A Switched System Model for Stability Analysis of Distributed Power Control Algorithms for Cellular Communications

Ayanendu Paul, Mehmet Akar, Urbashi Mitra and Michael G. Safonov
Department of Electrical Engineering-Systems
The University of Southern California
Los Angeles, CA, 90089 USA
E-mail: {ayanendp, akar, ubli, safonov}@usc.edu

***Abstract*— In this paper, we examine the well known Distributed Power Control (DPC) algorithm proposed by Foschini and Miljanic [1] and show via simulations that it may fail to converge in the presence of time–varying channels and handoff, even when the feasibility of the power control problem is maintained at all times. Simulation results also demonstrate that the percentage of instability is a function of the variance of shadow fading, interference and the target signal to interference plus noise ratio. In order to better explain these observations and provide a systematic framework to study the stability of distributed power control algorithms in general, we present the problem in the context of switched systems, which can capture the time variations of the channel and handoffs. This formulation leads to interesting stability problems, which we address using common quadratic Lyapunov functions and M–matrices.**

Keywords: Power control, wireless networks, switched systems, time–varying channel

I. Introduction

Controlling transmitted power in a wireless network is important to mitigate the near–far effect, meet minimum quality of service requirements and prolong battery life of mobile users. A common goal of any power control algorithm is to maintain an acceptable signal to interference plus noise ratio (SINR) for individual users while minimizing the total transmitted power. This objective can be met by using centralized power control algorithms [2], [3], [4]. However, centralized algorithms are not practical as they require complete information on the link gains. Therefore a partially distributed version of these algorithms was stated in [5], [6].

A completely Distributed Power Control (DPC) algorithm was first proposed by Foschini and Miljanic in [1]. Unlike [5], [6], this algorithm takes into account the receiver thermal noise and is implemented in a fully distributed fashion, that is, each user updates its transmitted power using only local measurements of its own achieved SINR and its past transmitted power. It is shown that this algorithm converges to a unique equilibrium power vector provided that there exists a feasible solution to the power control problem [1]. Later, several other fully distributed power control algorithms were proposed in [7], [8]. The algorithms [1]–[8] did not take the possibility of handoffs into account, which was studied in [9], [10]. Although the research reported in [1]–[10] addressed several important aspects of power control, they all assumed that the link gains were constant.

In reality, the channel is changing rapidly due to various factors, including fading, handoff, user mobility, etc. In this paper, we study the well known DPC algorithm [1] in the context of these channel variations. The obvious question that arises is: Given this rapid variation in channel gains and handoffs, would the distributed algorithm of [1] still track the equilibrium power (which would also be time–varying due to channel variations), or at least be bounded? In this paper, we show that the answer is indeed no in some cases. In order to better explain our observation, we present the problem in the context of switched systems which can capture the time variations of the channel and handoffs. We study the DPC algorithm using this model and show that it may diverge even if the stability condition of the algorithm mentioned in [1], namely the feasibility of the power control problem, is satisfied at all times.

The rest of the paper is organized as follows. In Section II, we briefly describe the power control problem and the distributed algorithm proposed in [1]. In Section III, we propose a switched system model to study distributed power control algorithms under time–varying channels and handoffs. Using this model, we analyze a two mobile, two base station scenario in Section IV and examine the possible effects of shadow fading and handoffs on DPC. Finally, we present the conclusions and discuss some future work in Section V.

II. Problem Description

We consider a network of N mobile users communicating on the same channel. We assume that the i^{th} mobile user is connected to the i^{th} base station, $i = 1, \ldots, N$. If two mobile users, say i and j are assigned to the same base station, then the indices i and j refer to the same physical base station. We study only the uplink, although the results are applicable to downlink as well. All values are in linear scale unless otherwise mentioned.

Let $g_{ij}(t)$ represent the channel gain between the j^{th} transmitter and the i^{th} receiver, and $p_i(t)$ denote the power transmitted by the i^{th} user. The achieved SINR for the i^{th} user can then be expressed as

$$\gamma_i(t) = \frac{g_{ii}(t)p_i(t)}{\sum_{j\neq i} g_{ij}(t)p_j(t) + \nu_i(t)}, \tag{1}$$

where $\nu_i(t)$ is the thermal noise at the i^{th} receiver.

