

Robust Stability Constrained Model Predictive Control

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Abstract— This paper proposes a robust model predictive control (MPC) scheme to asymptotically stabilize an uncertain linear plant with polytopic model uncertainty description. Quadratic robust stability constraints are explicitly imposed as contractive constraints on the predicted state at each sampling time. The feasibility of these constraints can be detected either off-line or at the first step of on-line optimization. The feasibility is independent of the selection of the optimization objective function and its parameters. Therefore, the objective function can be formulated to satisfy other criterion such as the performance requirements. The simulation study shows the effectiveness and features of the proposed method.

I. INTRODUCTION

Stability constrained model predictive control (SCMPC), sometimes also called contractive model predictive control, is one of the approaches to guarantee the closed-loop stability of a predictive control system. In this approach, a stability constraint (or contractive constraint) is explicitly imposed on the predicted state at each step of the on-line optimization. The first control step solved from the optimization is actually implemented and the whole process repeats at the next sampling time. The stability constraint is often constructed in quadratic form and can be shown as a Lyapunov function in different format ([1], [3], [4], [5], [6], [8], [10], [11]). When model uncertainties are included in the consideration, many robust MPC techniques solve the min-max optimization of a quadratic objective function over an infinite prediction horizon ([7],[12]). The infinite horizon objective function naturally becomes the Lyapunov function and the contractive constraint. Therefore, feasibility of the MPC calculation often depends on the selection of the objective function and its associated parameters. The baseline control solved from the first step optimization typically also depends on the initial condition of the state variable even if no control input and plant output constraints are imposed. This restrictiveness is pointed out in the work of [9]. Moreover, in most process control applications, especially in low-level regulation and set-point tracking control where MPC finds most of its applications, the higher priority should be placed on the

closed-loop stability and process response speed. Minimizing the “worst” case objective function value is less important, since the objective function for the true plant is not necessarily optimized.

In this paper we extend the concept proposed in the work of [4] and [5], and address the issue of robust stabilization of an uncertain linear plant described by a set of polytopic linear models using the SCMPC approach. Full state measurement is assumed and a set of Lyapunov equation type of stability constraints are explicitly imposed at each sample time. The objective function can be chosen as any type of convex function in finite prediction horizon. The feasibility of the robust stability constraints does not depend on the parameters in the objective function. The proposed approach guarantees closed-loop stability if the robust stability constraints can be constructed through a feasible off-line linear matrix inequality (LMI) solution, or, in the input constrained case, a feasible on-line LMI optimization at the first step.

This paper is organized as follows: Section II briefly introduces preliminaries and notations. Section III presents the robust SCMPC algorithm for input-unconstrained linear systems. Section IV describes the corresponding robust SCMPC algorithm for control input constrained systems. Section V, followed by the concluding section, illustrates the proposed algorithms by numerical examples.

II. PRELIMINARIES AND NOTATIONS

Assuming a multi-input uncertain linear plant is described by the following state space equation

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

where $x \in \mathfrak{R}^n$, $u \in \mathfrak{R}^m$, and A and B have appropriate dimensions. It is also assumed that the uncertain plant (A , B) is bounded by the convex hull of a set of linear time invariant (LTI) models (A_j, B_j) ($j = 1, \dots, H$). That is,

$$A = \sum_{j=1}^H \alpha_j \cdot A_j, \quad B = \sum_{j=1}^H \alpha_j \cdot B_j \quad (2)$$

where $\sum_{j=1}^H \alpha_j \leq 1$ and $0 \leq \alpha_j \leq 1$.

The use of polytopic model uncertainty representation assumes the true plant is the weighted average of all models in the uncertainty set. It is practically useful, especially in process control applications. For example, if the original model can be described by nonlinear differential (or difference) equations, then a set of LTI models can be obtained by linearizing the nonlinear model at different

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operating points. Models at any other operating points can be approximated as the linear interpolation of models obtained at linearization points. The same thing can be said when a set of LTI models are obtained at different operating points, and perhaps at different times, by experimental methods.

