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Friction and Output Backlash Compensation of Systems using Neural Network and Fuzzy Logic

Jun Oh Jang, Min Kyong Son, and Hee Tae Chung

***Abstract*—A friction and output backlash compensator is designed for systems by the fuzzy logic (FL) and the neural network (NN). The classification property of FL system and the function approximation ability of the NN make them the natural candidate for the rejection of errors induced by the friction and output backlash. The tuning algorithms are given for the fuzzy logic parameters and the NN weights, so that the friction and output backlash compensation scheme becomes adaptive, guaranteeing small tracking errors and bounded parameter estimates. Formal nonlinear stability proofs are given to show that the tracking error is small. The NN friction and FL output backlash compensator is simulated on a system to show its efficacy.**

I. Introduction

VERY accurate control is required in mechanical devices such as xy positioning tables [1], overhead crane mechanisms [2], robot manipulators [3], etc. Actuator and sensor in control systems often have nonsmooth nonlinear characteristics such as deadzone, backlash, and friction. Some common examples are mechanical connections, hydraulic servo-values and electric servo-motors, magnetic suspensions and bearings, and some biomedical systems. These nonlinear characteristics have been studied in many classical and modern control textbooks. One important result is the describing function approach for analyzing the stability of a closed loop system with such nonlinearities which can be used to design linear control schemes with certain robustness properties despite the presence of such nonlinearities. However, a linear controller alone cannot cancel the nonlinearity effects to achieve desired system tracking performance. The search for new approaches for control of systems with nonsmooth nonlinearities is of major practical interest. Because deadzone, backlash, and friction characteristics are usually poorly known and may vary with time, a desirable controller should be able to adaptively cancel them so that the system performance can be improved.

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Recently, an adaptive inverse approach has been developed for solving such a control problem. The development of such an approach was initiated in [4] for adaptive control for systems with unknown deadzones at the input of a known and smooth dynamics with full state measurement. Following this adaptive control of systems with a nonsmooth nonlinearity at the input or output of a linear dynamics has lead to solutions to problems with input deadzone, input backlash, output deadzone, and output backlash [5]. Fuzzy logic compensation of systems with input deadzone [6], input backlash, and output backlash are proposed. The NN compensation of systems with friction [7] and backlash [8] is proposed. In these problems there is only one nonlinear block in cascade with a linear block as the plant to be controlled.

In this paper we present the NN friction and FL output backlash compensation of systems. The NN and FL function approximation properties, and ability of fuzzy logic systems to discriminate information based on regions of the input variables, makes them an ideal candidate for compensation of non-analytic actuator nonlinearities [9]. A design procedure is given that results in a PD tracking loop with an adaptive fuzzy logic system using dynamic inversion for output backlash and an adaptive NN friction compensation in feed forward loop. We investigate the performance of the fuzzy friction and output backlash compensator in a system through the computer simulations.

II. NN Friction and FL Output Backlash Compensation

An NN friction compensator is designed for friction nonlinearity. Relevant features of the NN include their ability to model arbitrary differential nonlinear functions, and their intrinsic on-line adaptation and learning capabilities. Also, a FL compensator is designed for the output backlash nonlinearity. It is shown that the fuzzy logic approach includes and subsumes approaches based on switching logic and indicator functions [5]. This brings these references very close to fuzzy logic work in [10], and potentially allows for more exotic compensation schemes

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for actuator non- linearities using more complex decision (e. g. membership) functions.

2.1 Friction and output backlash nonlinearity

The friction and output backlash of systems is shown in Fig. 1. Friction models have been extensively discussed in the literature [11]. Nevertheless, there is considerable disagreement on the proper model structure. In the Coulomb friction model there is a constant friction torque opposing the motion when velocity is zero. For zero velocity the striction will oppose all motions as long as the torques are smaller in magnitude than the striction torque. The friction, in general, can be written as

$$T_f(\omega) = \alpha_0 \operatorname{sgn}(\omega) + \alpha_1 e^{-\alpha_2|\omega|} \operatorname{sgn}(\omega) \qquad (1)$$

with constants $\alpha_i > 0$, $i = 0,1,2$. In practical control systems the width of the friction is unknown, so that compensation is difficult.