The objective of power control in a wireless network is to update the power levels $p_i(t)$ in a distributed fashion, so that the actual SINR for each user exceeds an acceptable level, *i.e.,*

$$\gamma_i(t) \geq \gamma, \;\; i = 1, ..., N, \tag{2}$$

where γ is referred to as the target SINR. Foschini and Miljanic proposed a distributed algorithm to satisfy the above inequalities for all users [1]. The algorithm, which is referred to as DPC, can be represented in continuous–time as

$$\dot{p}_i(t) = -\left[1 - \frac{\gamma}{\gamma_i(t)}\right] p_i(t), \; i = 1, \ldots, N, \tag{3}$$

or equivalently using vector notation,

$$\dot{p}(t) = -B(t)\; p(t) + \;\eta(t), \tag{4}$$

where $p(t) = [p_1(t), p_2(t), ..., p_N(t)]^T$ is the transmitted power vector; $\eta(t) = [\eta_1(t), \eta_2(t), ..., \eta_N(t)]^T$ is the normalized noise vector with $\eta_i(t) = \gamma\; \nu_i(t)/g_{ii}(t)$; and $B(t) = [b_{ij}(t)]_{N\times N}$ is the normalized propagation matrix, whose components are given by

$$b_{ij}(t) \;\triangleq \begin{cases} 1, & \text{if i = j,} \\ -\gamma \frac{g_{ij}(t)}{g_{ii}(t)}, & \text{if i} \neq \text{j.} \end{cases} \tag{5}$$

Under static channel conditions, *i.e.*, when $B(t) = B$ and $\eta(t) = \eta$ are constants, a necessary and sufficient condition for the stability of the DPC algorithm is that all eigenvalues of B are in the right half plane. When the power control problem is feasible, *i.e.*, if there exists a positive power vector p such that the inequalities in (2) are satisfied, we have

$$Bp = \eta. \tag{6}$$

Due to the special structure of the matrix B, *i.e.*, $b_{ii} = 1 > 0, b_{ij} \leq 0, i \neq j$, and the fact that it satisfies (6) for some positive vectors p and η imply that B is an M–matrix[1]. Hence, all of its eigenvalues are in the right half plane, implying that (4) is stable [1], [12]. In that case, the power vector converges to the unique equilibrium

$$p^\star = B^{-1}\eta > 0 \tag{7}$$

under static channel conditions [1].

[1]A real $N\times N$ matrix $B = [b_{ij}]$ is said to be an M–matrix if $b_{ij} \leq 0,\; i \neq j$ and if all principle minors of B are positive [11], [12], [13].

As discussed in the Introduction section, most available power control algorithms assume constant link gains [1]–[10]. However, wireless channels are highly time–varying due to fading, handoff and user mobility; therefore both $B(t)$ and $\eta(t)$ in (4) are functions of time. In such time–varying cases, the simple requirement that all eigenvalues of $B(t)$ are in the right half plane (*i.e.,* B(t) is an M–matrix) at all times is not a sufficient condition for the boundedness of the power vector.

III. Switching Based Problem Formulation

In this section, we pose the distributed power control problem in the framework of a switched system. Wireless communication channels vary rapidly with time due to fading and user mobility. Furthermore, the link gain between a mobile user and its assigned base station changes considerably with a handoff. Thus the propagation matrix $B(t)$ is a function of channel fading, mobile user location and base station assignment. To capture all these variations, in this paper we modify (4) as

$$\dot{p}(t) = -B(\,\chi(t), t)p(t) \;\; + \eta(\chi(t), t), \tag{8}$$

where $\chi(t)$ is a switching signal that characterizes the handoff decision. The system matrix $B(\,\chi(t), t)$ assumes values from the set $\{B_1(t),\; B_2(t), ..., B_K(t)\}$, which represents all K possible base station assignments where each $B_i(t)$ is a continuously time–varying matrix that models the effects of path loss, shadowing, multi–path fading and user mobility for a particular base station assignment. Similarly, $\eta(\chi(t), t)$ can take values from the set $\{\overline{\eta}_1(t),\; \overline{\eta}_2(t), \ldots, \overline{\eta}_K(t)\}$, which represents the possible thermal noise vectors.

