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Nonlinear Reconfiguration for Asymmetric Failures in a Six Degree-of-Freedom F-16

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***Abstract*— In this paper we consider an F-16 fighter aircraft subject to asymmetric actuator failures. To address non-symmetric faults it is not possible to decouple the longitudinal and lateral dynamics. It is necessary to deal with a full six degree of freedom airframe. First, we outline an automated procedure to assemble symbolic and simulation models of complex aircraft. The symbolic model can be manipulated in various ways and used for both linear and nonlinear control system design. In the event of actuator failures, the failed surfaces not only cease to function as viable inputs but also impose persistent disturbances on the system. As previously shown, the problem of designing a reconfigured controller can be formulated as a nonlinear disturbance rejection problem. We apply this method to design a controller for the F-16.**

I. Introduction

All systems are prone to failures, in spite of regular maintenance. In complex and critical systems like the F-16, failure could lead to catastrophic consequences. Hence fault tolerant control systems have received considerable attention in the flight control literature. Fault tolerant methods can be broadly classified as 'on-line' and 'off-line'. In on-line algorithms (e.g., adaptive control approaches), the control laws are computed online and in real time. Although, this obviates the need to know the nature of the faults a priori, it requires substantial on-line computational power and hence could lead to stability issues especially for large and complex systems. In off-line schemes all critical failures are envisioned 'a priori' and appropriate control laws are designed off-line and stored in memory. Although the memory needs can be extensive it has the advantage of rapid availability and guaranteed performance.

In this paper we treat fault tolerant control of the F-16, in the event of non-symmetric failures, as a reconfigurable controller based on nonlinear regulator theory, extending a line of inquiry initiated in [1], [2]. A set of off-line laws are designed for all anticipated failure scenarios. We assume that a Fault Detection and Identification (FDI) mechanism detects and isolates the faults (with some time delay) and makes the switch to the appropriate controller. When a failure occurs the failed actuator not only ceases to be an effective control surface but also adds a persistent disturbance acting on the system. Thus, the structure of the failed and nominal plant are entirely different. A reconfigurable controller design based on regulator theory addresses this dual problem and guarantees stability and reasonable performance of the impaired plant. For many flight control systems the necessary and sufficient conditions for the existence of reconfigurable control laws are satisfied because of the redundancy of the control surfaces.

In the event of non-symmetric failures, the conventional decoupling of longitudinal and lateral equations is not valid and the impaired aircraft model should reflect a coupled nonlinear system. Moreover, if the delay in the detection process is substantial, the vehicle will diverge further from the equilibrium point before the reconfigured controller is engaged. Hence, a design based on a nonlinear model provides a larger window of safety and better performance. The range limits on control actuators present a significant impediment to post fault stability and performance. They are included in the model. The work reported here is part of an ongoing project in which we hope to characterize the range of severity of the faults that can be tolerated.

Because of the extensive algebraic manipulation required to apply such control design methods to this relatively complex model we exploit modern computer algebra tools. We treat the aircraft as a rigid body with six degrees of freedom and automatically generate symbolic mathematical and simulation models using computer algebra tools. The mathematical model is formulated in terms of Poincaré's equation and reduced to a state space form as appropriate. Because of the flexibility of the tools at our disposal, the model can be switched very easily between body and wind axes. In addition, we can easily switch between quaternion and Euler angle representations of angular configuration. We use the latter here. A linear model can also be developed about an equilibrium point. We use *Mathematica* to assemble both a symbolic (mathematical) model, that is used for design, and a computational (simulation) model in the form of a SIMULINK S-function.

We describe the six degree of freedom symbolic mathematical and simulation model construction in Section II. The reconfigured controller design is presented in Section

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III. Section IV presents the design and simulation results for an impaired F-16 with a stuck left elevator and Section V contains some concluding remarks.

II. Dynamics of the F-16

A. Creating Symbolic Models

In order to work effectively with the nonlinear 6 DOF aircraft, we have developed a set of mathematical models and computer simulation models of the F-16 aircraft using the symbolic computing program *Mathematica* [4] supplemented with the modelling and control design package *ProPac* [5]. While there are several simulation models available for the F-16, our process is unique in that we build a symbolic model that can be used for control system design (either linear or nonlinear) as well as a simulation model in the form of optimized C-code that compiles as a SIMULINK S-function. The symbolic model can be manipulated in various ways using standard *Mathematica* or specialized *ProPac* constructions. For example, linearized models can be derived or even parameter dependent linear families of models [6] can be obtained.