We start by defining a Lyapunov function for the plant as $V(x) = x^T P x$, where P is a positive definite matrix and x is the state variable. Following usual definitions, the weighted 2-norm on matrix P is defined as: $\|x\|_p = \sqrt{x^T P x}$. The prediction horizon (and also the control horizon) is represented by N . The state and control variables predicted at time k are denoted by $x^p(k+i|k)$ and $u^p(k+i-1|k)$ respectively ($i = 1, \dots, N$). The $x_j^p(k+i|k)$ is used to represent the predicted state variable made at time k using the model (A_j, B_j) , where $j = 1, \dots, H$. The measured state variable at time k is $x(k|k) = x(k)$.

Suppose a constant gain control law can be expressed by $u(k) = K \cdot x(k)$, where K is the static state feedback control gain. A sufficient condition to guarantee closed loop stability for the system (A, B) is the existence of positive definite matrices P and Q such that the following set of Lyapunov inequalities are satisfied (see Theorem 1 proof)

$$(A_j + B_j K)^T P (A_j + B_j K) - P \leq -Q \quad (j = 1, \dots, H) \quad (3)$$

By defining $S = P^{-1}$ and $L = K \cdot S$, (3) can be written as the following quadratic inequalities

$$\begin{bmatrix} S^T A_j^T + L^T B_j^T & S^T \\ S & Q \end{bmatrix} \begin{bmatrix} A_j S + B_j L \\ S \end{bmatrix} - S \leq 0 \quad (4)$$

By Schur Complement (see [2]), the satisfaction of (4) is equivalent to successfully solving the following set of linear matrix inequalities (LMI) in terms of matrix variables S, L , and Q^{-1} (for $j = 1, \dots, H$).

$$\begin{bmatrix} S & S^T A_j^T + L^T B_j^T & S^T \\ A_j S + B_j L & S & 0 \\ S & 0 & Q^{-1} \end{bmatrix} \geq 0 \quad (5)$$

From now on, we assume that the optimization objective function in the following robust SCMPC development is a generic convex function $\Phi(x^p(k+i), u^p(k+i-1))$, unless otherwise specified.

III. MAIN RESULT: INPUT-UNCONSTRAINED SCMPC

We start the development with the control input unconstrained case. The control input constrained case will be discussed in the next section. The constraint on plant output is not included in this paper, since it can be solved

in a similar way that the control input constraint is being handled. Only the regulation problem is discussed in this paper, since set-point tracking control can be easily formulated by using augmented state space equations. Output feedback with state estimation is expected to be a natural extension to this work with appropriate modifications.

The input-unconstrained robust SCMPC is formulated as the following algorithm.

Algorithm 1 (input unconstrained case):

Step 1: Solve the LMI (5) off-line. If it is infeasible, stop here. The proposed scheme can not apply directly. If it is feasible, go to the next step 2.

Step 2: Set $x_j^p(k|k) = x(k)$

Solve the following optimization at each sampling time $k = 1, 2, \dots$

$$\min_{u^p(k|k), \dots, u^p(k+N-1|k)} \Phi(x^p(k+i|k), u^p(k+i-1|k))$$

$$\text{S.t.} \quad x_j^p(k+i+1|k) = A_j x_j^p(k+i|k) + B_j u^p(k+i|k) \quad (6)$$

$$\|x_j^p(k+1|k)\|_p^2 - \|x(k)\|_p^2 \leq -\beta_j \cdot \|x(k)\|_Q^2 \quad (7)$$

$$(i = 0, 1, \dots, N, j = 1, \dots, H, 0 < \beta_j \leq 1)$$

Step 3: Apply $u(k) = u^p(k|k)$, and repeat from Step 2.

We denote equations (6) as the prediction state constraint and inequalities (7) as the robust stability constraint. As assumed earlier, Φ is a convex function. The robust stability constraints (7) are quadratic constraints and β_j are tuning constant. Therefore, the overall on-line optimization is a convex programming. The following lemma 1 is introduced to facilitate future discussions.

Lemma 1. If Step 1 in Algorithm 1 (off-line calculation) is feasible, then the robust stability constraints (inequality (7)) in Step 2 is feasible for all $k = 0, 1, \dots$

Proof. If step 1 (inequality (5)) is feasible, then the equivalent (3) is feasible. Since $\beta_j \leq 1$, (7) is feasible. The feasibility of (7) stands at every sampling time after step 1, since the off-line computed $u(k) = Kx(k)$ can always be selected by the optimization algorithm to satisfy the inequality (7). Therefore, the feasibility carries over to all subsequent steps.