In this paper, we would like to examine the stability properties of (8), given the special structure of the system matrix $B(\,\chi(t), t)$. To this end, consider the homogenous system

$$\dot{p}(t) = -B(\,\chi(t), t)p(t). \tag{9}$$

Unfortunately, stability properties of (8) may not immediately follow from the stability properties of the homogenous system in (9). To illustrate this, consider the following time–varying system [14]

$$\dot{x}(t) = -\frac{1}{t+2}x(t) + f(t), \tag{10}$$

where $x(t)$ is the state of the system and $f(t)$ is the input. For the unforced system (with $f(t) = 0$), the solution is given by

$$x(t) = \frac{2}{t+2}x_0,$$

where x_0 is the initial condition [14]. Clearly, the system is stable for all initial conditions. However, the complete solution of the system with the input $f(t) = 1$ is

$$x(t) = \frac{2}{t+2}x_0 + \frac{1}{2}(t+2) - \frac{2}{t+2}$$

and $\lim_{t\to\infty} x(t) = \infty$ [14]. Therefore, the output of the system is seen to be unbounded although the homogenous system is stable and $f(t)$ is bounded.

The above example illustrates some of the challenges in studying the stability properties of the time–varying system in (8). To gain further insights into the problem, we first assume that the matrices $B_1(t), \ldots, B_K(t)$ and the noise vectors $\eta_i(t), \ldots, \eta_K(t)$ are constants and study the possible effect of hard handoff on the stability of the system. Handoff is the activity of changing the controlling base station for a particular mobile user to maintain satisfactory quality of service. In hard handoff, a mobile user is controlled by a single base station at any given time and connection with a new base station can be made only after breaking the connection with the previous controlling base station [15]. An extension of hard handoff is soft handoff where the mobile can communicate with more than one base station at a given time [16]. However, in this paper, we focus on hard handoff only.

The stability problem might then be restated as follows. Consider the switched system

$$\Sigma_s : \dot{p}(t) = -B(t)p(t) + \eta(t),$$
$$B(t) \in \mathcal{B} = \{B_1, \ldots, B_K\}, \;\; \eta(t) \in \tilde{\eta} = \{\overline{\eta}_i, \ldots, \overline{\eta}_K\}, \tag{11}$$

where B_i and $\overline{\eta}_i$ are the system matrices and the thermal noise vectors for the subsystems

$$\Sigma_i : \; \dot{p}(t) = -B_i p(t) + \overline{\eta}_i \;, \; i = 1, 2, \ldots, K. \tag{12}$$

The individual subsystems of (12) correspond to all possible base station assignments. The network is assumed to have a feasible power vector at each time and for all handoff assignments. This implies that each $B_i, \; i = 1, ..., K$ is an M-matrix and the associated subsystem $\Sigma_i, \; i = 1, \ldots, K$ is asymptotically stable. Hence, the power vector converges to its equilibrium value, given by $B_i^{-1}\overline{\eta}_i$, for each Σ_i [1].

From the above discussion, the natural question that arises is whether the stability of the individual subsystems in (12) imply the boundedness of the power vector in (11). There are two factors that may lead to undesired effects:
(i) Switching among the subsystems, and
(ii) The fact that the subsystems in general have different equilibrium points, given by $B_i^{-1}\overline{\eta}_i, i = 1, \ldots, K$.

If the subsystems have the same equilibrium, *i.e.*, $B_i^{-1}\overline{\eta}_i = p^\star, i = 1, \ldots, K$, where $p^\star$ is constant, then the stability properties of (11) and (12) can be examined assuming $\eta(t) = 0$ and $\overline{\eta}_i = 0, \; i = 1, \ldots, K$. However, even when this is true, the following example shows that the stability of the individual subsystems in (12) is not sufficient for the stability of (11).