The aircraft is considered as a rigid body with a 6-DOF joint at the reference center of gravity location. Consider a reference frame fixed to the aircraft at the reference center of gravity location with the X, Y and Z axes in the forward, right wing and downward direction respectively. The position and orientation of this reference frame with respect to an inertial fixed frame comprise the generalized coordinate vector $\mathbf{q} = [\phi, \theta, \psi, x, y, z]^T$, where (x, y, z) gives the position and (ϕ, θ, ψ) are the Euler angles. The joint velocities, comprised of the angular velocities (p, q, r) and the linear velocities (u, v, w) relative to the X, Y and Z body axes respectively make up the quasi-velocity vector $\mathbf{p} = [p, q, r, u, v, w]^T$.

We considered a model with six control inputs, namely thrust T, left δ_{el} and right δ_{er} elevators, left δ_{al} and right δ_{ar} ailerons, and a rudder δ_r. The control surface angles are limited as follows: elevators $|\delta_{er}|, |\delta_{el}| \leq 0.436 \text{ rad } (25°)$, ailerons $|\delta_{ar}|, |\delta_{al}| \leq 0.375 \text{ rad } (21.5°)$, and rudder $|\delta_r| \leq 0.524 \text{ rad } (30°)$.

The nondimensional aerodynamic force (C_x, C_y, C_z) and moment (C_l, C_m, C_n) coefficients are expressed as multivariate nonlinear functions and were adapted from [10] (see also [11] for background on polynomial aerodynamic formulation) .

$$
\begin{aligned}
C_x &= \frac{1}{2}C_x(\alpha, \delta_{el}) + \frac{1}{2}C_x(\alpha, \delta_{er}) + C_{x_q}(\alpha)\widetilde{q} \\
C_y &= C_y(\beta, \frac{\delta_{al} - \delta_{ar}}{2}, \delta_r) + C_{y_p}(\alpha)\widetilde{p} + C_{p_r}(\alpha)\widetilde{r} \\
C_z &= \frac{1}{2}C_z(\alpha, \beta, \delta_{el}) + \frac{1}{2}C_z(\alpha, \beta, \delta_{er}) + C_{z_q}(\alpha)\widetilde{q} \\
&\quad - \frac{1}{2l_a}\frac{\delta_{al} - \delta_{ar}}{2}C_{l_{\delta_a}}(\alpha, \beta) \\
C_l &= C_l(\alpha, \beta) + C_{l_p}(\alpha)\widetilde{p} + C_{l_r}(\alpha)\widetilde{r} \\
&= +C_{l_{\delta_a}}(\alpha, \beta)\frac{\delta_{al} - \delta_{ar}}{2} \\
&\quad + C_{l_{\delta_r}}(\alpha, \beta)\delta_r + \frac{l_e}{2}((C_z)_{\delta_{er}=0} - (C_z)_{\delta_{el}=0}) \\
C_m &= \frac{1}{2}C_m(\alpha, \delta_{el}) + \frac{1}{2}C_m(\alpha, \delta_{er}) + C_{m_q}(\alpha)\widetilde{q} \\
&= +C_z(x_{cg_{ref}} - x_{cg}) \\
C_n &= C_n(\alpha, \beta) + C_{n_p}(\alpha)\widetilde{p} + C_{n_r}(\alpha)\widetilde{r} \\
&= +C_{n_{\delta_a}}(\alpha, \beta)\frac{\delta_{al} - \delta_{ar}}{2} + C_{n_{\delta_r}}(\alpha, \beta)\delta_r \\
&\quad - C_y(x_{cg_{ref}} - x_{cg})(\frac{\bar{c}}{b}) \\
\text{where} \quad \widetilde{p} &= pb/2V, \quad \widetilde{q} = q\bar{c}/2V, \quad \widetilde{r} = rb/2V.
\end{aligned}
$$

The following physical data was obtained from [12], [13] and [14]. $I_x = 9496\ slug - ft^2$, $I_y = 55814\ slug - ft^2$, $I_z = 63100\ slug - ft^2$, $I_{xz} = 982\ slug - ft^2$, $m = 637.14\ slugs$, $S = 299.992\ ft^2$, $b = 30\ ft$, $\bar{c} = 11.32\ ft$, $l_t = 0\ ft$, $l_e = 5.56\ ft$, $l_a = 6.39\ ft$.

