# Adaptive Compensation of Morphing Actuator Failures

Shuhao Chen

Dept. of Naval Architecture and Marine Engineering, Univ. of Michigan, Ann Arbor, MI 48109 *Gang Tao* and *Juntao Fei* Dept. of Electrical and Computer Engineering, Univ. of Virginia, Charlottesville, VA 22903 *Suresh M. Joshi* 

Mail Stop 132, NASA Langley Research Center, Hampton, VA 23681

### Abstract

In this paper, we develop an adaptive approach for compensating uncertain failures of morphing actuators used for flight control of morphing aircraft. Morphing actuators are special actuators that operate at only two states: "on" and "off", and are used in a large quantity. Failures of such actuators are characterized by some actuators being stuck at one state and unable to be changed by applied control inputs. We present a novel failure model for such actuators, by considering the failure percentage as an uncertain parameter, and derive a piecewise-linear characteristic to describe such failures. We develop an adaptive inverse of such characteristic to compensate for the uncertain actuator failures. Updated from a parameter projection adaptive law and combined with a state feedback control law, such an adaptive inverse compensation scheme is able to ensure desired closed-loop stability and tracking properties in the presence of uncertain actuator failures. An application of this compensation scheme to the control of an ICE aircraft model with morphing actuators is studied and simulation results are presented to illustrate the effectiveness of the actuator failure compensation design.

### **1** Introduction

Morphing aircraft are novel concept air vehicles that can achieve high performance under different flight conditions and environment via virtual geometry change. Such geometry change is realized by the new shape change actuators (effectors) which replace the conventional actuators such as ailerons or rudders. Some special features of the shape change actuators are that they operate only at two states: "on" and "off", represented by their normalized value: 1 and 0, and that they are used in large numbers to fulfill certain actuation function.

The features of morphing actuators make the morphing aircraft control problems, including the actuator failure compensation problem, quite different from the control problems for conventional aircraft. There have been some design methods to deal with actuator failures, such as the multiple models, switching and tuning designs [3], [5], adaptive designs [1], [2], [4], fault diagnosis method [10], and robust fault accommodation [6]. However, such designs may not be applicable

to the actuator failure problem of morphing aircraft. For actuator failures that may occur to morphing aircraft, new models are needed to describe the failures, and new compensation schemes are needed to handle such failures. In [9], we addressed the morphing actuator failure model and its adaptive compensation for the case wherein the morphing actuators only fail at the "off" (that is, the 0 value) state. In this paper, we address the more general case in which the actuators may fail at both the "on" and "off" states. While failures at the "off" state produce no additional force and moment, the failures at the "on" state do. Therefore, new actuator failure compensation schemes, for both the nominal case with known failure parameters and the adaptive case with unknown failure parameters, are needed. It is the goal of this paper to present such an adaptive actuator failure compensation scheme for morphing aircraft flight control. To derive such an adaptive control scheme, new solutions to some key issues such as morphing actuator failure modeling, failure compensation parametrization, implementation error analysis, and adaptive control algorithm are presented in detail.

## 2 **Problem Formulation**

**Morphing Aircraft and Actuators**. Morphing aircraft use small shape-change effectors (called morphing actuators in this paper) in large numbers to fulfill the flight control task without flap. The morphing aircraft model used in this paper is the representative Innovative Control Effector (ICE) aircraft [8], whose wing span is depicted in Figure 1. The effector suite under study includes four arrays on each wing: the upper-surface leading-edge (ULE) array, the upper-surface trailing-edge (UTE) array , the upper surface wingtip (UTip) array, and the lower-surface trailing-edge (LTE) array.

The aircraft control system turns the shape change effectors "on" or "off", in normalized numerical values, either 1 or 0, to generate the needed control signal. More effectors are turned on to produce larger force as needed. For the control of lateral-directional dynamics, a negative control input is implemented by turning on the actuators on the other wing.

