Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

A Nonlinear Programming Approach for Control Allocation†

Vishnu L. Poonamallee
Stephen Yurkovich*
Andrea Serrani
Department of Electrical Engineering
The Ohio State University, Columbus, OH

David B. Doman
Michael W. Oppenheimer
Air Force Research Laboratory
WPAFB, OH

Abstract— **In this paper, a novel nonlinear programming based control allocation scheme is developed. The performance of this nonlinear control allocation algorithm is compared with that of other control allocation approaches, including a mixed optimization scheme, a redistributed pseudo-inverse approach, and a direct allocation (geometric) method. The control allocation methods are first compared using open-loop measures such as the ability to attain commanded moments for a prescribed maneuver. The methods are then compared in closed-loop with a dynamic inversion-based control law. Next, the performance of the different algorithms is compared for different reference trajectories under a variety of failure conditions. Finally, we perform some preliminary studies employing "split actuators" that increase available control authority under failure conditions. All studies are conducted on a re-entry vehicle simulation.**

I. INTRODUCTION

One of the primary limitations of traditional control allocation algorithms is the dependence on assumptions of linearity. Most approaches are based on the assumption that the control variables and rates are linear functions of the effector deflections. However, the forces and moments produced by aerodynamic control effectors are often nonlinear functions of the effector deflection.

The concept of control allocation has therefore become increasingly more important as flight control systems employ multiple actuators, with numerous constraints, to achieve multiple control objectives. There are several objectives that must be satisfied by these algorithms. The algorithms must be computationally fast since they must run in real time and they must provide a guarantee that they will compute a solution in the time allotted by the flight control system. To be considered flight-worthy, the algorithm must also reliably produce smoothly varying actuator commands that do not chatter from one time step to the next. Another important goal is to make reconfiguration possible when one or more control surfaces fail. Ideally, the algorithm should also be able to deliver a set of effector inputs that minimize the difference between actual and desired commands in the event that the latter cannot be produced.

Several control allocation and control mixing algorithms have been developed, and a few survey papers exist [2], [5] that point out the advantages and disadvantages of the control allocation schemes. Existing control allocation algorithms are capable of dealing with systems where moments are linearly related to control effector positions, and have the ability to account for position constraints. Therefore, most existing algorithms assume that a linear relationship exists between the controlled variables (CVs) and the effector variables. In cases where this assumption fails, errors in the control allocation schemes must be mitigated by the robustness resulting from feedback control laws. A reasonable goal, therefore, is to free some of the burden on the feedback portion of the control law by increasing the accuracy of the control allocator. Recently, a piecewise linear control allocation approach has been introduced that effectively accommodates separable nonlinearities using piecewise linear programming methods [8]. The effects of the nonlinear relationships between forces, moments and control surfaces are directly considered in this paper, and a straightforward nonlinear control allocation law is proposed. In this work, actuator dynamics are assumed to have a negligible effect; the case of non-negligible actuator dynamics is addressed in a companion paper [7]. Nonlinear programming techniques are used to find the effector positions given the nonlinear relationship between the moments and the effector positions. While nonlinear programming does not offer guarantees of convergence in a finite period of time, it does provide a performance metric against which other methods can be compared and is a reasonable first step that can be used to assess the potential benefits of nonlinear control allocation approaches.

The results of this approach are compared to several linear techniques, including a mixed optimization control law, a redistributed pseudo inverse method, and a direct control allocation law. The algorithms are tested in both closed and open loop architectures. Because of the penalty for adding additional weight, re-entry vehicles have little hardware redundancy. Due to the availability of a limited number of control surfaces, a control effector failure can severely affect the vehicle's performance and safety. Hence, failures are explicitly addressed in this paper, and their effect on control allocation are observed in a comparative study. Preliminary studies employing "split effectors" to increase control authority in failure situations are also investigated.

The re-entry vehicle considered in this study has four aerodynamic control surfaces (right/left tails and right/left flaps), and is capable of flying through different flight regimes, spanning a wide envelope of speeds and altitude.

†This work is supported by the AFRL/AFOSR Collaborative Center of Control Sciences at the Ohio State University.
*Corresponding author. Email: yurkovich.1@osu.edu

Such a vehicle would be carried to orbit, reenter the atmosphere at hypersonic speeds, and finally land horizontally.