The backlash characteristic $B(\cdot)$ with input z and output y : $y = B[z(t)]$ is described by two parallel straight lines, upward and downword sides of $B(\cdot)$, connected with horizontal line segments. Mathematically, the backlash is modeled as

$$\dot{y}(t) = B(y, z, \dot{z})$$
$$= \begin{cases} \dot{z}(t), & if \quad \dot{z}(t) > 0 \quad and \quad y(t) = z(t) - d_+ \\ & or \quad \dot{z}(t) < 0 \quad and \quad y(t) = z(t) - d_- \\ 0, & otherwise \end{cases} . \qquad (2)$$

One can see that backlash is a first order velocity driven dynamic system, with inputs z and $\dot{z}$, and state $\dot{y}$. It contains its own dynamics, therefore its compensation requires the design of the dynamic compensator. Whenever the motion $\dot{z}(t)$ changes its direction, the motion $\dot{y}(t)$ is delayed from motion of $\dot{z}(t)$. A graphical inverse of the backlash characteristic is shown in Fig. 2, which contains vertical jumps.

2.2 NN friction and FL output backlash compensator

A rigorous framework for NN applications in friction compensation and FL output backlash compensation are described.

A three layer NN in Fig. 3 has a network output given by

$$y_{NN} = \sum_{m=1}^{N_2} [w_{1m} \cdot \sigma(\sum_{l=1}^{N_1} v_{lm} \cdot x_l + v_{1l}) + w_{1,0}] \qquad (3)$$

with notation $\sigma(\cdot)$, the activation function, v_{lm} , the interconnection weights from first to second layer, w_{1m} , the interconnection weights from second to third layer, N_1, the nunber of neurons in the first layer, and N_2 , the number of neurons in second layer. v_{0l} and w_{01} are threshold offsets.

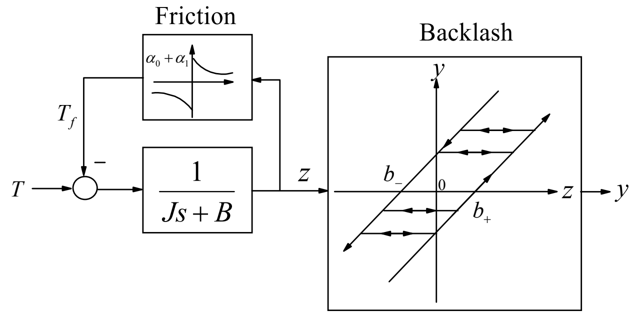


Fig. 1. Friction and output backlash of systems.

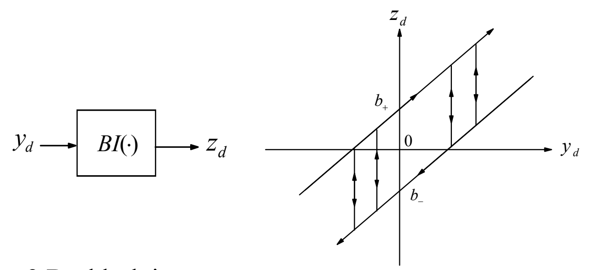


Fig 2 Backlash inverse.

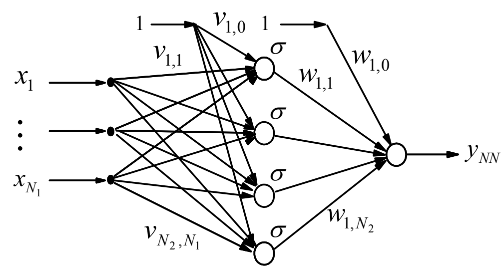


Fig. 3. Neural network.