A nonlinear model of the ICE morphing aircraft describing both the longitudinal and lateral-directional dynamics was developed in [8]. In this paper, we only consider the linearized



Figure 1: Shape-change actuator arrays of the ICE aircraft [8].

lateral-directional dynamics, which is described by a linear time-invariant plant

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.1}$$

where  $x(t) = [v, p, r, \phi]^T(t) \in \mathbb{R}^4$ , and  $u(t) \in \mathbb{R}^4$  represents the control input from the four morphing actuators arrays described above, and

$$B = [b_1, b_2, b_3, b_4], \ b_i \in \mathbb{R}^4$$
(2.2)

with each  $b_i$  denoting the actuation vector for one actuator array. The state variables are: v (velocity along the y-axis of the body coordinate system), p (roll rate), r (yaw rate), and  $\phi$  (roll angle). The actuation signal  $b_i u_i$  is to be approximated by a sum of actuation signals generated by a set of actuation vectors associated with a set of morphing actuators. We should note that although we only consider 4 states and 4 inputs, the results presented in this paper can be generalized to plants of any dimensions of states and inputs.

**Nominal Feedback Control**. The first problem of morphing aircraft control is how a nominal control signal for each actuation array is given. By assuming that the pair (A, B) is known, and x(t) is available, we use the nominal control law

$$u(t) = u_d(t) = K_1 x(t) + K_2 r_d(t)$$
(2.3)

where  $K_1 \in \mathbb{R}^{4 \times 4}$  is a state feedback gain matrix,  $K_2 \in \mathbb{R}^{4 \times n_r}$ and  $r_d(t) \in \mathbb{R}^{n_r}$ ,  $n_r \leq 4$ , is a reference input vector signal, which leads to the closed-loop system

$$\dot{x}(t) = Ax(t) + Bu^{0}(t) = (A + BK_{1})x(t) + BK_{2}r_{d}(t)$$
  
=  $A_{m}x(t) + B_{m}r_{d}(t).$  (2.4)

All eigenvalue of  $A_m$  are in the open left-half complex plane. The aircraft model under study is controllable and the nominal control law (2.3) can achieve arbitrary pole placement.

**Control Signal Implementation**. Let  $v(t) = [v_1(t), v_2(t), v_3(t), v_4(t)]^T$  be an applied control signal vector to be implemented. There are different schemes to implement v(t). We consider the same control signal implementation scheme as that in [9]: let  $v_i(t)$  be the *i*th component of v(t), then (i) positive  $v_i(t)$  is implemented by the morphing actuators in array *i* on the right wing and negative  $v_i(t)$  is implemented by those on the left; and (ii) actuators are activated in the order of increasing *j*, that is, the first  $n_i$  actuators from one end of the actuator array *i* is activated to approximate  $v_i(t)$ , with the number  $n_i$  being determined by the magnitude of  $v_i(t)$ .

Assume that there are totally  $2N_i$  actuators in array i,  $N_i$  on each wing (left and right), and for  $b_{lij}$  and  $b_{rij}$ , i = 1, 2, 3, 4,  $j = 1, 2, ..., N_i$ , the regional actuation vectors of the morphing actuators on the left and right wings,

$$b_{lij} = \alpha_{lij}b_i, \ b_{rij} = \alpha_{rij}b_i \tag{2.5}$$

for some constants  $\alpha_{lij} < 0$ ,  $\alpha_{rij} > 0$ , i = 1, 2, 3, 4,  $j = 1, 2, ..., N_i$ . Due to the fact that the morphing actuators on left and right wings are symmetrically distributed and their effects on the lateral-directional motion are opposite, we have

$$b_{lij} = -b_{rij}, \ \alpha_{lij} = -\alpha_{rij}, \ j = 1, 2, \dots, N_i.$$
 (2.6)

Then, the control law implementation scheme is

$$v_i(t) \approx \begin{cases} \sum_{j=1}^{n_i} \alpha_{rij} & \text{if } v_i(t) \ge 0\\ \sum_{i=1}^{n_i} \alpha_{lij} & \text{if } v_i(t) < 0 \end{cases}$$
(2.7)

where  $n_i \le N_i$  is the number of activated actuators. Introducing the indicator function of an event *X* as

$$\chi[X] = \begin{cases} 1 & \text{if } X \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$
(2.8)

and defining

$$\alpha_{ij}(t) = \chi[v_i(t) \ge 0]\alpha_{rij} + \chi[v_i(t) < 0]\alpha_{lij} \qquad (2.9)$$

we can express (2.7) as

$$v_i(t) \approx \sum_{j=1}^{n_i} \alpha_{ij}(t).$$
(2.10)

For the considered control problem, we make the following basic assumption:

(A1) There are enough actuators with enough density to implement a desired control signal v(t) for meeting desired stability and tracking performance requirements.