II. Dynamic Inversion

To set this work in proper context, we first consider the feedback loop within which the control allocation algorithm functions. An inner-loop control law is designed using dynamic inversion [1] so that the vehicle tracks body-rate commands generated by an outer-loop guidance and control system. A nonlinear control law is fashioned that globally reduces the dynamics of selected controlled variables (CVs) to a set of integrators. A closed-loop implicit model following system is then constructed to make the CVs exhibit a desired set of dynamics that replace the existing dynamics. Such a control scheme has several advantages, including greater generality for re-use across different flight regimes and greater flexibility for handling different flight models.

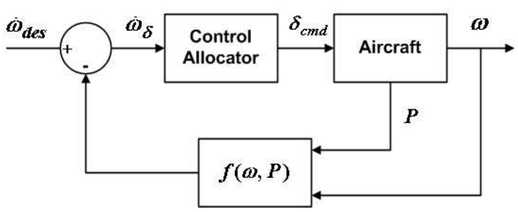

Fig. 1. Dynamic Inversion

The overall control scheme is depicted in Figure 1. The body-axis angular acceleration commands $\dot{\omega}_{des}$ are generated as the output of an inner-loop prefilter block, which is used to shape the dynamic response. An outer-loop guidance and control system generates body-axis angular acceleration commands, which drive the inner-loop. The inner-loop dynamic inversion control law is designed so that the vehicle tracks these body-rate commands. The prefilter is selected so that the closed-loop system has the properties of a desired model when the dynamic inversion is perfect. As such, the prefilters and dynamic inversion combination forms the implicit model following system [9]. The reentry vehicle rotational dynamics can be written as

$$\dot{\omega} = f(\omega, P) + g(P, \delta) \tag{1}$$

where $\omega = [p \ q \ r]^T$ is the angular velocity vector, δ is the vector of actuator deflections, and P denotes measurable or estimable quantities that influence the body rate states. The parameter vector P includes variables such as Mach number, angle of attack, sideslip angle and vehicle mass properties such as moments of inertia. Equation (1) expresses the body-axis rotational accelerations as a sum that includes control dependent accelerations $g(P, \delta)$ and accelerations due to the wing-body, $f(\omega, P)$. It is assumed that the mass properties of the vehicle are time-invariant, so that the inertia matrix I satisfies $\dot{I} = 0$ and equation (1) can be written as

$$\dot{\omega} = I^{-1}(G_B - \omega \times I\omega),$$

$$G_B = G_{WB}(\omega, P) + G_\delta(P, \delta) = \begin{pmatrix} L \\ M \\ N \end{pmatrix}_{WB} + \begin{pmatrix} L \\ M \\ N \end{pmatrix}_{\delta},$$

$G_{WB}(\omega, P)$ is the moment generated by the wing-body aerodynamics, and G_δ is the moment produced by the control effectors. In this representation, L, M, and N are the moment vector components in a body-axis coordinate system. Thus,

$$f(\omega, P) = I^{-1}(G_{WB}(\omega, P) - \omega \times I\omega),$$
$$g(P, \delta) = I^{-1}G_\delta(P, \delta)\,.$$

In this work we employ a simulation of the re-entry vehicle, based on a large array of flight conditions generated by Missile Datcom. The aerodynamic database provides force and moment coefficient data taken at a moment reference point (MRP), located at the center of gravity of the empty vehicle. Control derivative information is extracted from the tables in the database for the values of Mach number, angle of attack and sideslip angle encountered along the trajectory.