The NN equation may be conveiently expressed in a vector format by defining $\hat{W} = [w_{1,1}, w_{1,2}, \cdots, w_{1,N_2}]^T$, $\sigma(\cdot) = [\sigma_1(\cdot), \sigma_2(\cdot), \cdots, \sigma_4(\cdot)]^T$, $\sigma_i(\cdot) = \sigma(\cdot)$, and a matrix format defining $\hat{V}^T = [V_{lm}]$ Then,

$$y_{NN} = \hat{W}^T \sigma(\hat{V}^T x) . \qquad (4)$$

A general function, f , can be modeled by an NN as:

$$f(x) = W^T \sigma(V^T x) + \varepsilon(x) \qquad (5)$$

where W and V are constant ideal weight of the current weight $\hat{W}$ and $\hat{V}$ so that ε is bounded by a known contant ε_N , and ε is reconstruction error due to the NN structure. For notational convenience, define the matrix of all the weights as

$$Z = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix} . \qquad (6)$$

For practical situations, we assume that the ideal parameters are bounded by known positive values so that $\| W \| < W_M$, $\| V \| < V_M$, or $\| Z \| < Z_M$ with Z_M known, where $\| \cdot \|$ is a norm. Define the parameter deviation or the parameter estimation error as:

$$\widetilde{W} = W - \hat{W}\ ,\ \widetilde{V} = V - \hat{V} \tag{7}$$

and the second layer output error for a given x as:

$$\widetilde{\sigma} = \sigma - \hat{\sigma} = \sigma(V^T x) - \sigma(\hat{V}^T x). \tag{8}$$

The Taylor series expansion of the second layer output for a given x may be written as:

$$\sigma(V^T x) = \sigma(\hat{V}^T x) + \dot{\sigma}(\hat{V}^T x)\widetilde{V}^T x + O(\widetilde{V}^T x) \tag{9}$$

with $\dot{\sigma}(\hat{z}) \equiv \dfrac{d\sigma(z)}{dz}\Big|_{z=\hat{z}}$, and $O(\cdot)$, sum of higher order terms. Denoting $\hat{\dot{\sigma}} = \dot{\sigma}(\hat{V}^T x)$, we have

$$\widetilde{\sigma} = \dot{\sigma}(\hat{V}^T x)\widetilde{V}^T x + O(\widetilde{V}^T x) = \hat{\dot{\sigma}}\widetilde{V}^T x + O(\widetilde{V}^T x). \tag{10}$$

Now, define NN functional estimate of (5) by:

$$\hat{f}(x) = \hat{W}^T \sigma(\hat{V}^T x) \tag{11}$$

with $\hat{W}$, $\hat{V}$ the current estimated values of the ideal weights W , V as provided by the training algorithm subsequently to be disscussed.

In our control problem the backlash parameters are unknown and the internal signal $z(t)$ is not available for measurement. The control objective is to design a feedback control for the system which stabilizes the closed loop system and makes the plant output $y(t)$ track a given bounded reference signal $y_d(t)$. The mapping BI(.) : $y_d(t) \to z_d(t)$, defines a backlash inverse

$$B(BI(y_d(\tau))) = y_d(\tau) \Rightarrow B(BI(y_d(t))) = y_d(t) \tag{12}$$

for any $t > \tau$. Because of the dynamic nature of backlash, the backlash inverse is defined with the initialization $B(BI(y_d(\tau))) = y_d(\tau)$.

To offset the deleterious effects of backlash, we introduce the idea of the fuzzy backlash inverse scheme. A fuzzy inverse backlash compensator using dynamic inversion would be discontinuous and depend on the region within which $\dot{y}_d$ occurs. It would be naturally described using the rules

$$\begin{aligned}
&\text{If (} \dot{y}_d \text{ is positive) then (} z_d = y_d + \hat{b}_+ \text{)}\\
&\text{If (} \dot{y}_d \text{ is zero) then (} z_d = y_d + \hat{b}_0 \text{)}\\
&\text{If (} \dot{y}_d \text{ is negative) then (} z_d = y_d + \hat{b}_- \text{)}
\end{aligned} \tag{13}$$

where $\hat{b} = [\hat{b}_+\ \hat{b}_0\ \hat{b}_-]^T$ is an estimate of the backlash width parameter vector $b = [b_+\, b_0\, b_-]^T$.