An actuation error inevitably occurs when the nominal feedback control signal is implemented by the morphing actuators. As in [9], we refer to this error as the implementation error. From (2.10), the implementation error for array i is

$$e_{1i}(t) = \left(\sum_{j=1}^{n_i} \alpha_{ij}(t) - \nu_i(t)\right) b_i \tag{2.11}$$

which gives

$$\|e_{1i}(t)\|_2 \le |\sum_{j=1}^{n_i} \alpha_{ij}(t) - v_i(t)| \|b_i\|_2.$$
 (2.12)

Following the analysis procedure in [9], we have

$$|\sum_{j=1}^{n_i} \alpha_{ij}(t) - \nu_i(t)| \le \delta_{1i}$$

$$(2.13)$$

for some constant  $0 < \delta_{1i} < \max\{|\alpha_{ij}(t)|\}$  (that is,  $0 < \delta_{1i} < \max\{|\alpha_{lij}|\} = \max\{|\alpha_{rij}|\}$ ) and independent of  $v_i(t)$ . Hence,

$$\|e_{1i}(t)\|_2 \le \delta_{1i} \|b_i\|_2 \stackrel{ riangle}{=} \varepsilon_{1i}$$
 (2.14)

and the total implementation error

$$e_1(t) = \left(\sum_{i=1}^4 \left(\sum_{j=1}^{n_i} \alpha_{ij}(t)\right) b_i\right) - Bv(t) = \sum_{i=1}^4 e_{1i}(t) \quad (2.15)$$

is bounded as

$$||e_1(t)||_2 \le \sum_{i=1}^4 ||e_{1i}(t)||_2 \le \sum_{i=1}^4 \varepsilon_{1i} \stackrel{\triangle}{=} \varepsilon_1$$
 (2.16)

for some constant  $\varepsilon_1 > 0$  independent of v(t).

Actuator Failures. Actuator failures, which may happen during system operation, introduce large actuation error to the control system and make the nominal control signal  $v(t) = u_d(t)$  designed from (2.3) unable to achieve the desired performance requirements and must be redesigned.

In this paper, we consider the adaptive actuator failure compensation problem for the case when actuators may fail at either 1 or 0, that is, when an actuator fails, it is either "on" or "off". Although there may be different actuator failure patterns in different situations, we consider only the failure pattern that the failed actuators are uniformly distributed such that the actuator failures can be modeled in terms of failure percentage. This is an actuator failure model of practical significance. Since in each array, the actuators are of the same type, they should have the same possibility of failure, due to the large number of actuators, the distribution of failed actuators among the total  $N_i$  of them appears to be uniform.

Letting  $\alpha_{li}$  and  $\alpha_{ri}$  be the failure percentage parameters for failures at 0 on the left and right wings, and  $\beta_{li}$  and  $\beta_{ri}$  for failures at 1, respectively, and defining

$$m_{ri} = 1 - \alpha_{ri} - \beta_{ri}, \ m_{li} = 1 - \alpha_{li} - \beta_{li}$$
 (2.17)

where  $m_{ri}$  and  $m_{li}$  represent the percentage of functioning actuators on the right wing and left wing respectively, then the actuator failures can be approximately modeled as

$$m_i \approx m_{ai} = \begin{cases} m_{ri}n_i + \beta_{ri}N_i + \beta_{li}N_i & \text{if } v_i(t) \ge 0\\ m_{li}n_i + \beta_{li}N_i + \beta_{ri}N_i & \text{if } v_i(t) < 0 \end{cases}$$
(2.18)

where  $m_i$  is the number of actually activated actuators, and  $n_i$  is the desired number of actuators to be activated to implement an applied control input  $v_i(t)$ .

Unlike the case studied in [9], where  $0 \le m_i \le n_i$ , the number  $m_i$  from (2.18) may be larger than  $n_i$ , or less than 0, because the actuators failed at 1 in the same wing but outside the chosen  $n_i$  give an additional number of activated actuators, and those on the other wing give a negative number of activated actuators in the sense of control effect. A negative  $m_i$  means that more actuators on the opposite wing are actually activated than those on the desired one, and the sign of the actual control signal is also different from that of  $v_i(t)$ .