As stated previously, most conventional control allocation approaches require that the control dependent portion of the model is affine in the control. Thus, in order to compare the nonlinear approach to conventional linear approaches, we develop a linear approximation of the control dependent portion as

$$G_\delta(P, \delta) \approx G_\delta(P)\delta\,.$$

The model used for the design of the dynamic inversion control law, for use with linear control allocators, has the form

$$\dot{\omega} = f(\omega, P) + I^{-1}G_\delta(P)\delta$$

and our objective is to find a control law that provides direct control over $\dot{\omega}$ so that $\dot{\omega} = \dot{\omega}_{des}$. Therefore, the inverse control law must satisfy

$$\dot{\omega}_{des} - f(\omega, P) = I^{-1}G_\delta(P)\delta\,. \tag{2}$$

Similarly, for the nonlinear control allocator, the control law must satisfy

$$\dot{\omega}_{des} - f(\omega, P) = I^{-1}G_\delta(P, \delta)\,. \tag{3}$$

III. Linear Control Allocation

A. Overview

Since there are three controlled variables and four control effectors, a control allocation law can be used to ensure that Equation (2) is satisfied. Equation (2) can be represented as,

$$B\delta = d_{des}\,, \tag{4}$$

where $B = I^{-1}G_\delta(P)$ and $d_{des} = \dot{\omega}_{des} - f(\omega, P)$ are given, and δ is to be determined such that the following constraints are satisfied

$$\begin{array}{c} \delta_{min} \leq \delta \leq \delta_{max} \\ \dot{\delta}_{min} \leq \dot{\delta} \leq \dot{\delta}_{max}\,. \end{array} \tag{5}$$

The above inequalities express the position and rate limits of the actuators. Given these constraints, an exact solution may not exist, despite the redundancy. Furthermore, a solution (either exact or approximate) cannot be assumed to be unique. If a failure occurs, a new control input must be found that accounts for the modified entries of the matrix B, or changes in rate or position limits.

B. Mixed Optimization with Intercept Correction Problem

The mixed optimization with intercept correction (MOIC) problem considers both error minimization and the control minimization problem [2]. The error minimization problem can be formulated as follows. Given a matrix B, find a vector δ such that $J = \| B\delta - d_{des} \|_1$ is minimized subject to the constraint (5). The control minimization problem involves minimization of $J = \| \delta - \delta_p \|_1$, where δ_p is a preferred control vector position. The mixed optimization approach essentially combines the error and control minimization problems into a single one, and is formulated as finding a control vector δ such that,

$$J = \| B\delta - d_{des} \|_1 + \lambda \| \delta - \delta_p \|_1$$

is minimized, subject to the constraints (5). If the parameter $\lambda > 0$ is small, priority will be given to error minimization over control minimization, as desired. The LP problem which results from this type of performance index can be solved using the simplex algorithm [2].

Included in this control allocation scheme is the intercept correction term [4]. This term takes into account some of the nonlinearities in the aerodynamic data and manifests itself by simply modifying d_{des}. Instead of using a linear relationship to represent $G_\delta(P, \delta)$, an affine relationship is utilized such that

$$G_\delta(P, \delta) = G_\delta(P)\delta + \epsilon(P, \delta) \tag{6}$$

so that equation 2 becomes

$$\dot{\omega}_{des} - f(\omega, P) = I^{-1} G_\delta(P)\delta + I^{-1}\epsilon(P, \delta)\,. \tag{7}$$

Then, it is easily seen that $d_{des} = \dot{\omega}_{des} - f(\omega, P) - I^{-1}\epsilon(P, \delta)$ and $B = I^{-1} G_\delta(P)$.

In the results to follow, the intercept correction term is included only in the mixed optimization intercept correction scheme and is not used in the pseudo-inverse or direct allocation approaches.

C. Redistributed Pseudo-Inverse

The redistributed pseudo-inverse (RPI) method [5] involves finding the control vector δ that minimizes

$$J = W_\delta \| \delta - \delta_p \|_2^2 \tag{8}$$

subject to $B\delta = d_{des}$. For the unconstrained case, the solution is given by a biased weighted pseudo-inverse,

$$\delta = \delta_p + W_\delta^{-1} B^T (B W_\delta^{-1} B^T)^{-1} (d_{des} - B\delta_p)\,. \tag{9}$$

In presence of constraints of the kind (5), this approach may produce solutions that violate the effector limits. The following method is used to handle unattainable solutions. First, a control vector that solves the unconstrained problem (8) is found using (9). If the solution violates the constraints, the individual commands that saturate are clipped at their respective limit, their contributions are subtracted from d_{des}, and the inverse is computed again. The procedure is repeated until a feasible solution is found or until all the components have saturated. The major problem with this method is that it is not able to make use of the full control authority. Otherwise, it is simple, and often effective.