To make this intuitive notion mathematically precise for analysis define the membership function's

$$\begin{aligned}
X_+(\dot{y}_d) &= \begin{cases} 0, & \dot{y}_d < 0 \\ 1, & 0 < \dot{y}_d \end{cases}\\
X_0(\dot{y}_d) &= \begin{cases} 0, & \dot{y}_d \neq 0 \\ 1, & \dot{y}_d = 0 \end{cases}\\
X_-(\dot{y}_d) &= \begin{cases} 1, & \dot{y}_d < 0 \\ 0, & 0 < \dot{y}_d \end{cases}.
\end{aligned} \tag{14}$$

One may write the fuzzy inverse compensator as

$$z_d = y_d + z_F \tag{15}$$

where z_F is given by the rule base

$$\begin{aligned}
&\text{If (} \dot{y}_d \in X_+(\dot{y}_d) \text{) then (} z_F = \hat{b}_+ \text{)}\\
&\text{If (} \dot{y}_d \in X_0(\dot{y}_d) \text{) then (} z_F = \hat{b}_0 \text{)}\\
&\text{If (} \dot{y}_d \in X_-(\dot{y}_d) \text{) then (} z_F = \hat{b}_- \text{).}
\end{aligned} \tag{16}$$

The output of the fuzzy logic system with this rule base is given by

$$z_F = \frac{\hat{d}_+ X_+(\dot{y}_d) + \hat{d}_0 X_0(\dot{y}_d) + \hat{d}_- X_-(\dot{y}_d)}{X_+(\dot{y}_d) + X_0(\dot{y}_d) + X_-(\dot{y}_d)}. \tag{17}$$

The estimates $\hat{b}_+$, $\hat{b}_0$, $\hat{b}_-$ are, respectively, the control representive value of $X_+(\dot{y}_d)$, $X_0(\dot{y}_d)$, and $X_-(\dot{y}_d)$. This may be written (note $X_+(\dot{y}_d) + X_0(\dot{y}_d) + X_-(\dot{y}_d) = 1$) as

$$z_F = \hat{b}^T X(\dot{y}_d) \tag{18}$$

where the fuzzy logic basis function vector given by

$$X(\dot{y}_d) = \begin{bmatrix} X_+(\dot{y}_d) \\ X_0(\dot{y}_d) \\ X_-(\dot{y}_d) \end{bmatrix} \tag{19}$$

is easily computed given any value of $\dot{y}_d$. It should be noted that the membership functions (18) are the indicator functions and $X(\dot{y}_d)$ is similar to the regressor [5].

The fuzzy backlash inverse compensator may be expressed as follows

$$\begin{aligned} z_d &= y_d + z_F \\ &= y_d + \hat{b}^T X(\dot{y}_d) \end{aligned} \tag{20}$$

where $\hat{b}$ is estimated backlash width.

Since $z(t)$ is not available, we choose its estimate to be :

$$\begin{aligned} \bar{z}(t) &= y(t) + \bar{z}_F \\ &= y(t) + \hat{b}^T X(\dot{y}) \end{aligned} \tag{21}$$

where the fuzzy basis function vector given by

$$X(\dot{y}) = \begin{bmatrix} X_+(\dot{y}) \\ X_0(\dot{y}) \\ X_-(\dot{y}) \end{bmatrix} \tag{22}$$

is easily computed given any value of $\dot{y}$.

III. Adaptive NN Friction and FL Backlash Compensation of Systems

In this section we will show how to provide the NN friction and FL output backlash compensation for friction and backlash in systems. The proposed control structure is shown in Fig. 4.

The dynamics of systems can be written as

$$J\dot{z} + B_v z + T_f + T_d = T\ ,\quad y = B[z(t)] \tag{23}$$

where $y(t)$ is the system output, J is the mass, B_v is the damping, T_f is the friction, T_d is the bounded unknown disturbance, and T is the system input. It is assumed that $|T_d| < \tau_d$, with τ_d, a known positive constant.

Given the reference signal z_d, the tracking error is expressed by $e = z_d - z$. Differentiating the tracking error and using (23), the dynamics of the system may be written in terms of the tracking error as:

$$J\dot{e} = -B_v e - T + f(x) + T_d \tag{24}$$

where the nonlinear plant function is defined as:

$$f(x) = J\dot{z}_d + B_v z_d + T_f \,. \tag{25}$$

Vector x contains all the time signals needed to compute $f(x)$,and may be defined for instance as $x \equiv [e \quad z_d \quad \dot{z}_d]^T$. It is noted that the function $f(x)$ contains all the potentially unknown functions, except for J, B_v appearing in (25) - these latter terms cancel out in the stability proof.