The generalized force vector is $Q = [L, M, N, X, Y, Z]^T$ where

$$L = \frac{1}{2}\rho V^2 SC_l, M = \frac{1}{2}\rho V^2 SC_m + l_t T, N = \frac{1}{2}\rho V^2 SC_n$$

$$X = \frac{1}{2}\rho V^2 SC_x + T, Y = \frac{1}{2}\rho V^2 SC_y, Z = \frac{1}{2}\rho V^2 SC_z$$

The model is then generated (see [5], [7] and [8] for more details) in the form of Poincaré's equations [5].

$$
\begin{aligned}
\dot{\mathbf{q}} &= V(\mathbf{q})\mathbf{p} \\
M(\mathbf{q})\dot{\mathbf{p}} + C(\mathbf{q})\mathbf{p} + Q(\mathbf{p}, \mathbf{q}, \mathbf{u}) &= 0
\end{aligned} \tag{1}
$$

The function $Q(\mathbf{p}, \mathbf{q}, \mathbf{u})$, the generalized force vector, contains the aerodynamics and the input vector is $\mathbf{u} = [T, \delta_{el}, \delta_{er}, \delta_{al}, \delta_{ar}, \delta_r]^T$. We can adjoin a set of output equations

$$\mathbf{y} = g(\mathbf{p}, \mathbf{q}) \tag{2}$$

Finally Eqs. (1) and (2) are automatically coded using *ProPac* (refer [5], [7] and [8] for more details). The output is a C file that can be compiled using any standard C compiler. In this way we create a .dll file that defines the SIMULINK S-function. We should note that differential equations of the form (1) require that the symmetric positive definite matrix $M(\mathbf{q})$ needs to be inverted at each integration step. This is done efficiently using the Lapac routine *dvsop*. All required supporting subroutines are linked during the compilation process.

It is also possible to invert $M(\mathbf{q})$ symbolically and create the S-function for the resulting state space system. For the rigid airframe with relatively simple inertial dependencies on $\mathbf{q}$ and parameters (like center of mass location), it is not clear which approach is more efficient. We have done both and they work very well. In the alternative approach we first convert from body to wind coordinates, i.e., $u, v, w \mapsto V, \alpha, \beta$, using the transformation

$$
\begin{aligned}
u &= V \cos\alpha \cos\beta \\
v &= V \sin\beta \\
w &= V \sin\alpha \cos\beta
\end{aligned} \tag{3}
$$

We do this mainly to show the flexibility of our tools. Then, we have the state space system

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) \tag{4}$$

$$\mathbf{y} = g(\mathbf{x}) \tag{5}$$

where $\mathbf{x} = [\phi\ \theta\ \psi\ x\ y\ z\ p\ q\ r\ V\ \alpha\ \beta]^T$. We construct the C file for (4) again using *ProPac* and then compile as before. It is also possible to find an equilibrium point for (4) and symbolically compute the required Jacobians to assemble a linear model. An equilibrium flight condition is $\phi = 0\,rad,\ \theta = 0.0872665\,rad,\ \psi = 0\,rad,\ p = 0\,rad/s,\ q = 0\ rad/s,\ r = 0\ rad/s,\ u = 349.897\ ft/s,\ v = 0\ ft/s,\ w = 30.612\ ft/s, T = 1595.46\ lb,\ \delta_{el} = -0.0267235\ rad,\ \delta_{er} = -0.0267235\ rad,\ \delta_{al} = 0\ rad,\ \delta_{ar} = 0\ rad,\ \delta_r = 0\ rad$, which corresponds to level flight at sea level with $\rho = 0.0023769\ slug/ft^3,\ g = 32.1302\ ft/s^2$ and the center of gravity location coinciding with the reference gravity position [12]. This trim condition also satisfies the rate of climb constraint and the coordinated turn constraint [12].

B. The nominal system

We consider the F-16 at the afore mentioned equilibrium conditions. This corresponds to level flight at a constant velocity of $351.233 ft/s$ with zero heading, zero flight path angle and zero roll, ϕ. By transforming the body frame representation of velocity (V, α, β) to the space frame we get, correspondingly, the space frame velocity (V, γ, Ψ). Here γ is the flight path angle and Ψ is the heading. At the above equilibrium $\gamma = 0, \Psi = 0$. It is noted that it is an unstable equilibrium and the system needs a stabilizing controller. For control design purpose, we could drop the coordinates x, y since they decouple from the remaining equations and essentially, we wish to regulate velocities and attitude. Also, we can drop z if we choose to neglect density variations with altitude. Thus, we have the following plant.