D. Direct Control Allocation

In the direct allocation (DA) proposed by Durham [6], the objective is to find a control vector δ that results in the best approximation to the commanded moment d_{des} in a given direction. The Attainable Moment Set (AMS) is defined as the set of all moments that can be produced by a set of control effectors constrained within a set of known limits. If a desired moment lies outside the boundary of the AMS, then it is not attainable; in such a case, the solution which lies on the boundary of the AMS and preserves the direction of d_{des} is used. If the desired moment lies within the AMS, the control effector solution is scaled down such that equation (4) is satisfied.

IV. Nonlinear Control Allocation

The assumption that aerodynamic control effectors produce moments that are linear is often violated in practice. The impact of this assumption becomes especially important in the event of failure of one or more control effectors. The effectors can then be forced to operate in highly nonlinear regions of the moment-deflection curves. To relax the linearity assumption, we propose a nonlinear control allocation (NCA) method that minimizes the sum of weighted square distances between the commanded moments and the corresponding moment functions:

$$\begin{aligned} min_L[w_l(L_d - L(\delta))^2 + w_m(M_d - M(\delta))^2 \\ + w_n(N_d - N(\delta))^2] \end{aligned} \tag{10}$$

subject to

$$\begin{aligned} \delta_{min} \le \delta \le \delta_{max} \\ \dot{\delta}_{min} \le \dot{\delta} \le \dot{\delta}_{max} \end{aligned}$$

where w_l, w_m, and w_n are weighting factors that can be used to weight the relative importance of achieving the individual moments.

The functions $L(\delta)$, $M(\delta)$ and $N(\delta)$ are defined via a nonlinear curve fit based on information obtained from the aerodynamic database. Therefore, the approach uses a two-stage search. From the specified desired moments, the volume of AMS is searched for the points closest to the desired moment. The minimum and the maximum range of the deflections are calculated to produce a resulting moment closest to the desired one. This point is used to initialize an optimization algorithm that uses a Sequential Quadratic Programming (SQP) approach to solve for the control deflections that yield the desired moment to within some prespecified tolerance. The algorithm has been implemented using the gradient-based optimization tools available in

Matlab. The following set of basis functions are used to curve fit the aerodynamic data:

$$Moment = \sum_{i=1}^{4} a_i \delta_i + \sum_{i=1}^{4} b_i \delta_i^2 + \sum_{i=1}^{4} c_i \delta_i^3 + k_i \qquad (11)$$

where $a_i,\ b_i,\ c_i$ and k_i represent the coefficients of each polynomial approximation.

The basis function structure was obtained after a careful investigation of the moment versus deflection plots over a wide range of operating conditions, as provided by the aerodynamic database. To illustrate the nonlinearity inherent in the relationship between moment and effector deflection, consider the pitching moment for a tail effector depicted in Figure 2. It is evident from the plot that for a portion of this moment-versus-deflection curve, a linear approximation is adequate to represent the mapping (approximately in the region of -10° to 10° of deflection). However, failure conditions can drive the effectors to a region where the linear approximation is no longer valid.

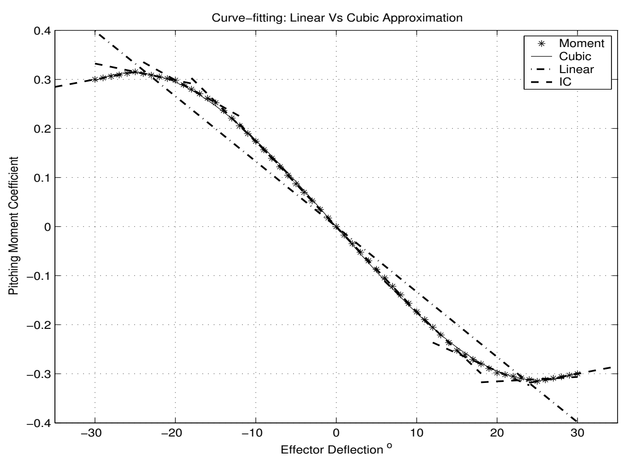

Fig. 2. Curve-fitting result for moment-versus-deflection curve. The curve marked with "*" represents the moment obtained from the aerodynamic database, the solid curve shows the curve fit using (11), the dashed line shows an approximation for this mapping that is used in the MOIC, and the cross-hatched line shows the linear approximation that is used for the RPI and DA.