It is also important to note that (2.18) is only an approximation of the actuator failure model. One simple but obvious reason is that, in general, the right hand of (2.18) is not an integer but  $m_i$  should always be an integer. We may also wonder how small the error given in (2.18) is so that a failure pattern can be classified as uniformly distributed. Therefore, here we give a more accurate definition of the uniformly distributed actuator failures: the failed actuator are so distributed such that (i) for the number of activated actuators,

$$|m_i - m_{ai}| \le k_i \tag{2.19}$$

for some chosen constant  $k_i > 0$  independent of  $n_i$ , and (ii) for the actual control input,

$$|u_i(t) - w_i(t)| \le \delta_{2i} \tag{2.20}$$

for some chosen constant  $\delta_{2i}$  independent of  $n_i$  and  $v_i$ , with  $u_i(t)$  be the actual control input produced by the  $m_i$  activated actuators, and

$$w_{i}(t) = \begin{cases} m_{ri} \sum_{j=1}^{n_{i}} \alpha_{ij}(t) + m_{0i} & \text{if } v_{i}(t) \ge 0\\ m_{li} \sum_{j=1}^{n_{i}} \alpha_{ij}(t) + m_{0i} & \text{if } v_{i}(t) < 0 \end{cases}$$
(2.21)

where

$$m_{0i} = \beta_{ri} \alpha_i - \beta_{li} \alpha_i \tag{2.22}$$

and  $\alpha_i$  be the total positive control input that array *i* can produce by activating all actuators on the right wing, that is,

$$\alpha_i = \sum_{j=1}^{N_i} \alpha_{rij} = -\sum_{j=1}^{N_i} \alpha_{lij}.$$
 (2.23)

From (2.20), the actuation error caused by the modeling error for array i is

$$e_2(t) = \sum_{i=1}^4 e_{2i}(t) \stackrel{\triangle}{=} \sum_{i=1}^4 b_i(u_i(t) - w_i(t))$$
(2.24)

which is bounded by a constant  $\varepsilon_2$ :

$$\|e_2(t)\|_2 \le \sum_{i=1}^4 \|b_i\|_2 \delta_{2i} \stackrel{\triangle}{=} \varepsilon_2.$$
 (2.25)

We can express the actuator failure model in terms of the values of signals with a piecewise-linear nonlinearity  $N(\cdot)$ :

$$u_i(t) \approx N(v_i(t)) = \begin{cases} m_{ri}v_i(t) + m_{0i} & \text{if } v_i(t) \ge 0\\ m_{li}v_i(t) + m_{0i} & \text{if } v_i(t) < 0 \end{cases}$$
(2.26)

where  $v_i(t)$  is the designed (applied) control input, and  $u_i(t)$  is the actual control input to the morphing aircraft plant (2.1). This failure model is approximately depicted in Figure 2.

The total actuation error of the model (2.26) is

$$e_{3i}(t) = b_i(u_i(t) - N(v_i(t)))$$
  
=  $b_i(u_i(t) - w_i(t) + w_i(t) - N(v_i(t)))$   
=  $\begin{cases} e_{2i}(t) + m_{ri}e_{1i}(t) & \text{if } v_i(t) \ge 0\\ e_{2i}(t) + m_{li}e_{1i}(t) & \text{if } v_i(t) < 0. \end{cases}$  (2.27)



Figure 2: Morphing actuator failure characteristic.

The control objective is to design a control signal v(t) based on the nominal control signal  $u_d(t)$  for the actuator arrays, such that despite the actuator failures, the nominal control signal is appropriately implemented and the desired state tracking is achieved with only small tracking error.

**Inverse Design for Failure Compensation**. The morphing actuator failure model (2.26) is similar to a piecewise-linear characteristic, with different gains in different signal regions.

