mathbf{x}) - \beta \cdot \mathbf{s}_{\mathrm{m}})^T \begin{bmatrix} \nabla \mathbf{f} \\ -\mathbf{s}_{\mathrm{m}} \end{bmatrix} \right\|, \text{ else stop.}$$

This algorithm has in addition two outputs: γ and α, and they might be useful for detection and classification of faults.

IV. Simulation

In this section the two algorithms' abilities to solve the described inverse problem are tested by a number of different simulations. The used input signals (e_{f} and e_{r}) to this simulation are two sine signals with a small difference in the frequency so that the two input signals are not fully correlated. The frequency and amplitude of these sine signals are chosen in a way that the maximum variance value from one sample to another is at least: 0.014μm, see Remark 1. The orthogonal projection method is tested first. Starting with a simulation without any faults, see Fig. 7. The signal with faults is constructed based on the

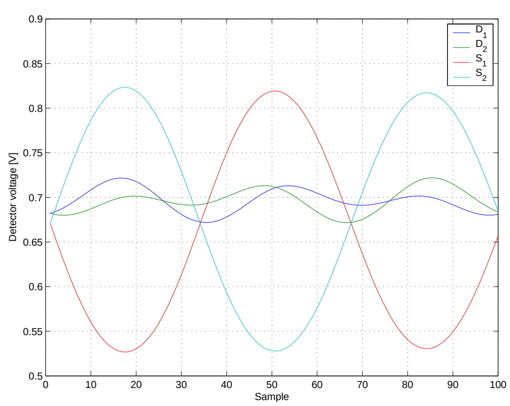

Fig. 7. The four simulated detector signals without any faults.

model in (17), where $k[n]$ represents surface defects, and the signal is illustrated in Fig. 8. Using the fault model and the fault signal in Fig. 8, the simulation series of samples with surface defects are computed and illustrated

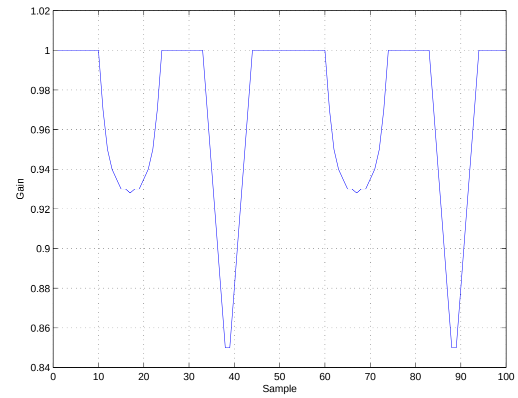

Fig. 8. The $\alpha[n]$ series, which models the surface faults of a CD used in these simulations.

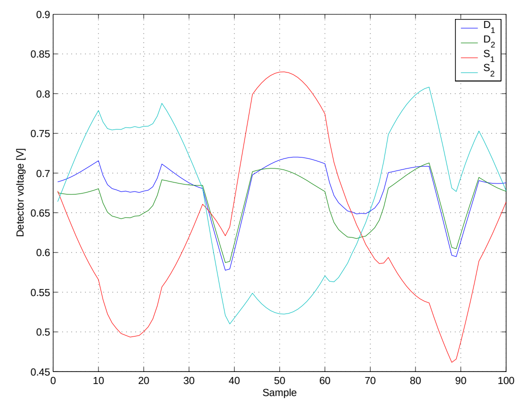

Fig. 9. Simulation of the four detector signals with surface faults.

in Fig. 9. The next step in the simulation is to apply the signal from Fig. 9 to the algorithm with the orthogonal projection. In Fig. 10 the series of $\mathbf{f}(\tilde{\mathbf{x}})$ are illustrated. From this figure it can be seen that this algorithm has a problem achieving good estimates of $\mathbf{x}$ at faulty samples. The reason for this is that the faults are not orthogonal to $\mathcal{D}$. From this simulation it is clear that the orthogonal projection method is not appropriate for the type of faults described in this paper. One final simulation of this method is performed, where the fault is almost orthogonal to $\mathcal{D}$. If the input time series of $\mathbf{x}$ is 0 in every element, the used fault model will be almost an orthogonal fault, but not completely. However, since these faults were not completely orthogonal they introduce a small distance in the solution, which is correlated with the faults. This means that even though the fault is almost orthogonal to $\mathcal{D}$, it is not enough for the orthogonal projection based algorithm to give a good solution to the inverse problem. The next simulation is of the scaling projection algorithm. In this simulation the detector signals shown in Fig. 9 are fed to the algorithm. The results are shown in Figs. 11. From this