Levenberg-Marquardt or Gauss-Newton methods can be used to obtain the curve fit. It is important to note that the coefficients are obtained off-line for different operating regimes specified by Mach number, angle of attack, sideslip and altitude. They are stored in databases for future retrieval. Cubic interpolation is then employed to extract the appropriate coefficients corresponding to the operating point. Position and rate constraints are included in the control allocation law by applying the most restrictive at the current operating condition. Specifically, the performance index is minimized subject to $\underline{\delta} \leq \delta \leq \overline{\delta}$, where $\underline{\delta}$ and $\overline{\delta}$ are the most restrictive lower and upper bounds on the control effector deflection, given by

$$\overline{\delta} = min(\delta_{max}, \Delta T \dot{\delta}_{max} + \delta)$$
$$\underline{\delta} = max(\delta_{min}, -\Delta T \dot{\delta}_{max} + \delta)$$

where $\delta_{max}, \delta_{min}$ are the lower and upper position limits vector, $\dot{\delta}_{max}$ is a vector of effector rate limits, and ΔT is the inner-loop flight control system sampling rate.

The proposed two-stage nonlinear algorithm (curve fit and nonlinear programming optimization) successfully accounts for the nonlinearities inherent in the re-entry vehicle model. Superior performance is possible, as would be expected, over the linear control allocation schemes, particularly in the event of failures. The main disadvantage of this method lies in its computational complexity: its feasibility for utilization in real-time control allocation schemes is a subject of current investigation.

A. *Results and Comparison*

A test trajectory is used here to compare the different control allocation schemes. The trajectory is a rather stringent test for the control allocation schemes, since it drives the system into the nonlinear regions in the pitch, roll and yaw moment characteristics. The performance of the different control allocation schemes is shown in Figures 3 and 4. For these results, the control allocator was isolated (open-loop). Hence, there were no feedback loops being closed on the inner-loop. In Figure 3 the dotted trace displays the desired moment or acceleration to be produced by the control effectors (corresponds to d_{des}). Each control allocation scheme computes a set of control effector deflections at each time instant. These deflections are then used to interrogate the aerodynamic database to find the moments produced by the control effectors for each allocation algorithm. For the sake of clarity, the results for the redistributed pseudo inverse technique (which are the poorest of the methods compared) are omitted. The mixed optimization with intercept correction scheme implemented in this comparative study follows the work of [2], but implements a local slope of the control moment curve with an added intercept term to account more accurately for the nonlinear behavior of aerodynamic control effectors (see [4]). From Figure 3, it is clear that once the nonlinear region is reached the nonlinear technique, as expected, tracks the desired moments quite well. Table I summarizes the comparative performance for the nominal test trajectory (no failures) in Figure 3. The mean squared error corresponds to the error in the commanded moment and the moment obtained by substituting the allocated controls into the nonlinear aerodynamic database and summing the contributions of the individual moment effectors. The average number of control effector saturations occurred is also given. Figure 4 illustrates the error performance of the three methods.

B. *Actuator Failures*

As mentioned previously, when an actuator failure occurs, control allocation schemes must be able to accommodate the increased demands on the remaining effectors in order to produce desired and attainable moments. Among the various possible failures, those addressed in this study are:

TABLE I

PERFORMANCE (NO FAILURE): MSE-MEAN-SQUARE ERROR FOR MOMENT COEFFICIENTS; MOIC-MIXED OPTIMIZATION WITH INTERCEPT CORRECTION; NCA-NONLINEAR CONTROL ALLOCATION; RPI-REDISTRIBUTED PSEUDO INVERSE; DA-DIRECT ALLOCATION

Method	MSE Roll	MSE Pitch	MSE Yaw	Saturations
MOIC	3.55e-003	1.77e-001	1.23e-002	25
NCA	2.84e-004	1.48e-003	1.44e-006	4
RPI	1.83e-004	5.38e-001	1.68e-003	50
DA	9.10e-005	4.31e-002	3.83e-005	24