In our control scheme, an adaptive inverse  $NI(\cdot)$  is to be used to cancel the nonlinearity  $N(\cdot)$ , that is,  $v_i(t) = \widehat{NI}(u_{di}(t))$ , where  $u_{di}(t)$  is the desired control input from the nominal feedback control (2.3). If the failure percentage parameters are known, the ideal inverse  $NI(\cdot)$  for  $N(\cdot)$  is designed as

$$v_i^*(t) = NI(u_{di}(t)) = \begin{cases} \frac{u_{di}(t) - m_{0i}}{m_{ri}} & \text{if } u_{di}(t) \ge m_{0i} \\ \frac{u_{di}(t) - m_{0i}}{m_{li}} & \text{if } u_{di}(t) < m_{0i}. \end{cases}$$
(2.28)

Such an inverse signal  $v_i^*(t)$ , when applied to the actuator arrays, activates appropriate numbers of actuators such that the actual control signal generated by the morphing actuators is the desired control signal  $u_{di}$  within sufficient accuracy.

#### **3** Adaptive Control Scheme

In this section, we develop an adaptive control scheme and feedback control when the failure percentage parameters are unknown, to achieve closed-loop stability and tracking.

Adaptive Inverse Design. When failure percentage parameters are unknown, we use the adaptive inverse

$$v_i(t) = \widehat{NI}(u_{di}(t)) = \begin{cases} \frac{u_{di}(t) - \widehat{m_{0i}}(t)}{\widehat{m_{ri}}(t)} & \text{if } u_{di}(t) \ge \widehat{m_{0i}}(t) \\ \frac{u_{di}(t) - \widehat{m_{0i}}(t)}{\widehat{m_{li}}(t)} & \text{if } u_{di}(t) < \widehat{m_{0i}}(t) \end{cases}$$

$$(3.1)$$

where  $\widehat{m_{ri}}(t)$ ,  $\widehat{m_{0i}}(t)$ , and  $\widehat{m_{li}}(t)$  are the online estimates of the failure percentage parameters  $m_{ri}$ ,  $m_{0i}$ , and  $m_{li}$ , and

$$u_d(t) = [u_{d1}(t), u_{d2}(t), u_{d3}(t), u_{d4}(t)]^T = K_1 x(t) + K_2 r_d(t)$$
(3.2)

is the desired control signal from the feedback control law (2.3). Since  $m_{ri} > 0$  and  $m_{li} > 0$ , we can always use parameter



Figure 3: Inverse failure compensation characteristic.

projection to ensure  $\widehat{m_{ri}}(t) > 0$ , and  $\widehat{m_{li}}(t) > 0$ . Therefore,  $u_{di}(t) \ge \widehat{m_{0i}}(t)$  is equivalent to  $v_i(t) \ge 0$  and  $u_{di}(t) < \widehat{m_{0i}}(t)$  is equivalent to  $v_i(t) < 0$ .

Since the estimates rather than the true values of failure percentage parameters are used in the adaptive design, the actual control signal implemented by the morphing actuators is different from the control signal  $u_{di}(t)$  designed from the nominal feedback control law (2.3).

To parameterize the adaptive inverse and the associated control error, we introduce the indicator functions

$$\chi_r(t) = \chi[0 < v_i(t)], \ \chi_l(t) = \chi[v_i(t) < 0]$$
 (3.3)

and the parameter vectors  $\theta_i^*$ ,  $\theta_i(t)$  and the related regressor vector  $\omega_i(t)$ :

$$\theta_i^* = [m_{ri}, m_{0i}, m_{li}]^T \tag{3.4}$$

$$\boldsymbol{\theta}_{i}(t) = [\widehat{m_{ri}}(t), \widehat{m_{0i}}(t), \widehat{m_{li}}(t)]^{T}$$
(3.5)

$$\boldsymbol{\omega}_i(t) = [\boldsymbol{v}_i \boldsymbol{\chi}_r(t), 1, \boldsymbol{v}_i \boldsymbol{\chi}_l(t)]^T.$$
(3.6)

Then we have

$$N(v_i(t)) = \Theta_i^{*T} \omega_i(t), \ u_{di}(t) = \Theta_i^T(t) \omega_i(t).$$
(3.7)

With the adaptive inverse (3.1), the control error is

$$u_{i}(t) - u_{di}(t) = u_{i}(t) - N(v_{i}(t)) - (\theta_{i}(t) - \theta_{i}^{*})^{T} \omega_{i}(t).$$
(3.8)