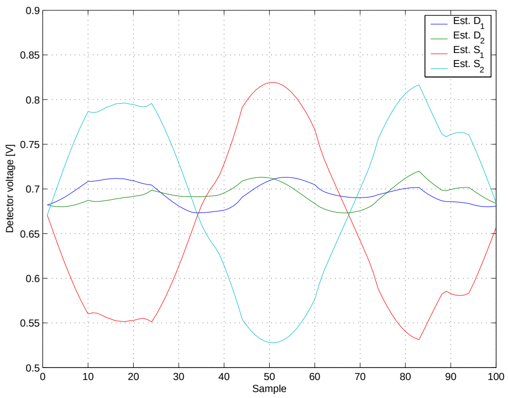

Fig. 10. Illustration of the four detector signals as they are if no fault occurs, and the ones estimated by the use of the orthogonal projection method.

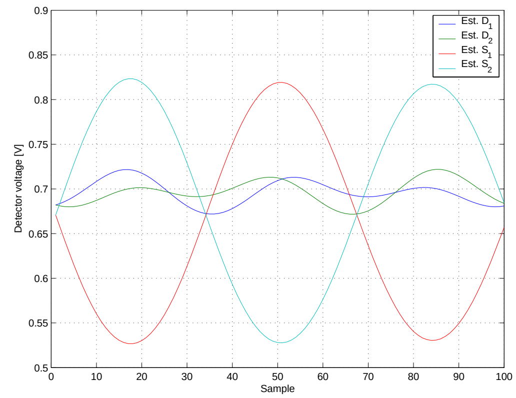

Fig. 11. Illustration of the estimated four detector signals. Notice the surface defects are removed from the signals.

plot it is clear that the scaling projection method is well suited for solving this inverse problem with the given fault model.

V. Conclusion

The simulations show that the orthogonal projection method is well suited for solving the given inverse problem if there are no faults or the faults are orthogonal to the set. However, in cases of other types of faults it is possible to achieve a better solution of the inverse problem by adjusting the algorithm to the given fault structure. In the CD player a good model of the faults would be the one described in this paper. Solving the inverse problem with these types of faults, the described scaling projection algorithm achieves very good results as illustrated in the simulations. The solution of this inverse problem gives estimates of: focus and radial distances and the two pairs of fault parameters, (e_{f}, e_{r}, r_{f}, r_{r}, γ and α).

VI. Acknowledgement

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VII. References

[1] W. Bouwhuis, J. Braat, A. Huijser, J. Pasman, G. van Rosmalen, and K. Schouhamer Immink, *Principles of Optical Disc Systems*. Adam Hilger Ltd, 1985.

[2] S. G. Stan, *The CD-ROM drive*. Kluwer Academic Publishers, 1998.

[3] Philips, *Product specification: Digital servo processor DSIC2, TDA1301T*, Philips Semiconductors, March 1994.

[4] P. Andersen, T. Pedersen, J. Stoustrup, and E. Vidal, "Method for improved reading of digital data disc," International patent, no. WO 02/05271 A1, 2001.

[5] E. Vidal, K. Hansen, R. Andersen, K. Poulsen, J. Stoustrup, P. Andersen, and T. Pedersen, "Linear quadratic control with fault detection in compact disk players," in *Proceedings of the 2001 IEEE International Conference on Control Applications*, Mexico City, Mexico, 2001.

[6] P. Odgaard, J. Stoustrup, P. Andersen, and H. Mikkelsen, "Modelling of the optical detector system in a compact disc player," in *Proceedings of the 2003 American Control Conference*, Denver, USA, June 2003.

[7] ——, "Extracting focus and radial distances, fault features from cd player sensor signals by use of a kalman estimator," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, December 2003.

[8] ——, "Estimation of residuals and servo signals for a compact disc player," 2003, submitted for publication.

[9] P. Odgaard, J. Stoustrup, P. Andersen, M. Wickerhauser, and H. Mikkelsen, "A simulation model of focus and radial servos in compact disc players with disc surface defects," 2003, submitted for publication.

[10] E. Vidal, P. Andersen, J. Stoustrup, and T. Pedersen, "A study on the surface defects of a compact disk," in *Proceedings of the 2001 IEEE International Conference on Control Applications*, Mexico City, Mexico, 2001.