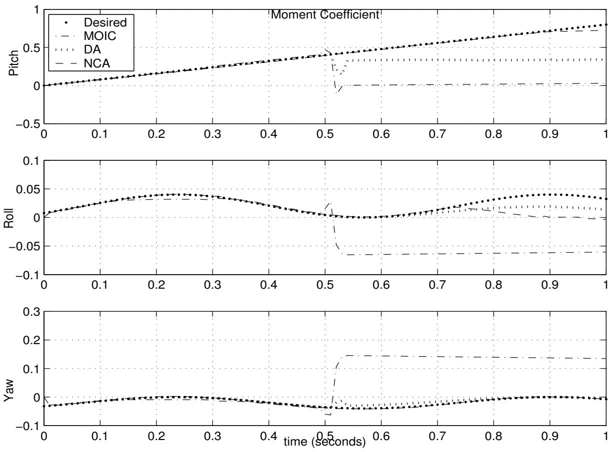

Fig. 3. Comparison of different control allocation schemes for moment coefficient trajectories. MOIC, DA, and NCA correspond to the results generated by the mixed optimization with intercept correction, direct allocation, and nonlinear control allocation schemes, respectively.

1) Lock-up failure: The failed actuator remains in a fixed position, irrespective of the command to the device.
2) Surface Loss: The actuator is still functioning properly, but the vehicle control surface is partially or completely damaged.
3) Floating surface failure: The actuator position floats with the angle of attack, that is, the control surfaces are intact but are simply floating.

Figures 5 and 6 shows the open loop performance (control allocation scheme only) of the allocation schemes in the event of a particular failure: the tail surface effectiveness is decreased in such a way that the actuator can move only from -2 to 2 degrees. Figure 7 shows the closed loop responses with the dynamic inversion control law under the same failure. In this case, the control allocation scheme is used in conjunction with the dynamic inversion control law and prefilters to form a closed inner-loop.

In Figure 7, the curve labelled "desired" is the resulting moment inside the feedback loop when the output of the nonlinear CA is used (results using mixed optimization give unreasonably large moment commands and have been omitted). Table II gives results for this particular failure. It is apparent that the nonlinear control allocation scheme provides superior performance compared to the linear schemes. Similar results have been obtained for various other failures, such as stuck actuators and so forth. Current investigations aim at clarifying the interactions between the control law

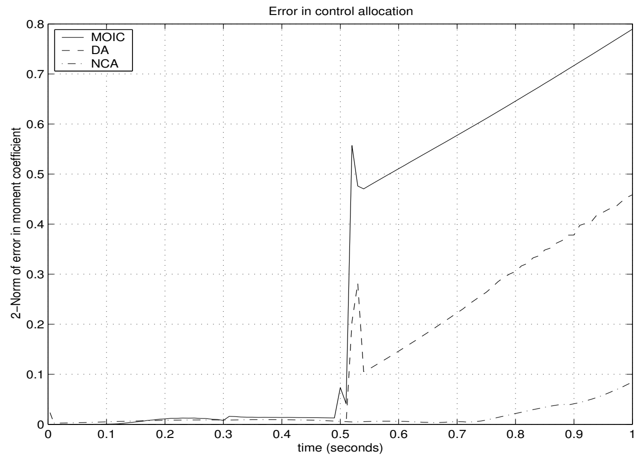

Fig. 4. Comparison of different control allocation schemes for moment coefficient trajectories: Error in moment coefficient ($\| d - \epsilon(P, \delta) \|_2$)

and the control allocator in closed loop under various failure conditions.