We assume that the desired trajectory is bounded in the sense, for instance, that

$$\left| \begin{matrix} z_d \\ \dot{z}_d \end{matrix} \right| \le Y_d \tag{26}$$

where Y_d is a known constant.

For each time t, $x(t)$ is bounded by

$$|x| \le c_1 Y_d + c_2 |e| \tag{27}$$

for computable positive constants c_i.

A robust compensation scheme for unknown terms in $f(x)$ is provided by selecting the tracking controller

$$T = \hat{f}(x) + K_f e - v \tag{28}$$

with $\hat{f}(x)$, an estimate for the nonlinear terms $f(x)$, $v(t)$ a robustifying term, and $K_f > 0$.

Select a control input torque using (11) and (28) as:

$$T = K_f e + \hat{W}^T \sigma(\hat{V}^T x) - v \,. \tag{29}$$

Using (5) and (29), the closed loop error dynamics (24) become:

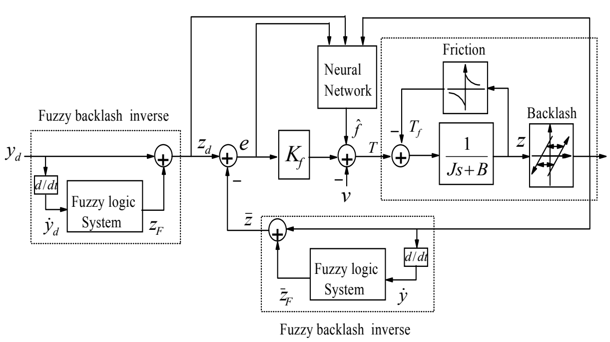


Fig. 4. NN friction and FL output backlash compensation.

$$\begin{aligned} J\dot{e} = &-(K_f + B_v)e + W^T\sigma(V^T x) - \hat{W}^T\sigma(\hat{V}^T x) \\ &+ \varepsilon + T_d + v \end{aligned} \,. \tag{30}$$

Adding and subtracting $W^T\hat{\sigma}$ yields:

$$J\dot{e} = -(K_f + B_v)e + \tilde{W}^T\hat{\sigma} - \hat{W}^T\tilde{\sigma} + \varepsilon + T_d + v \,. \tag{31}$$

Adding and subtracting again $\hat{W}^T\tilde{\sigma}$ yields:

$$J\dot{e} = -(K_f + B_v)e + \tilde{W}^T\hat{\sigma} + \hat{W}\tilde{\sigma} + \tilde{W}^T\tilde{\sigma} + \varepsilon + T_d + v \,. \tag{32}$$

Using the Taylor series approximation for $\tilde{\sigma}$, the closed loop error system becomes

$$J\dot{e} = -(K_f + B_v)e + \tilde{W}^T(\hat{\sigma} - \dot{\hat{\sigma}}V^T x) + \hat{W}^T\dot{\hat{\sigma}}\tilde{V}^T x + \delta + v \tag{33}$$

where the disturbance δ is

$$\delta = \tilde{W}^T\dot{\hat{\sigma}}\tilde{V}^T x + W^T O(\tilde{V}^T x) + (\varepsilon + T_d) \,. \tag{34}$$

It is important to note that the NN reconstruction error ε, the plant disturbance T_d, and the higher order terms in the taylor series expansion of f all have exactly the same influence as disturbance in the error system.

The higher order terms in the Taylor series are bounded by

$$\| O(\tilde{V}^T x) \| \le c_3 + c_4 Y_d \|\tilde{V}\|_F + c_5 \|\tilde{V}\|_F |e| \tag{35}$$

where c_i are computable positive constants. The disturbance term (34) is bounded according to

$$\|\delta\| \le C_0 + C_1 \|\tilde{Z}\|_F + C_2 \|\tilde{Z}\|_F |e| \tag{36}$$

with C_i computable known positive constants.

For the NN training algorithm to improve the tracking performance of the closed loop system it is required to demonstrate that the tracking error, e is suitably small, a bound on the tracking error is derived by the following theorem.