Theorem 1. If the robust stability constraints in Algorithm 1 is feasible at every sampling step, then the system (1) controlled by using Algorithm 1 is asymptotically stable.

Proof : By assumption, the state variable for the uncertain system (1) can be represented by

$$x(k+1) = \sum_{j=1}^H (\alpha_j A_j) \cdot x(k) + \sum_{j=1}^H (\alpha_j B_j) \cdot u(k)$$

$$\begin{aligned}
&= \sum_{j=1}^H \alpha_j (A_j x(k) + B_j u(k)) \\
&= \sum_{j=1}^H \alpha_j x_j^p(k+1|k) \quad (j=1, \dots, H)
\end{aligned}$$

By Schur Complement, the robust stability constraints (7) can be written as the following LMI.

$$\begin{bmatrix} \|x(k)\|_p^2 - \beta_j \|x(k)\|_Q^2 & x_j^p(k+1|k)^T P \\ P x_j^p(k+1|k) & P \end{bmatrix} \geq 0$$

Multiplying the above inequalities by α_j , and adding them by the index j from 1 to H , we have

$$\begin{bmatrix} (\sum_{j=1}^H \alpha_j) \cdot \|x(k)\|_p^2 & \sum_{j=1}^H \alpha_j x_j^p(k+1|k)^T P \\ \sum_{j=1}^H \alpha_j P x_j^p(k+1|k) & (\sum_{j=1}^H \alpha_j) P \\ - \left[\begin{array}{c|c} \sum_{j=1}^H (\alpha_j \beta_j) \cdot \|x(k)\|_Q^2 & 0 \\ 0 & 0 \end{array} \right] & 0 \end{bmatrix} \geq 0$$

By applying Schur Complement again on the above inequality, we have

$$\begin{aligned}
& \left(\sum_{j=1}^H \alpha_j x_j^p(k+1|k) \right)^T P \left(\sum_{j=1}^H \alpha_j x_j^p(k+1|k) \right) - \\
& \left(\sum_{j=1}^H \alpha_j \right) \cdot \|x(k)\|_p^2 + \left(\sum_{j=1}^H \alpha_j \right) \cdot \left(\sum_{j=1}^H \alpha_j \beta_j \right) \cdot \|x(k)\|_Q^2 \leq 0
\end{aligned}$$

Since $\sum_{j=1}^H \alpha_j \leq 1$, it leads to

$$\|x(k+1)\|_p^2 - \|x(k)\|_p^2 \leq - \left(\sum_{j=1}^H \alpha_j \right) \cdot \left(\sum_{j=1}^H \alpha_j \beta_j \right) \cdot \|x(k)\|_Q^2$$

Since Q is positive definite and $\alpha_j \geq 0, \beta_j > 0$, it follows that $\|x(k)\|_p \rightarrow 0$ as $k \rightarrow \infty$. Therefore, $x(k) \rightarrow 0$ as $k \rightarrow \infty$. \square

The features of Algorithm I can be summarized in the following remarks.

Remark 1. As stated in Lemma 1, if the LMI calculated in step 1 is feasible, then the on-line optimization (Step 2) is feasible throughout. The feasibility of the off-line LMI does not depend on the format and parameters of the objective function selected in Step 2. Therefore, the objective function and its parameters can be selected to accommodate performance requirements. Note that all features of [5] are now directly carried on to here where model uncertainty is included.

Remark 2. In this input unconstrained case, the off-line solved control law $u(k) = Kx(k)$ does not depend on the initial condition $x(0)$ either. Therefore, in addition to simply satisfying the feasibility of (5), the off-line computed baseline constant gain feedback control can even be chosen according to some performance criteria.

Remark 3. The baseline control is solved off-line. However, since the control action $u(k)$ is directly solved from the on-line optimization, most likely $u(k)$ will not be the same as $u(k) = Kx(k)$. In addition to the stability

constraint (7), the actual applied $u(k)$ also depends on the objective function format and its parameters. The parameters β_j , which can be viewed as tuning factors, also play a role in determining the contraction rate. A smaller β_j implies possible looser contraction (slower response) than what can be caused by applying the baseline control calculated in step 1, and vice versa.