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$$

where $\mathbf{x} = [\phi\ \theta\ \psi\ p\ q\ r\ u\ v\ w]^T$ The open loop eigenvalues of this 9^{th}-order system at the equilibrium point are

$$\{-0.7578,\ -0.0104958 \pm 0.514127i, -0.29475,\\ -0.0571886 \pm 0.0839696i,\ 0.0782493,\ -0.00266475, 0.\}$$

III. Reconfigured Controller Design

Suppose now that some of the control surfaces are stuck. The obvious fallout is that the jammed surfaces can no longer be used as inputs. Their dynamics can be expressed as

$$\dot{\mathbf{u}_f} = 0 \tag{6}$$

These failed surfaces pose an additional problem in that they now acts as persistent disturbances on the plant. Thus the plant dynamics are altered as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}_e, \mathbf{u}_f) \tag{7}$$

where $\mathbf{u}_e$ are the still effective control inputs. In essence we have partitioned the original control vector $\mathbf{u} = [\mathbf{u}_f, \mathbf{u}_e]^T$. If no action is initiated, not only will the impaired system be unable to maintain its original performance, but may become unstable with potentially catastrophic consequences.

In formulating the reconfigured controller problem, we acknowledge that it may not be possible to achieve the original performance; however, we still wish to regulate some very critical variables. Thus we can pose the the problem as follows:

Output Regulator Problem: Given the plant (7) having disturbances $\mathbf{u}_f$ with dynamics (6), and measurements (5), determine an output feedback stabilizing controller so that the variables

$$\mathbf{z} = h(\mathbf{x}) \tag{8}$$

have a prescribed ultimate-state value which, for convenience, we take to be zero.

Also, for convenience, we assume that $(\mathbf{x}, \mathbf{u_e}, \mathbf{u_f}) = (0, 0, 0)$ is an equilibrium point of (7), (6) corresponding to $\mathbf{y} = 0, \mathbf{z} = 0$. We make the following standing assumptions.

1) The number of controls is greater than or equal to the number of regulated variables, $\dim \mathbf{u_e} \geq \dim \mathbf{z}$.
2) The system $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}_e, 0)$ is exponentially stabilizable at $\mathbf{x} = 0, \mathbf{u}_e = 0$.
3) The system (7), (6) with measurements (5) is exponentially detectable at $\mathbf{x} = 0, \mathbf{u}_e = 0, \mathbf{u}_f = 0$

A complete discussion of the (local) nonlinear regulator problem for constant disturbances is given in [9]. A summary of the key results from [9] needed here is given in the following theorem.

Theorem 3.1: The output regulator problem is solvable if and only if there exist mappings $\mathbf{x} = \pi(\mathbf{u}_f)$ and $\mathbf{u}_e = c(\mathbf{u}_f)$, with $\pi(\mathbf{u}_f) = \mathbf{0}$ and $c(\mathbf{u}_f) = \mathbf{0}$, both defined in the neighborhood of the origin satisfying the conditions

$$\begin{aligned} \mathbf{0} &= f(\pi(\mathbf{u}_f), \mathbf{u}_f, c(\mathbf{u}_f)) \\ \mathbf{0} &= h(\pi(\mathbf{u}_f), \mathbf{u}_f) \end{aligned} \tag{9}$$

Furthermore, there exist functions $k_f(\mathbf{x})$ and $\ell_c(\widehat{\mathbf{x}}_c, \mathbf{y}$ such that the output regulator problem has a solution of the form

$$\mathbf{u}_e = c(\mathbf{u}_f) + k_f(\mathbf{x} - \pi(\mathbf{u}_f)) \tag{10}$$

$$\dot{\widehat{\mathbf{x}}}_c = f_c(\widehat{\mathbf{x}}_c, \mathbf{u}_e) + \ell_c(\widehat{\mathbf{x}}, \mathbf{y}) \tag{11}$$

where

(i) $\widehat{\mathbf{x}}_c$ is an estimate of the composite state $\mathbf{x}_c = [\mathbf{x}, \mathbf{u}_f]^T$
(ii) k_f is a state feedback that exponentially stabilizes $\dot{\mathbf{x}} = f(\mathbf{x}, k_f(\mathbf{x}), 0)$.
(iii) ℓ_c is any function that satisfies the conditions: (1) $\ell_c(0, 0) = 0$, (2) $\ell_c(\mathbf{x}_c, g(\mathbf{x}_c)) = f_c(\mathbf{x}_c)$, and (3) The origin is an exponentially stable equilibrium point of $\dot{\mathbf{x}}_c = \ell_c(\mathbf{x}_c, 0)$.

Proof: We omit the proof which follows directly from the results in [9]. ■

Remark 3.2 (Computing the Regulating Functions): The mappings $\mathbf{x} = \pi(\mathbf{u}_f)$ and $\mathbf{u}_e = c(\mathbf{u}_f)$ can be considered as Taylor series expansions in $\mathbf{u}_f$ (see [15])

$$\begin{aligned} \pi(\mathbf{u}_f) &= \pi'(\mathbf{0})\mathbf{u}_f + \pi''(\mathbf{0})\mathbf{u}_f^2 + \pi'''(\mathbf{0})\mathbf{u}_f^3 + \cdots \\ c(\mathbf{u}_f) &= c'(\mathbf{0})\mathbf{u}_f + c''(0)\mathbf{u}_f^2 + c'''(0)\mathbf{u}_f^3 + \cdots \end{aligned} \tag{12}$$

and the unknown coefficients can be determined by substitution the expansion (12) into (9).