The next theorem provides an algorithm for tuning the NN friction compensator.

Theorem 1*:* Given the system (23), select the tracking control (29) and choose the robustifying signal

$$v(t) = -K_z(\|\tilde{Z}\|_F + Z_M)e \tag{37}$$

and gain

$$K_z > C_2 \tag{38}$$

with C_2 the known constant in (36). Let NN weight tuning be provided by

$$\dot{\hat{W}} = \eta_1\hat{\sigma}e - \eta_1\dot{\hat{\sigma}}\hat{V}^T xe - k\eta_1 |e| \hat{W} \tag{39}$$

$$\dot{\hat{V}} = \eta_2 x(\dot{\hat{\sigma}}^T\hat{W}e)^T - k\eta_2 |e| \hat{V} \tag{40}$$

with any constant $\eta_1 > 0$, $\eta_2 > 0$ and scalar design parameter $k > 0$. Then the tracking error e evolves with a practical bounds given specially by the right hand sides of (A6), provided that $\bar{z}(t) = z(t)$.

Proof : See Appendix.

Given the fuzzy inverse compensator with rule base (16), the throughput of the compensator plus output backlash is given by

$$y = y_d - \tilde{b}^T X(\dot{y}_d) + \tilde{b}^T \delta_o \tag{41}$$

where the fuzzy inverse estimation error is given by $\tilde{b} = b - \hat{b}$ and the modeling mismatch term δ_o is bounded so that $|\delta_o| < \delta_{oM}$ for some scalar δ_{oM}.

We choose the fuzzy logic system tuning algorithm for the estimated backlash inverse

$$\dot{\hat{b}} = X(\dot{y}_d)e - k_o \hat{b}\,|e| \tag{42}$$

where the scalar $k_o > 0$ and $e = z_d - \bar{z}$. The second term in (42) is a term of the Narendra's e mod, which is used in adaptive control to provide robustness to disturbances.

System tuning algorithm is robust with respect to the modeling mismatch term δ_o in (41) and the components of $\tilde{b}$ stay in a convex set to which the true widths b belong, for implementing the fuzzy output backlash inverse $BI(\cdot)$. System tuning algorithm ensure that the widths $\hat{b}$ and the error, $y_d - y$ are bounded. So are z_d and z. These properties are sufficient to ensure that all closed loop signals are bounded in the presence of the unknown output backlash $B(\cdot)$.

IV. SIMULATION RESULTS

In this section, we illustrate the effectiveness of an NN friction and FL output backlash compensator by computer simulations. One considers a system with friction and output backlash nonlinearity as

$$J = 0.015,\ B_v = 0.95,\ \alpha_0 = 0.055,\ \alpha_1 = 0.01,$$
$$\alpha_2 = 3.0,\ b_+ = 0.06 \text{ and } b_- = -0.07. \tag{43}$$

The system response without friction and output backlash nonlinearity by a PI controller is shown in Fig. 5. The parameters of the PI controller are chosen as $K_p = 3$ and $K_I = 4$. The system response with friction and backlash nonlinearity is included in Fig. 6. The performance is degraded by the nonlinearity. Therefore, we use the NN friciton and FL output backlash compensator in order to compensate for friction and output backlash effects. The input vector x can be taken as $x = [e(k), \mathrm{sgn}(\bar{z}(k)), z_d(k)]$. The sigum $\mathrm{sgn}(\cdot)$ is needed for Coulomb friction terms. The NN parameter $N_1 = 3$, $N_2 = 4$, and $\eta_1 = \eta_2 = 2$. The system response with the friction and output backlash compensator is shown in Fig. 7. The proposed method exhibits an improvement in its response compared with the only PI controller.