IV. MAIN RESULT: INPUT CONSTRAINED SCMPC

We consider only input constrained robust SCMPC in this section. Given $x(0)$ as the initial condition, since the future state $x(k)$ will be contracted by the robust stability constraints, the feasible region for the constraints becomes an invariant set. Therefore, the calculated Lyapunov function value at the first step serves as the boundary for this invariant set. If $x(k)^T P x(k) \leq r$ (with r determined at the beginning) and the robust stability constraint (7) is always feasible for the future control $u(k)$, then we have $x(k+i)^T P x(k+i) \leq r$ (for all $k > 0$). This can be cast as an LMI initial condition constraint at the first step of the robust SCMPC calculation.

$$\begin{bmatrix} r^{-1} & r^{-1} x(k)^T \\ r^{-1} x(k) & S \end{bmatrix} \geq 0 \quad (8)$$

Assuming the control input is constrained by

$$|u(k+i)| \leq U_{\max}, \quad (9)$$

where $U_{\max} = [u_{1,\max}, \dots, u_{m,\max}]^T$. Then, it follows that ([7])

$$\begin{aligned}
\max_{i \geq 0} |u_j(k+i|k)|^2 &= \max_{i \geq 0} |(LS^{-1} x(k+i|k))_j|^2 \\
&\leq \|(LS^{-1/2})_j\|^2 \cdot \|S^{-1/2} x(k+i|k)\|^2 \\
&\leq \|(LS^{-1/2})_j\| \cdot r \leq (LS^{-1} L^T)_{jj} \cdot r
\end{aligned}$$

where $(\cdot)_{jj}$ means the diagonal elements of the corresponding matrix. Therefore, if

$$LS^{-1} L^T \leq r^{-1} \text{diag}\{u_{1,\max}, u_{2,\max}, \dots, u_{m,\max}\} \quad (10)$$

then (9) holds.

The inequality (10) can be cast as the following LMI

$$\begin{bmatrix} r^{-1} \cdot M & L \\ L^T & S \end{bmatrix} \geq 0 \quad (11)$$

where $M = \text{diag}\{u_{1,\max}, u_{2,\max}, \dots, u_{m,\max}\}$

Now the input constrained robust SCMPC can be stated as the following algorithm.

Algorithm II (input constrained case)

Step 1. Solve the LMI (5), (8), and (11) at the beginning step. If it is infeasible, stop here. The proposed scheme can not be applied directly. If it is feasible, go to the next step 2.

Step 2. Set $x_j^p(k|k) = x(k)$

Solve the following optimization at each sampling time k ($k = 1, 2, \dots$)

$$\min_{u^p(k|k), \dots, u^p(k+N-1|k)} \Phi(x^p(k+i|k), u^p(k+i-1|k))$$

S.t. (6), (7), and (9).

Step 3. Apply $u(k) = u^p(k|k)$, and repeat from Step 2.

Lemma 2. If Step 1 in Algorithm II (first step on-line calculation) is feasible, then the robust stability constraint (inequality (7)) in Step 2 is feasible for all $k = 0, 1, \dots$

We omit the proof since it is similar to the Lemma 1 proof. The key is that the first step on-line computed $u(k) = Kx(k)$ can always be selected by the optimization algorithm to satisfy the future feasibility.

Theorem 2. If the robust stability constraints in Algorithm II is feasible at every sampling step, then the system (1) controlled by using Algorithm II is asymptotically stable.

The proof is omitted since it is the same as the Theorem 1 proof. The key is that the feasibility of the robust stability constraints guarantees the closed-loop system stability.

Remark 4. Algorithm II preserves all features in Algorithm I, except that the admissible initial condition set is smaller than \mathfrak{R}^n due to control input constraints.

Remark 5. Algorithm II implies if the initial state starts from the feasible constraint region (invariant set), then the future feasibility and stability are all guaranteed. It does not resolve the issue when the initial state starts from outside this invariant set.