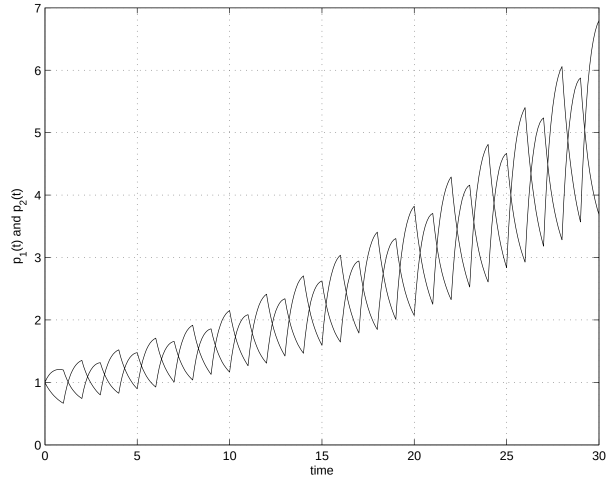

Fig. 1. System state response in Example 1

Example 1: Consider the switched system in (11) with $N = 2, \; K = 2, \; \eta_1 = \eta_2 = 0$ and the system matrices

$$B_1 = \begin{bmatrix} 1 & -1.7 \\ -0.4 & 1 \end{bmatrix} \;\text{ and }\; B_2 = B_1^T.$$

It can be shown that all eigenvalues of B_1 and B_2 are positive and hence both subsystems are stable. However, if we switch among the subsystems periodically every 1.5 seconds or less, the resulting switched system of (11) is unstable for any non-zero initial condition. Fig. 1 shows the evolution of the power vector for the initial condition $p(0) = [1, \; 1]^T$ and periodic switching among the subsystems every 1 sec. Clearly the switched system is unstable. $\square$

In this paper, stability analysis of the switched system in (11) will be carried out using the Common Quadratic Lyapunov Function (CQLF) approach [17], [18]. The objective here is to find a quadratic Lyapunov function $v(p) = p^TQp$ where $Q = Q^T > 0$ such that $v(p)$ decreases along the trajectories of each subsystem $\Sigma_i, i = 1, \ldots, K$. Alternatively, the problem involves finding necessary and sufficient conditions for the existence of a positive definite matrix $Q = Q^T > 0$ such that the following linear matrix inequalities (LMI) are satisfied:

$$B_i^TQ + QB_i > 0, \; i = 1, \ldots, K. \tag{13}$$

We have the following results.

***Proposition** 1:* If there exists a symmetric positive definite matrix Q such that (13) holds, then the power vector in (11) is bounded under arbitrary switching.

Proof: Let $v(p) = p^TQp$ be a Lyapunov function candidate for (11). We have

$$\dot{v} = -p^T \left(B^T(t)Q + QB(t) \right) p \; + 2p^TQ\eta(t)$$

$$\leq -\overline{\lambda}||p||^2 + 2\lambda_M(Q)\eta_b||p||,$$

where $||p||$ is the norm of the power vector $p(t)$; the constants η_b and $\overline{\lambda}$ are given by

$$\eta_b = \max_{i=1,\ldots,K} ||\overline{\eta}_i||,$$

$$\overline{\lambda} = \min_{i=1,\ldots,K} \lambda_m(B_i^T Q + QB_i),$$

$\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalues, respectively. Since (13) is satisfied for $i = 1, ..., K$, we have $\overline{\lambda} > 0$. Therefore, we obtain $\dot{v} \leq 0$ for $||p|| > \frac{2\lambda_M(Q)\eta_b}{\overline{\lambda}}$. (QED) □

From Proposition 1, it follows that the solution of (11) will be bounded under arbitrary switching (which may be due to handoff) if the subsystems Σ_i, $i = 1, \ldots, K$ have a CQLF. Note that, in general, existence of a CQLF for the subsystems may not be necessary for the stability of a switched system [19], [18]. Given the matrices B_i, $i = 1, \ldots, K$, the existence of a CQLF can be checked numerically by solving the LMIs in (13) using MATLAB. However, without exact information about these matrices, the approach we pursue is to derive theoretical conditions for the existence of a CQLF by exploring the special structure of the system matrices. To this end, we first consider second order systems and state the following results.