The higher order terms not only improve the regulation but also allow for more delay in detecting the fault by which time the system could have wandered into nonlinear regimes. The number of variables that can be regulated cannot be greater than the number of available controls. However, this is not a sufficient condition for the existence of a solution to (9).

Remark 3.3 (Composite Observer Design): The primary purpose of incorporating an observer is to estimate the stuck actuator positions. If all the states cannot be measured, it should estimate those states as well. The observer is composite in the sense that it can estimate both the states and the stuck actuator positions.

In formulating the observer (17), we recognize that the impaired plant dynamics (6), (7) can be recast as

$$\dot{\mathbf{x}}_c = f_c(\mathbf{x}_c, \mathbf{u}_e) \tag{13}$$

where $\mathbf{x}_c = [\mathbf{x}, \mathbf{u}_f]^T$.

Remark 3.4 (The Functions k_f and ℓ_c): In the present work, we will choose

$$k_f(\mathbf{x}) = K_f \mathbf{x} \tag{14}$$

K_f is a matrix that asymptotically stabilizes $(A_f + B_f K_f)$ where $A_f = \frac{\partial f}{\partial \mathbf{x}}(0,0,0)$ and $B_f = \frac{\partial f}{\partial \mathbf{u}_e}(0,0,0)$. Consequently,

$$\mathbf{u}_e = c(\mathbf{u}_f) + K_f(\mathbf{x} - \pi(\mathbf{u}_f)) \tag{15}$$

Also, we choose

$$\ell_c(\widehat{\mathbf{x}}, y) = L_c(h(\widehat{\mathbf{x}}) - y) \tag{16}$$

where L_c is any matrix that asymptotically stabilizes $(A_c + L_c C)$ and the matrices A_c and C are given by $A_c = \frac{\partial f_c}{\partial \mathbf{x}_c}(0,0)$ and $C = \frac{\partial g}{\partial \mathbf{x}_c}(0)$. Thus, the observer is

$$\dot{\widehat{\mathbf{x}}}_c = f_c(\widehat{\mathbf{x}}_c, \mathbf{u}_e) + L_c(h(\widehat{\mathbf{x}}) - y) \tag{17}$$

During normal functioning, the plant operates with the nominal controller and observer. The outputs and the specific controls inputs are provided to each reconfigured controller and observer as well. When a failure occurs, the FDI mechanism detects and isolates the fault and switches to the appropriate controller. Since the observers are furnished with necessary information from the outset of the failure (in fact, the information is provided even before the failure occurs), it is able to provide a better estimate than if were to begin estimating only after the switching occurred.

IV. Nonlinear Reconfiguration of the F-16 with the Left Elevator Jammed

The controller for the nominal system was designed as a LQR controller with equal weighting on all states and control inputs. Our choice of the controller does not necessarily reflect the ones in practice or is it necessarily the ideal choice; our sole purpose is to stabilize the system at its equilibrium and we do not delve into other performance characteristics of the nominal plant in this paper. The resulting closed loop eigenvalues are

$$\{-20.6176,\ -9.67398 \pm 4.26946i,\ -5.26487,\ -1.41421,\ -0.20114 \pm 0.226455i,\ -1.30797,\ -0.558141,\ -0.0919829\}$$

It should be noted that we incorporated a simplified engine model in the above and all subsequent designs. The engine dynamics was modelled as a first order system with unit time constant, i.e.,

$$\dot{T} = T_c - T \tag{18}$$

where T_c is the commanded thrust and T is the actual thrust.

Let us consider a scenario when the left elevator is stuck at -0.05 radians, i.e., $\mathbf{u}_f = [\delta_{el}]$ and $\mathbf{u}_e = [T_c, \delta_{er}, \delta_{al}, \delta_{ar}, \delta_r]^T$. If we continue to employ a nominal controller the plant will become unstable and loose regulation. At the very least, we wish to maintain orientation of the impaired aircraft. We also wish to regulate velocity, in order to circumvent stall. Thus we have

$$h(x) = [\phi, \gamma, \Psi, V]^T$$

which has a steady state value of $[0\ rad,\ 0\ rad,\ 0\ rad,\ 351.3\ ft/s]^T$.