TABLE II

PERFORMANCE (WITH FAILURE)

Method	MSE Roll	MSE Pitch	MSE Yaw	Saturations
MOIC	1.174e-003	1.26e-002	2.08e-004	25
NCA	3.99e-003	4.43e-005	1.92e-004	0
RPI	5.80e-004	2.89e-004	4.24e-003	51
DA	1.10e-004	6.38e-003	3.23e-005	24

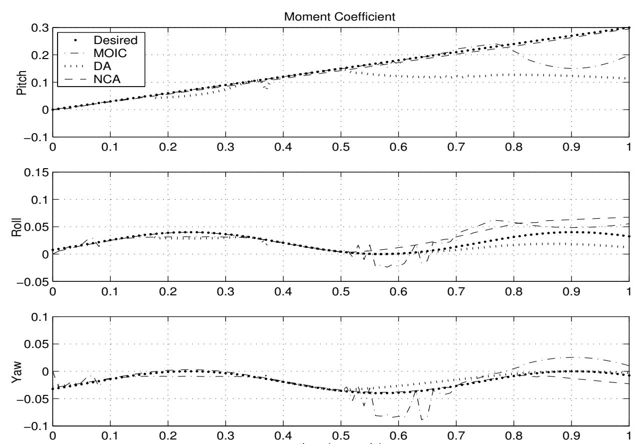

Fig. 5. Comparison of different control allocation schemes under tail effector failure, in terms of moment coefficients

C. Split Actuators

The re-entry vehicle model under investigation has only four control surfaces, implying that a failure (especially in the tail effector) can have significant adverse effects on performance, despite having three remaining healthy actuators. If the control actuator architecture were to be changed to accommodate what is known as a "split-surface" actuator structure, then control authority can be positively affected. Currently, we are investigating the performance of a redesign of this sort for the tail effector in the presence of failures. Specifically, the moment contributions from a split tail effector are assumed to be half that of the original

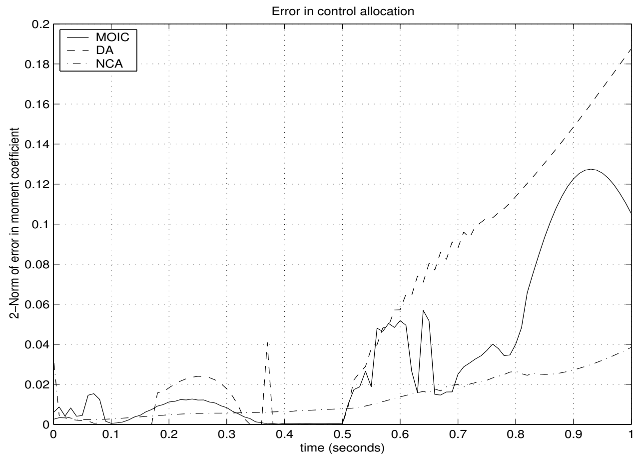

Fig. 6. Comparison of different control allocation schemes under tail effector failure, Error in moment coefficient $((\| d - \epsilon(P, \delta) \|_2))$

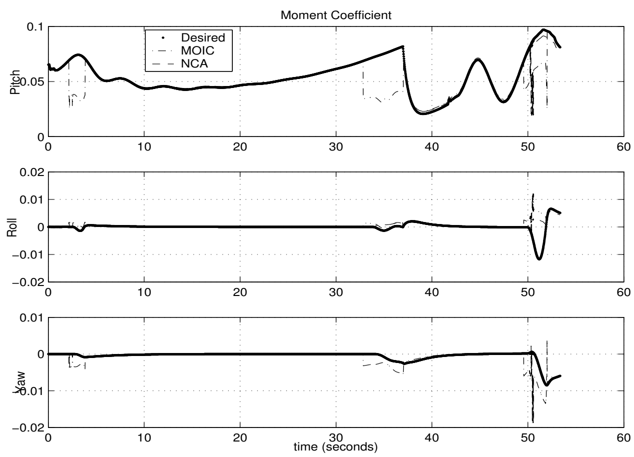

Fig. 7. Closed loop control allocation under effector failure with desired input from the dynamic inversion control law.

tail. The curve-fitting is redone to accommodate the split effectors and the optimization routine is executed as before. Figure 8 illustrates the performance of the nonlinear control allocation scheme under the failure condition of two split tails (one on each side) stuck at 4 degrees for a pitch moment trajectory in which the roll and yaw motions of the vehicle are held at zero (being identically zero, the coefficient trajectories for roll and yaw are not shown). As can be seen, the vehicle cannot maintain the desired pitch command while at the same time regulating the rolling and yawing moments to zero.