***Proposition** 2:* The following statements are equivalent for a 2 user case.

(i) The switched system Σ_s in (11) is bounded under *arbitrary* switching.
(ii) The matrix pencil $\sum_{i=1}^{K} \alpha_i(-B_i)$ is Hurwitz stable for all $\alpha_i \geq 0$, $i = 1, 2, \ldots, K$ and $\sum_{i=1}^{K} \alpha_i = 1$.
(iii) The matrix pencils $\alpha(-B_i) + (1-\alpha)(-B_j)$ are Hurwitz stable for all $\alpha \in [0, 1]$, and all $i, j = 1, 2, \ldots, K$, $i \neq j$.
(iv) A diagonal CQLF exists for every pair of subsystems Σ_i and Σ_j, $i, j = 1, 2, \ldots, K$, $i \neq j$.
(v) A diagonal CQLF exists for all of the subsystems Σ_i, $i = 1, 2, \ldots, K$.

Proof: Follows from Proposition 1 and the proof of Theorem 1 in [20]. □

Remark: If $B_i^{-1}\overline{\eta}_i = p^\star$, $i = 1, \ldots, K$, where $p^\star$ is constant, then the equivalence of the statements in Proposition 2 will still hold when (i) is replaced by the following stronger statement:

(i)$'$ The switched system Σ_s in (11) is stable under *arbitrary* switching.

Proposition 2 states necessary and sufficient conditions on the stability of (11) for the special class of second order systems. As a simple application of this result, reconsider Example 1. It can be shown that the matrix pencil $\alpha(-B_1) + (1-\alpha)(-B_2)$, $\alpha \in [0, 1]$, has eigenvalues in the right half plane for $\alpha = 0.5$ and hence the output of the switched system, as per Proposition 2, is not stable under sufficiently fast switching.

Even though a precise condition for the existence of a CQLF and hence for the boundedness of second order switched systems are given in Proposition 2, these results are yet to be extended to higher order systems.

IV. Handoff in two user case

In this section, we examine a simple scenario with two mobiles and two base stations, and study the stability of the DPC algorithm under shadow fading and hard handoff. Consider two mobile users, M_1 and M_2, and two base stations, B_1 and B_2, as shown in Fig. 2. Recall that $g_{11}(t)$ and $g_{21}(t)$ represent the channel gains from M_1 to B_1 and B_2, respectively. Similar notations $g_{12}(t), g_{22}(t)$ apply for M_2.

We assume that a mobile is connected to the base station with which its channel gain is largest at a given time [9], [10]. Depending on the signal strength, fading, effective SINR, mobility, etc., there might be four possible base station assignments with system matrices of (8) as follows:

(i) When $g_{11} > g_{21}$ and $g_{22} > g_{12}$, M_1 is connected to B_1, M_2 to B_2, and the system matrix is of the form

$$B_1 = \begin{bmatrix} 1 & -\gamma\frac{g_{12}(t)}{g_{11}(t)} \\ -\gamma\frac{g_{21}(t)}{g_{22}(t)} & 1 \end{bmatrix},$$

(ii) When $g_{21} > g_{11}$ and $g_{22} > g_{12}$, M_1 is connected to B_2, M_2 to B_2, and the system matrix is of the form

$$B_2 = \begin{bmatrix} 1 & -\gamma\frac{g_{22}(t)}{g_{21}(t)} \\ -\gamma\frac{g_{21}(t)}{g_{22}(t)} & 1 \end{bmatrix},$$

(iii) When $g_{11} > g_{21}$ and $g_{12} > g_{22}$, M_1 is connected to B_1, M_2 to B_1, and the system matrix is of the form

$$B_3 = \begin{bmatrix} 1 & -\gamma\frac{g_{12}(t)}{g_{11}(t)} \\ -\gamma\frac{g_{11}(t)}{g_{12}(t)} & 1 \end{bmatrix},$$

(iv) When $g_{21} > g_{11}$ and $g_{12} > g_{22}$, M_1 is connected to B_2, M_2 to B_1, and the system matrix is of the form

$$B_4 = \begin{bmatrix} 1 & -\gamma\frac{g_{22}(t)}{g_{21}(t)} \\ -\gamma\frac{g_{11}(t)}{g_{12}(t)} & 1 \end{bmatrix}.$$

It can be shown that for $\gamma \leq 1$, the individual system matrices $B_i, i = 1, ..., 4$, are M-matrices, hence all of their eigenvalues are in the right half plane (note that this is a conservative case, as even for $\gamma > 1$, the system matrices might be M–matrices, if the interfering channel gain is low). Hence, at each instant, for all possible base station assignments and channel gains, the power control problem is feasible.