**Error Equation**. From (2.1), (2.4) and (3.8), we have the closed-loop system as

$$\dot{x}(t) = A_m x(t) + B_m r_d(t) + e_3(t) - \sum_{i=1}^4 b_i (\theta_i(t) - \theta_i^*)^T \omega_i(t)$$
(3.0)

where  $e_3(t) = \sum_{i=1}^4 e_{3i}(t)$ . From (2.16), (2.25), and (2.27), we see that  $e_3(t)$  is bounded as

$$||e_3(t)||_2 \le \varepsilon_2 + \max_{i=1,\dots,4} \{m_{ri}, m_{li}\} \varepsilon_1 \stackrel{\triangle}{=} \varepsilon_3.$$
 (3.10)

It is clear that  $\varepsilon_3$  is also independent of  $n_i$  and  $v_i$ .

Introducing the reference model system

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_d(t)$$
 (3.11)

where  $A_m = A + BK_1$  and  $B_m = BK_2$ , and defining the tracking error  $e(t) = x(t) - x_m(t)$ , from (3.9) and (3.11), we have

$$\dot{e}(t) = A_m e(t) - \sum_{i=1}^4 b_i (\theta_i(t) - \theta_i^*)^T \omega_i(t) + e_3(t).$$
(3.12)

Based on this tracking error equation, we can design the adaptive laws for updating the parameters  $\theta_i(t)$ , i = 1, 2, 3, 4, for the adaptive inverse compensation scheme (3.1).

**Robust Adaptive Laws**. Given the physical meaning of  $m_{ri}$ ,  $m_{li}$ , and  $m_{0i}$ , we have

$$m_{ri}^0 \le m_{ri} \le 1, \ m_{li}^0 \le m_{li} \le 1, \ -m_{0i}^0 \le m_{0i} \le m_{0i}^0$$
 (3.13)

for some constant  $m_{ri}^0 > 0$ ,  $m_{li}^0 > 0$ ,  $m_{0i}^0 > 0$ , where  $m_{ri}^0$  and  $m_{li}^0$ are the lower bounds of percentage of functioning actuators in array *i* on the right wing and left wing respectively, and  $m_{0i} \ge$  $|\beta_{ri} - \beta_{li}|\alpha_i$  is the upper bound of control input produced by the actuators failed at 1. We assume that the bounds  $m_{ri}^0$ ,  $m_{li}^0$ , and  $m_{0i}^0$  are known and such knowledge is used in adaptive laws with parameter projection to guarantee the boundedness of parameter estimates  $\theta_i(t) = [\widehat{m_{ri}}(t), \widehat{m_{0i}}(t), \widehat{m_{li}}(t)]^T$ , i = 1, 2, 3, 4. We choose such adaptive laws as:

$$\widehat{m_{ri}}(t) = h_{ri}(t) + f_{ri}(t) \tag{3.14}$$

$$\widehat{m}_{0i}(t) = h_{0i}(t) + f_{0i}(t)$$
 (3.15)

$$\widehat{m}_{li}(t) = h_{li}(t) + f_{li}(t) \tag{3.16}$$

where

$$h_{ri}(t) = \gamma_{ri} v_i(t) \chi_r(t) e^{I}(t) P b_i, \, \gamma_{ri} > 0 \qquad (3.17)$$

$$h_{li}(t) = \gamma_{li} v_i(t) \chi_l(t) e^{t}(t) P b_i, \, \gamma_{li} > 0 \qquad (3.18)$$

$$h_{0i}(t) = \gamma_{0i}e^{I}(t)Pb_{i}, \qquad \gamma_{0i} > 0 \qquad (3.19)$$

with  $P = P^T > 0$  such that  $PA_m + A_m^T P = -Q$  for a chosen  $Q = Q^T > 0$ , and

$$f_{ri}(t) = \begin{cases} -h_{ri}(t) & \text{if } \widehat{m_{ri}}(t) \ge 1 \text{ and } h_{ri}(t) > 0, \text{ or} \\ & \text{if } \widehat{m_{ri}}(t) \le m_{ri}^{0} \text{ and } h_{ri}(t) < 0 \quad (3.20) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{0i}(t) = \begin{cases} -h_{0i}(t) & \text{if } \widehat{m_{0i}}(t) \ge m_{0i}^{0} \text{ and } h_{0i}(t) > 0, \text{ or} \\ & \text{if } \widehat{m_{0i}}(t) \le -m_{0i}^{0} \text{ and } h_{0i}(t) < 0 \quad (3.21) \\ 0 & \text{otherwise} \end{cases}$$