The mappings (12) were determined up to fourth order as

$$\begin{aligned}
\pi'(\mathbf{0}) &= [0, -0.02442, 0.04625, 0, 0, 0, 0.7475,\\
&\quad -16.2428, -8.5445, -154.32]^T\\
\pi''(\mathbf{0}) &= [0, -0.010077, -0.07437, 0, 0, 0, -0.1699,\\
&\quad 26.1208, -3.5678, 12771.2]^T\\
\pi'''(\mathbf{0}) &= [0, 0.00399384, 0.02516, 0, 0, 0, 0.9941,\\
&\quad -8.83007, 1.5052, -2913.57]^T\\
\pi^{iv}(\mathbf{0}) &= [0, -0.0336048, -0.03104, 0, 0, 0, -0.3273,\\
&\quad 10.8744, -11.9016, 1441.96]^T\\
c'(\mathbf{0}) &= [-154.32, -1.00374, -7.8114, 7.8114, 2.5385]^T\\
c''(\mathbf{0}) &= [12771.2, 0.2255, 1.0717, -1.0717, 0.1263]^T\\
c'''(\mathbf{0}) &= [-2913.57, -0.09627, -0.6211, 0.621096, 0.0519]^T\\
c^{iv}(\mathbf{0}) &= [1441.96, 0.7296, 2.968, -2.968, -0.8706]^T
\end{aligned}$$

The gain K_f was designed using a Riccati equation with weighting one on all the available control inputs and states except the ailerons δ_{al} and δ_{ar} which had weight twenty. The resulting closed loop system had eigenvalues

$$\{-13.2158 \pm 2.47417i,\ -4.72321,\ -1.88107,\ -1.41421,\ -0.27263 \pm 0.17751i,\ -0.0936843,\ -0.17613 \pm 0.207032\}$$

which are obviously in the left half plane. It was observed that any K_f that stabilizes the system does not guarantee system feasibility (because of the control magnitude constraints). The resulting reconfigured control law (15) is

$$\begin{aligned}
T_c &= 2264.58 + 738.406\delta_{el} + 17739.6\delta_{el}^2 - 3902.43\delta_{el}^3\\
&\quad +2039.23\delta_{el}^4 - 3.3236 \times 10^{-5}p + 1.57763 \times 10^{-4}\phi\\
&\quad +1.7144 \times 10^{-5}\psi + 0.06691q - 0.00151r - 0.4142T\\
&\quad +0.2459\theta - 3.5772 \times 10^{-3}u + 6.1589 \times 10^{-6}v\\
&\quad -9.6215 \times 10^{-4}w\\
\delta_{er} &= 273.56 + 5.1557\delta_{el} + 18.3237\delta_{el}^2 - 3.0544\delta_{el}^3\\
&\quad +13.8157\delta_{el}^4 - 0.440724p - 0.3273\phi - 0.110914\psi\\
&\quad +44.9042q + 1.87501r - 8.6514 \times 10^{-4}T\\
&\quad +118.165\theta - 0.85345u - 0.1694v + 0.53004w\\
\delta_{al} &= 5.65424 - 7.8331\delta_{el} + 1.4814\delta_{el}^2 - 0.401556\delta_{el}^3\\
&\quad +3.11407\delta_{el}^4 + 0.5537p + 0.1499\phi + 0.1571\psi
\end{aligned}$$

$$
\begin{aligned}
& +6.837q + 0.8419r - 1.7807 \times 10^{-5}T \\
& +2.10440\theta - 0.016936u - 0.00799v - 0.003072w \\
\delta_{ar} \quad = \quad & -5.65424 + 7.8331\delta_{el} - 1.4814\delta_{el}^2 + 0.401556\delta_{el}^3 \\
& -3.11407\delta_{el}^4 - 0.5537p - 0.1499\phi - 0.1571\psi \\
& -6.837q - 0.8419r + 1.7807 \times 10^{-5}T \\
& -2.10440\theta + 0.016936u + 0.00799v + 0.003072w \\
\delta_r \quad = \quad & -49.8355 - 13.2841\delta_{el} + 22.2369\delta_{el}^2 - 7.1283\delta_{el}^3 \\
& +7.7036\delta_{el}^4 - 2.658p - 2.573\phi - 0.02046\psi \\
& -12.1703q + 27.4859r + 1.564 \times 10^{-4}T \\
& -21.35030\theta + 0.1535u - 0.9652v - 0.0864w \qquad (19)
\end{aligned}
$$

For the equilibrium values of the states and the stuck elevator δ_{el}, (19) yielded the equilibrium values of $\mathbf{u}_e$ as expected.