In practical systems, the maximum transmitted power from a mobile is fixed; however in this paper we consider

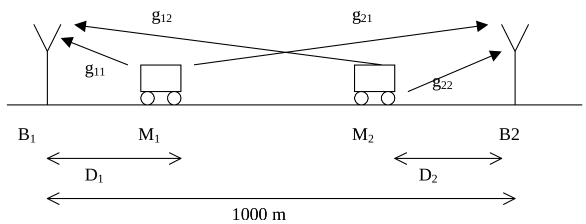

Fig. 2. Simulation scenario with two mobiles and two base stations

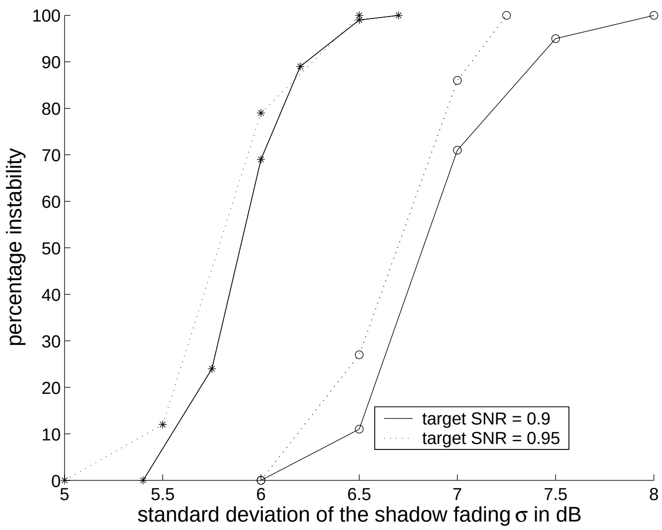

Fig. 3. Percentage instability as a function of standard deviation of fading for $D_2 = 0.4$ km (*) and $D_2 = 0.3$ km (o) and various target SINR

unconstrained DPC to show that the algorithm might diverge in some cases.

All values, unless otherwise noted, are in linear scale. The path loss exponent for channel gains is assumed to be 4 (note that in a practical scenario, even this parameter may be time–varying, thereby augmenting the effect of time–varying channel due to fading) and the lognormal shadowing is assumed with standard deviation of σ (in dB). The thermal noise is taken to be 1e-6.

We implement the DPC algorithm and examine the effect of time–varying channel gains and handoffs on the stability of the algorithm. We consider several simulation scenarios with $\gamma \in \{0.9, 0.95\}$, $D_1 = 300$ m, $D_2 \in \{300$ m, 400 m$\}$, and $\sigma \in [5, 8]$. Monte Carlo simulations are performed and the percentage of total number of simulations for which the transmitted powers become unbounded are plotted in Fig. 3.

It is seen that with the increase in the standard deviation of the fading process, the percentage instability of the algorithm increases, which confirms the fact that with low channel fading, the DPC algorithm indeed tracks the equilibrium power. Also, the percentage instability is a function of D_2 (the distance of M_2 from B_2). The target SINR also plays a major role in the stability problem. Not surprisingly, the higher the target SINR, the higher the