V. CONCLUSION

An NN friction and FL output backlash compensator has been proposed for systems. The classification property of FL system and the function approximation ability of the NN makes them a natural candidate for offsetting this sort of

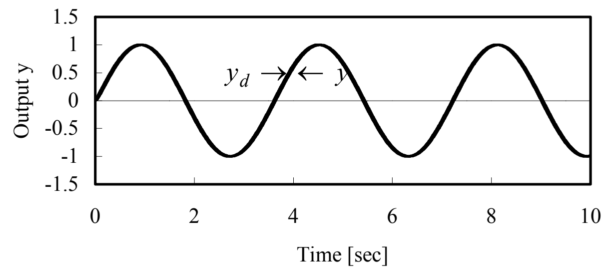
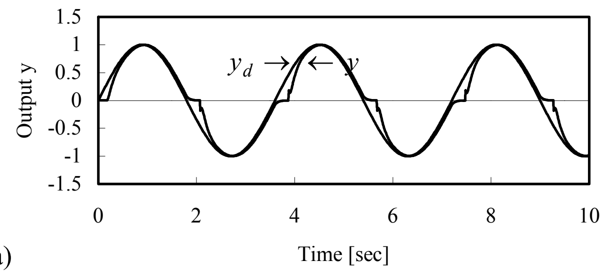
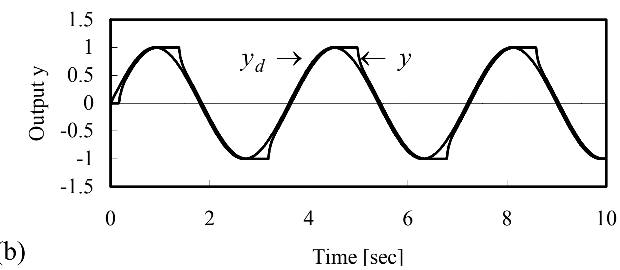


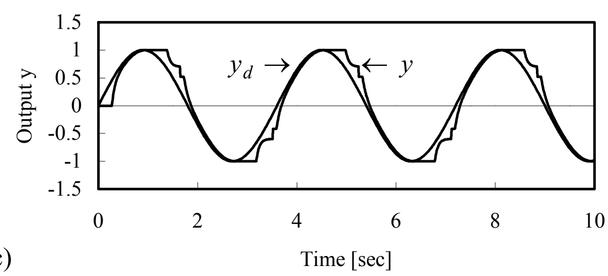
Fig. 5. System response without friction and backlash



(a)



(b)



(c)

Fig. 6. System response (a) with only friction (b) with only backlash (c) with friction and backlash.

actuator nonlinearity having a strong dependence on the region in which the arguments occurs. It was shown how to tune the FL parameters and the NN weights so that the unknown friction and backlash parameters are learned on line, resulting an adaptive friction and output backlash compensator. Simulation results show that significantly improved system performance can be achieved by our adaptive NN and FL control schemes.

APPENDIX

Proof of theorem 1

Define a Lyapunov function candidate for the error dynamics (33) as

$$L = \frac{1}{2}Je^2 + \frac{1}{2\eta_1}tr(\tilde{W}^T\tilde{W}) + \frac{1}{2\eta_2}tr(\tilde{V}^T\tilde{V}). \qquad (A1)$$

Differentiating and substituting now from the error system (A1) yields

$$\begin{aligned}\dot{L} = &-(K_f + B_v)e^2 + \frac{1}{2}\dot{J}e^2 + tr\tilde{W}^T(\frac{1}{\eta_1}\dot{\tilde{W}} + \hat{\sigma}e \\ &- \dot{\hat{\sigma}}\hat{V}^T xe) + \frac{1}{2}tr\tilde{V}^T(\frac{1}{\eta_2}\dot{\tilde{V}} + xe\hat{W}^T\dot{\hat{\sigma}}) + e(\delta + v)\end{aligned} \quad . \qquad (A2)$$

Since $\dot{\tilde{W}} = -\dot{\hat{W}}$ with W constant (and similary for $\dot{\tilde{V}}$), the tuning rules and the asumption $|\dot{J}| = 0$ gives

$$\begin{aligned}\dot{L} &= -(K_f + B_v)e^2 + k|e|tr\tilde{W}^T(W - \tilde{W}) \\ &\quad + k|e|tr\tilde{V}^T(V - \tilde{V}) + (\delta + v) \\ &= -(K_f + B_v)e^2 + k|e|tr\tilde{Z}^T(Z - \tilde{Z}) + e(\delta + v)\end{aligned} \quad . \qquad (A3)$$