V. CONCLUSIONS

In this paper a two-stage nonlinear control allocation technique has been proposed and tested for an experimental re-entry simulation. Standard nonlinear programming techniques were used to improve control allocation methods, and comparisons were drawn against currently used linear control allocation algorithms. Among linear techniques employing linear programming, the redistributed pseudo-inverse was found to exhibit large errors, with frequent saturation of the effectors. The mixed optimization with intercept correction scheme based on the simplex algorithm with an affine intercept correction, performs well in regions where the linear behavior is dominant with monotonic moment deflection relationships, but performs poorly when nonlinearities become significant. The nonlinear technique proposed herein gives better results in the nonlinear regions, and in presence of failures. Moreover, preliminary results for split actuator architectures have been investigated for similar scenarios. Future work along these lines will address the computational burden of these techniques, and its impact on real-time implementation.

TABLE III

AVERAGE MSE PERFORMANCE UNDER TWO DIFFERENT TRAJECTORIES AND SIX DIFFERENT FAILURES (12 TESTS)

Method	MSE Roll	MSE Pitch	MSE Yaw
MOIC	2.04e-004	2.02e-001	3.46e-004
NCA	1.15e-004	2.66e-002	1.40e-005
RPI	3.81e-004	2.47e-001	1.37e-003
DA	3.94e-004	1.66e-001	1.56e-003

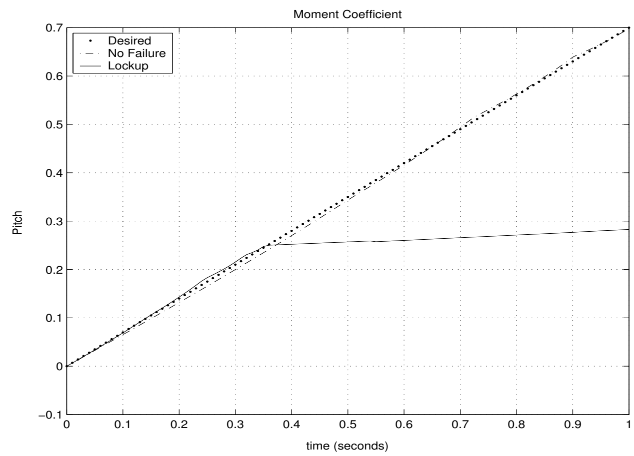

Fig. 8. Split tail actuator performance under effector failure, in terms of pitching moment coefficient.

REFERENCES

[1] D.B. Doman, A.D. Ngo, "Dynamic Inversion-Based Adaptive/Reconfigurable Control of the X-33 on Ascent", *AIAA Journal of Guidance Control and Dynamics*, Vol. , No. , 2002.

[2] M. Bodson, "Evaluation of optimization methods for control allocation, *AIAA Journal of Guidance, Control, and Dynamics*, 25(4):703–711, 2002.

[3] D. Enns, "Control allocation approaches," *AIAA Journal of Guidance, Control, and Dynamics*, 98-4109, 1998.

[4] D.B. Doman and M.W. Oppenheimer "Improving Control Allocation Accuracy for Nonlinear Aircraft Dynamics," *Proceedings of 2002 Guidance, Navigation and Control*, AIAA, 2002.

[5] A.B. Page and M.L.Steinberg "A closed comparison of control allocation methods," *Proceedings of 2000 Guidance, Navigation and Control*, AIAA 2000-4538,2000.

[6] W.Durham, "Contrained Control Allocation," *Journal of Guidance, Control and Dynamics*, Vol.16, No.4, July-August,1993.

[7] Y. Luo, A. Serrani, S. Yurkovich, D. B. Doman and M. W. Oppenhiemer, "Model predictive dynamic control allocation with actuator dynamics," *Proceedings of the 2004 American Control Conference*.

[8] M.A. Bolender, D.D. Doman, "Non-Linear Control Allocation Using Piecewise Linear Functions: A Linear Programming Approach", *Proceedings of 2003 Guidance, Navigation and Control*, AIAA, 2003.

[9] "Application of Multivariable Control Theory to Aircraft Control Laws," Tech. Report, WL-TR-96-3099, Wright Laboratory, WPAFB, OH.