$$f_{li}(t) = \begin{cases} -h_{li}(t) & \text{if } \widehat{m_{li}}(t) \ge 1 \text{ and } h_{li}(t) > 0, \text{ or} \\ & \text{if } \widehat{m_{li}}(t) \le m_{li}^{0} \text{ and } h_{li}(t) < 0 \quad (3.22) \\ 0 & \text{otherwise} \end{cases}$$

With  $\widehat{m_{ri}}(0) \in [m_{ri}^0, 1]$ , it can be verified that, for i = 1, 2, 3, 4, this adaptive scheme ensures that  $m_{ri}^0 \leq \widehat{m_{ri}}(t) \leq 1$ , and

$$(\widehat{m_{ri}}(t) - m_{ri})f_{ri}(t) \le 0. \tag{3.23}$$

Similar properties can be established for  $\widehat{m_{li}}(t)$  and  $\widehat{m_{0i}}(t)$ .

To give a compact form of adaptive laws, letting

$$f_i(t) = [f_{ri}(t), f_{0i}(t), f_{li}(t)]^T, \ \Gamma_i = \text{diag}\{\gamma_{ri}, \gamma_{0i}, \gamma_{li}\} \quad (3.24)$$

we can express (3.14)–(3.16) as

$$\dot{\theta}_i(t) = \Gamma_i \omega_i e^T(t) P b_i + f_i(t).$$
(3.25)

**Performance Analysis**. Considering the positive definite function

$$V = e^{T} P e + \sum_{i=1}^{4} (\theta_{i} - \theta_{i}^{*})^{T} \Gamma_{i}^{-1} (\theta_{i} - \theta_{i}^{*})$$
(3.26)

we have its time derivative as

$$\dot{V} = 2e^{T}P\dot{e} + 2\sum_{i=1}^{4} (\theta_{i} - \theta_{i}^{*})^{T}\Gamma_{i}^{-1}\dot{\theta}_{i}$$

$$\leq -e^{T}Qe + 2e^{T}Pe_{3}$$

$$\leq -\frac{1}{2}\lambda_{\min}\|e\|_{2}^{2} + 2\frac{\|P\|_{2}^{2}\varepsilon_{3}^{2}}{\lambda_{\min}} \qquad (3.27)$$

where  $\lambda_{\min}$  is the minimum eigenvalue of Q. Together with the boundedness of the parameter estimates (which is guaranteed by parameter projection), (3.27) implies that e(t) and all other closed-loop signals are bounded. The expression (3.27) indicates that the system tracking performance may be improved by proper choices of the design and implementation parameters  $\alpha_{lrj}$  and  $\alpha_{rij}$ , i = 1, 2, 3, 4,  $j = 1, 2, ..., N_i$ , which determine  $\varepsilon_3$ : the smaller  $\alpha_{lrj}$  and  $\alpha_{rij}$  are, the smaller  $\varepsilon_3$  is.

#### 4 Application to a Morphing Aircraft

We now apply the adaptive actuator failure compensation scheme to the lateral dynamic model of the ICE aircraft [8] to study its effectiveness. The linearized ICE aircraft lateraldirectional dynamics matrices [7] are

$$A = \begin{bmatrix} -0.0134 & 48.5474 & -632.3724 & 32.0756 \\ -0.0199 & -0.1209 & 0.1628 & 0 \\ -0.0024 & -0.0526 & -0.0252 & 0 \\ 0 & 1 & 0.0768 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.0431 & 0.0476 & -0.0401 & -0.0308 \\ -0.0076 & -0.0023 & -0.0022 & 0.0297 \\ 0 & 0 & 0 & 0 \end{bmatrix} . (4.1)$$

The plant is open-loop unstable.

We choose the gain matrices  $K_1$  and  $K_2$  to place the closed-loop poles at  $s = [-1.7678 \pm 1.7668j, -2.2502, -7.2498]^T$  and make  $B_m = [0 \ 1 \ 0 \ 0]^T$ . The lateral dynamics is stabilized and the damping and settling time are improved.