Since $\begin{aligned}tr\tilde{Z}^T(Z - \tilde{Z}) = &< \tilde{Z}, Z >_F - \|\tilde{Z}\|_F^2 \le \|\tilde{Z}\|_F \|Z\|_F \\ &- \|\tilde{Z}\|_F^2\end{aligned}$,

there results

$$\begin{aligned}\dot{L} = &-(K_f + B_v)|e|^2 + k|e| \cdot \|\tilde{Z}\|_F (Z_M - \|\tilde{Z}\|_F) \\ &- K_z(\|\tilde{Z}\|_F + Z_M)|e|^2 + \|\delta\| \cdot |e| \\ \le &-(K_f + B_v)|e|^2 + k|e| \cdot \|\tilde{Z}\|_F (Z_M - \|\tilde{Z}\|_F) \\ &- K_z(\|\tilde{Z}\|_F + Z_M)|e|^2 + |e|[C_0 + C_1\|\tilde{Z}\|_F \\ &+ C_2\|\tilde{Z}\|_F |e|] \\ \le &-|e|[(K_f + B_v)|e| + k\|\tilde{Z}\|_F (\|\tilde{Z}\|_F - Z_M) \\ &- C_0 - C_1\|\tilde{Z}\|_F]\end{aligned} \quad . \qquad (A4)$$

Thus $\dot{L}$ is negative as long as the term in brace is positive.

Defining $C_3 = Z_M + C_1/k$ and completing the square yields

$$\begin{aligned}&(K_f + B_v)|e| + k\|\tilde{Z}\|_F (\|\tilde{Z}\|_F - C_3) - C_0 \\ &= k(\|\tilde{Z}\|_F - C_3/2)^2 - kC_3^2/4 + (K_f + B_v)|e| - C_0\end{aligned} \qquad (A5)$$

which is guaranteed positive as long as either

$$|e| > \frac{C_3^2/4 + C_0}{K_f + B_v} \qquad (A6)$$

or

$$\|\tilde{Z}\|_F > C_3/2 + \sqrt{C_3^2/4 + C_0/k} \qquad (A7)$$

where $C_3 = Z_M + C_1/k$.

According to the Lyapunov theorem extension, tracking error decreases as long as the error is bigger than the right hand sides of Eq. (A6). This implies Eq. (A6) gives a practical bound on the tracking error

$$|e| \le \frac{C_3^2/4 + C_0}{K_f + B_v}. \qquad (A8)$$

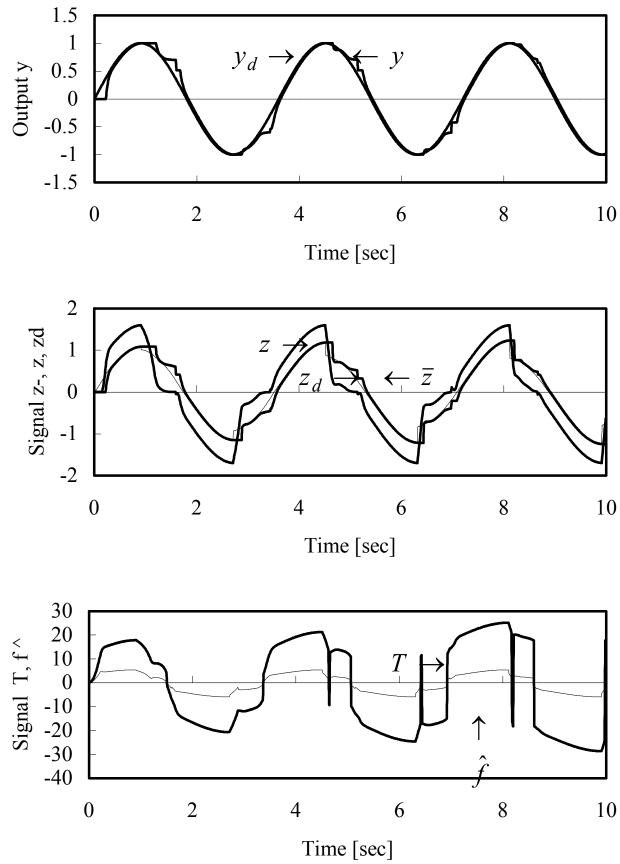


Fig. 7. System response with friction and backlash compensation

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