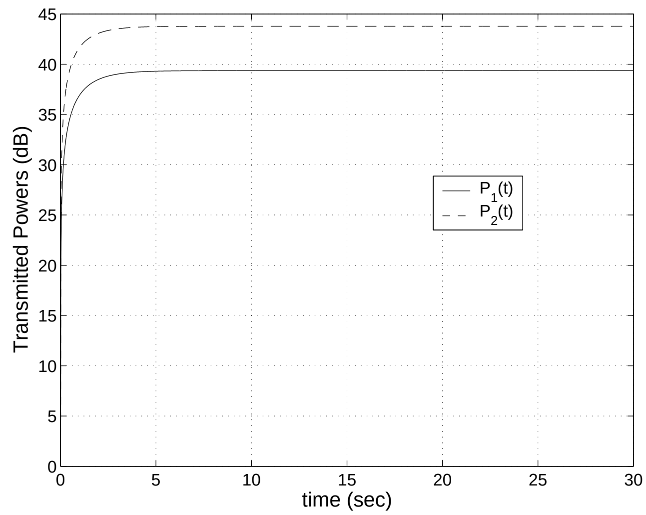

Fig. 4. Very low fading ($\sigma = 0.1$); DPC stable

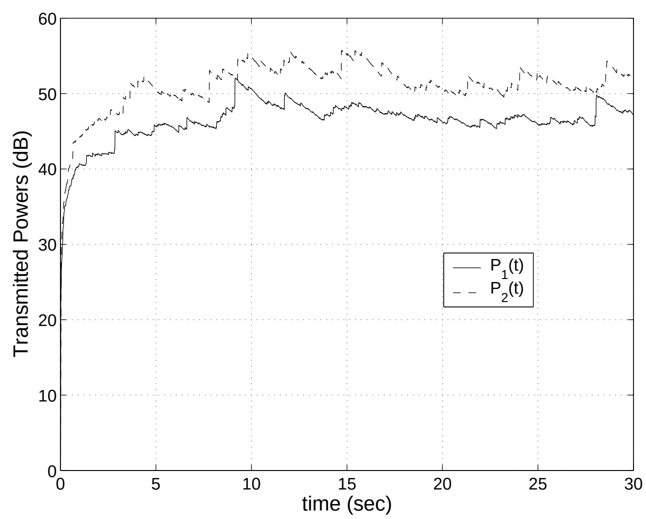

Fig. 5. Moderate fading ($\sigma = 5$); bounded power vectors

percentage instability.

Figs. 4–6 show the power vectors for the mobile users with $\sigma \in \{0.1, 5, 6\}$, $D_1 = 300$ m, $D_2 = 400$ m and $\gamma = 0.9$. The three figures correspond to channels with (i) very low ($\sigma = 0.1$), (ii) moderate ($\sigma = 5$) and (iii) high ($\sigma = 6$) fading. As expected, the first two cases are stable while for the last case, the DPC algorithm fails to converge.

V. Concluding Remarks and Future work

In this paper, it is seen that the stability of the well known DPC algorithm is jeopardized in the presence of time–varying channels and handoffs. Parameters that are observed to affect the dynamic properties adversely include the variance of fading process, interference levels and target SINR. We have also proposed a switched system framework to analyze power control algorithms under fading channels

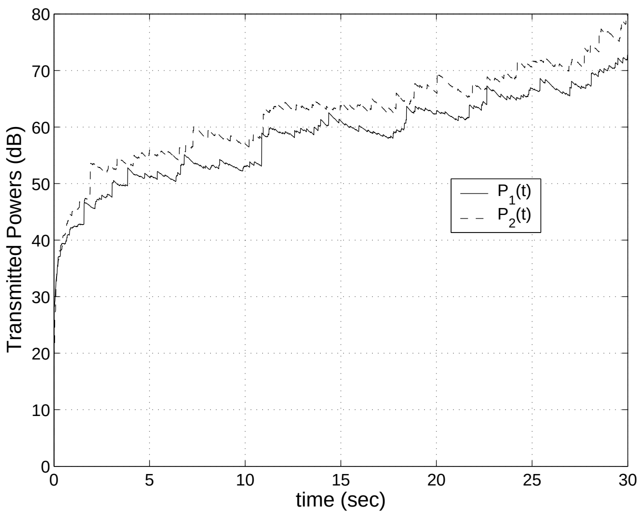

Fig. 6. High fading ($\sigma = 6$); power vectors unbounded

and handoffs, and linked it with the CQLF concept and M–matrices. Future research will be on extending the results in this paper to higher order systems and using these ideas to develop schemes to avoid possible instabilities.

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