The observer gain L_c, was chosen so that the eigenvalues of $(A_c + L_c C)$ are

$$
\begin{gathered}
\{-13.2472 \pm 13.2189i,\ -12.5935 \pm 12.543i, \\
-4.07242 \pm 3.9497i,\ -1.2974,\ -1.00908, \\
-1.2797 \pm 1.1940i,\ -1.009\}
\end{gathered}
$$

In Figures 1 through 2, the failure occurs at $t = 0$. The debilitated aircraft continues to operate with the nominal controller until a switch is made by a FDI mechanism at $t = 2$ seconds to the reconfigured controller. We observe that when the left elevator is stuck at -0.05 radians (approx $2.86°$ downwards), the impaired aircraft rolls towards the right (ϕ increases) and pitches upwards (γ decreases). The velocity also decreases. It was observed that with the nominal controller the F-16 would gain altitude while rolling and heading towards the right and eventually loose stability. When the reconfigured controller took over, the roll, flight path angle, heading and velocity was $0.1895\ rad,\ -0.0014\ rad,\ 0\ rad$ and $351.1502\ ft/s$ respectively. To accomplish the desired regulation the reconfigured controller had to provide increased thrust. The two aileron motions are anti-symmetrical and aid in correcting the roll. The simulations show the interplay between longitudinal and lateral dynamics. From the plots we see that the regulation is accomplished in approximately 55 seconds.

V. Conclusions

In this paper, we report on the application of nonlinear regulator theory to the problem of aircraft control system reconfiguration to accommodate jammed actuators. This work extends the formulation of [1], [2] and applies it to a real aircraft.

We have illustrated a unique and convenient method to build symbolic and simulation models for complex aircraft. The former can be used for control system design (either linear or nonlinear) and the latter is automatically created in the form of optimized C-code that compiles as a SIMULINK S-function.

The problem of stuck actuators not only reduces the number of control surfaces but also imposes additional disturbances on the system. By treating the reconfiguration problem as a nonlinear output regulation problem we are able to compensate for the uncertainty of the failed actuator

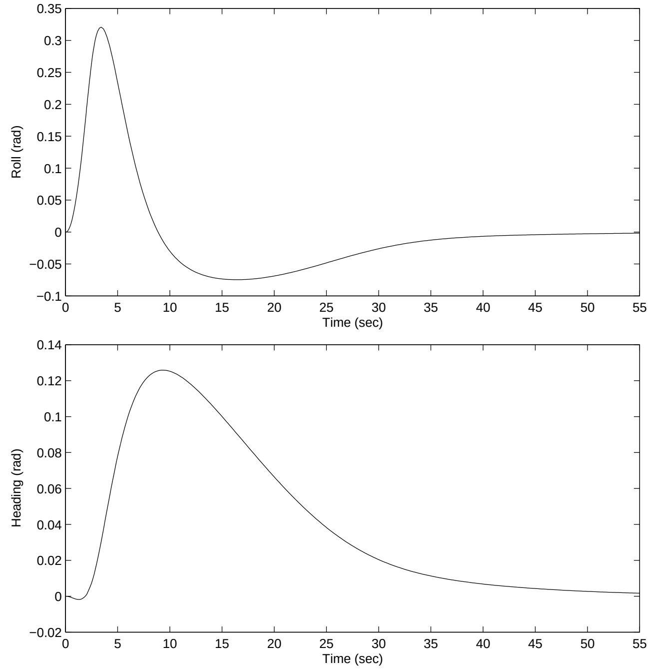

Fig. 1. Trajectories of the lateral regulated variables, roll and heading, for the impaired aircraft. The reconfigured controller takes over from the nominal controller at 2 sec.

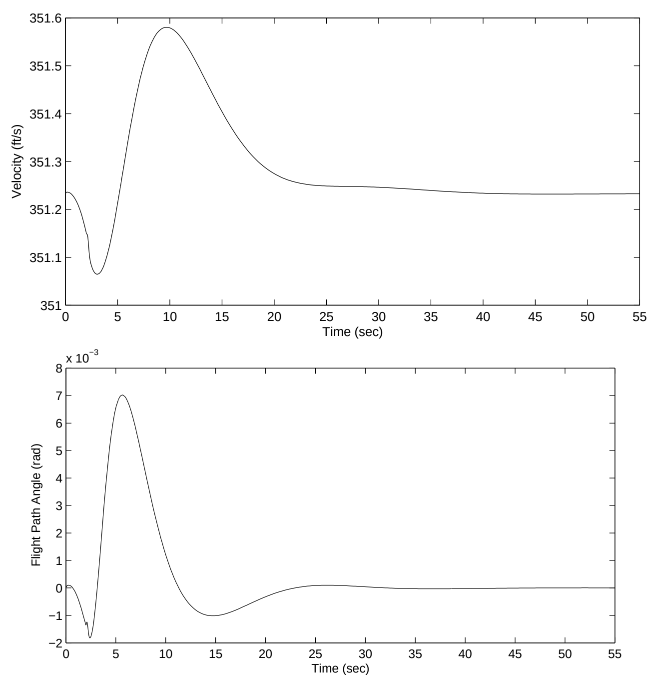

Fig. 2. Trajectories of the longitudinal regulated variables, flight path angle and velocity, for the impaired aircraft. The reconfigured controller takes over from the nominal controller at 2 sec.