The desired reference state vector  $x_m$  is generated by

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_d(t) \tag{4.2}$$

$$r_d(t) = \begin{cases} 0.7 & \text{if } 10i \le t < 10i + 5, \ i = 0, 1, 2, \dots \\ -0.7 & \text{otherwise.} \end{cases}$$
(4.3)

The system response without actuator failure compensation is shown in Figure 4, and the system response with adaptive failure compensation is shown in Figure 5. By comparing the simulation results, we see that the performance of system with adaptive actuator failure compensation is much better than that without failure compensation.



Figure 4: System response without failure compensation.



Figure 5: System response with failure compensation.

## 5 Concluding Remarks

In the paper, we presented an adaptive actuator failure compensation control scheme for state tracking for a morphing aircraft model with unknown morphing actuator failures. A novel practical morphing actuator failure model is formulated. A complete compensation design for both the "on" and "off" failures was given. An adaptive inverse was used to generate compensation control signals applied to the morphing actuator arrays such that the desired nominal control signal can be implemented correctly by the morphing actuator arrays. Adaptive laws updating the parameter estimates were derived to ensure that the adaptive actuator failure compensation scheme achieves the desired system performance: closedloop signal boundedness and (approximate) asymptotic tracking. Simulation results for adaptive compensation of morphing actuator failures of a linearized lateral dynamic ICE aircraft model verified the desired performance of the developed adaptive morphing actuator failure compensation design.

#### Acknowledgements

This research was partially supported by NASA Langley Research Center under Grant NCC-1-02006. The authors would like to thank Mr. David L. Raney for providing the data of aircraft dynamics matrices *A* and *B* used in the simulation study.

#### References

- F. Ahmed-Zaid, P. Ioannou, K. Gousman, and R. Rooney, "Accommodation of failures in the F-16 aircraft using adaptive control," *IEEE Control Systems Magazine*, vol. 11, no. 1, pp. 73–78, 1991.
- [2] M. Bodson and J. E. Groszkiewicz, "Multivariable adaptive algorithms for reconfigurable flight control," *IEEE Transactions on Control Systems Technology*, vol. 5, no. 2, pp. 217–229, 1997.
- [3] J. D. Boskovic and R. K. Mehra, "Stable multiple model adaptive flight control for accommodation of a large class of control effector failures," *Proceedings of the 1999 ACC*, pp. 1920–1924, San Diego, CA, 1999.
- [4] J. D. Boskovic, S.-H. Yu, and R. K. Mehra, "A stable scheme for automatic control reconfiguration in the presence of actuator failures," *Proceedings of the 1998 ACC*, pp. 2455–2459, Philadelphia, PA, 1998.
- [5] J. D. Boskovic, S.-H. Yu, and R. K. Mehra, "Stable adaptive fault-tolerant control of overactuated aircraft using multiple models, switching and tuning," *Proceedings of the 1998 AIAA GNC Conference*, pp. 739– 749, Boston, MA, 1998.
- [6] J. P. Gao, B. Huang, Z. D. Wang, and D. J. Fisher, "Robust reliable control for a class of uncertain nonlinear systems with time-varying multistate time delays," *International Journal of Systems Science*, vol. 32, no. 7, pp. 817–824, 2001.
- [7] Private communication with Dr. David L. Raney, Mail Stop 132, NASA Langley Research Center.
- [8] D. L. Raney, R. C. Montgomery, L. L. Green, and M. A. Park, "Flight control using distributed shape-change effector arrays," AIAA Paper 2000-1560, *Proceedings* of the 41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference & Exhibit, Atlanta, GA, 2000.
- [9] G. Tao, S. H. Chen, J. T. Fei, and S. M. Joshi, "An adaptive actuator failure compensation scheme for controlling a morphing aircraft model," *Proceedings of the* 42nd IEEE CDC, pp. 4926–4931, Maui, HI, 2003.
- [10] A. T. Vemuri and M. M. Polycarpou, "Robust nonlinear fault diagnosis in input-output systems," *International Journal of Control*, vol. 68, no. 2, pp. 343–360, 1997.