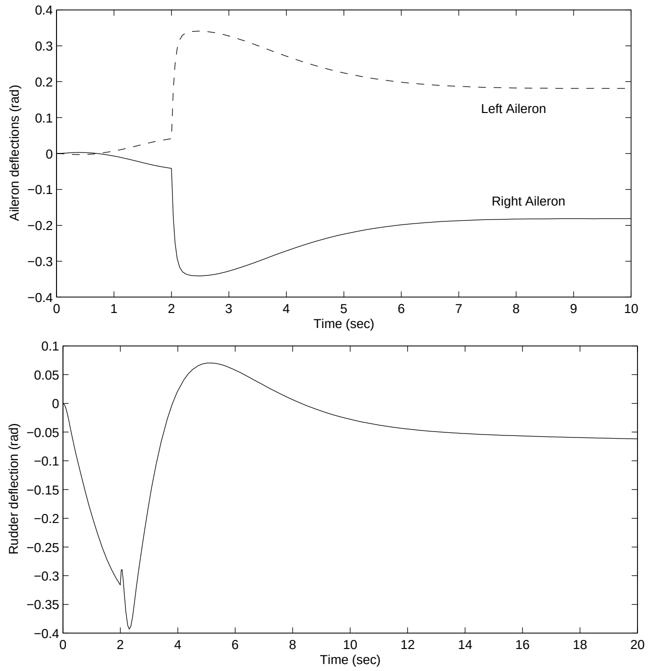

Fig. 3. Trajectories of the effective control inputs: rudder and aileron.

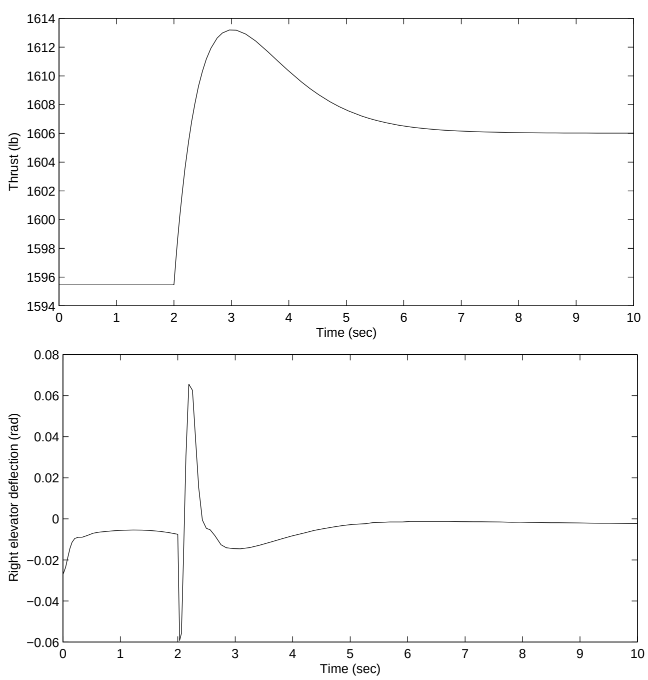

Fig. 4. Trajectories of the effective control inputs: thrust and right elevator.

position. Computation of the nonlinear controller is facilitated by working in a symbolic computing environment. In this regard, the relatively complex controller is automatically coded, just as the model, producing a separate SIMULINK S-function that is integrated into a closed loop simulation.

We have demonstrated that the approach is viable. Ultimately, we need to determine the severity of the faults that can be accommodated by the fault tolerant controller. Two factors are critical: the time for fault identification, and the post fault controller design. So far, we have focused mainly on the tools we need to address these issues. In the design presented here, we note that with a delay of 2 sec in fault detection we can tolerate a range of displacements for the failed elevator of about -0.06 rad to -0.01 rad. At the extreme, with a delay of 0 sec, we can tolerate about -0.07 rad to 0.02 rad. Our controller, however, has not been optimized. Current work is focused on two questions: How can we systematically evaluate the envelope of positions for single and multiple surface failures that can be accommodated by a given controller, and how can we design the post-fault controller to maximize that envelope?

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