In the situation where the convex objective function is selected in the quadratic, l_1 -norm, or linear format, then there is an alternative to cast the entire Algorithm II as the linear objective minimization with LMI constraints. Efficient algorithms and commercial software packages are available to handle this type of optimization problem. Note that the on-line calculated robust stability constraint (7) in Step 2 of the previous algorithms can be converted to the equivalent LMI constraints as follows

$$\begin{bmatrix} x(k)^T(P-Q)x(k) & x(k)^T A_j^T P + u^p(k|k)^T B_j^T P \\ PA_j x(k) + PB_j u^p(k|k) & P \end{bmatrix} \geq 0$$

($j = 1, \dots, H$) (12)

If a finite horizon quadratic objective function (over nominal model (A_q, B_q)) is taken as an example, the objective function can be selected as

$$\Phi = \sum_{i=1}^N \{x_q^p(k+i|k)^T W x_q^p(k+i|k) + u^p(k+i-1|k)^T R u^p(k+i-1|k)\} \quad (13)$$

where W and R are positive definite weighting matrices on the predicted state and control input.

Minimizing the objective function (13) is equivalent to solving the following constrained optimization problem.

$$\min_{u^p(k+i|k), w_i, v_i} J = \sum_{i=1}^N w_i + \sum_{i=1}^N v_i$$

S.t. $x_q^p(k+i|k)^T W x_q^p(k+i|k) \leq w_i$
 $u^p(k+i-1|k)^T R u^p(k+i-1|k) \leq v_i$
 $w_i \geq 0$
 $v_i \geq 0 \quad (i=1, \dots, N)$

The above optimization is equivalent to the following linear objective function optimization with LMI constraint.

$$\min_{u^p(k+i|k), w_i, v_i} J = \sum_{i=1}^N w_i + \sum_{i=1}^N v_i$$

S.t. $\begin{bmatrix} w_i & \tilde{A}_q(i)^T \\ \tilde{A}_q(i) & W^{-1} \end{bmatrix} \geq 0$ (14)

$$\begin{bmatrix} v_i & u^p(k+i-1|k)^T \\ u^p(k+i-1|k) & R^{-1} \end{bmatrix} \geq 0 \quad (15)$$

$$w_i \geq 0 \quad (16)$$

$$v_i \geq 0 \quad (i = 1, \dots, N) \quad (17)$$

where

$$\tilde{A}_q(i) = A_q^i x(k) + A_q^{i-1} B_q u^p(k|k) + \dots + B_q u^p(k+i-1|k)$$

Therefore, we can formulate the on-line LMI version robust SCMPC as the following algorithm.

Algorithm III (control input constrained case)

Step 1. Solve LMIs (5), (8), and (11) in the initial step. If it is infeasible, stop here. The proposed scheme can not be applied directly. If it is feasible, go to the next step 2.

Step 2. Set $x_j^p(k|k) = x(k)$

Solve the following optimization at each sampling time k ($k = 1, 2, \dots$)

$$\min_{u^p(k+i|k), w_i, v_i} J = \sum_{i=1}^N w_i + \sum_{i=1}^N v_i$$

S.t. (9), (12), (14), (15), (16), and (17).

Step 3. Apply $u(k) = u^p(k|k)$, and repeat from Step 2.

Lemma 2 and Theorem 2 also stand for the Algorithm III, since Algorithm III is a special case of Algorithm II in terms of the format of the objective function.

We conclude this section by making a final remark. In this paper, the control u is directly solved from the on-line optimization to keep the original model predictive control flavor. In the algorithm development and simulation study (next section), finite horizon objective functions are always used. An infinite horizon robust MPC formulation can be

treated as a special case of this finite horizon MPC development.

V. SIMULATION EXAMPLES

Example 1. The example models are taken from [7]. We assume the following

$$A_1 = \begin{bmatrix} 0.9347 & 0.5194 \\ 0.3835 & 0.8310 \end{bmatrix}, A_2 = \begin{bmatrix} 0.0591 & 0.2641 \\ 1.7971 & 0.8717 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} -1.4462 \\ -0.7012 \end{bmatrix}$$

Let the control input be limited by 1. The initial condition is $x_0 = [2.25 \ 2.8]^T$. When Algorithm II is applied, we find the following solution at the first step calculation

$$P = \begin{bmatrix} 0.00424 & 0.00172 \\ 0.00172 & 0.00193 \end{bmatrix},$$

$$Q = 10^{-5} \times \begin{bmatrix} 0.14139 & -0.01412 \\ -0.01412 & 0.07023 \end{bmatrix}, K = [0.212 \ 0.176].$$

However, when the objective function is chosen in the infinite horizon quadratic form and the weighting matrices on state and control variables are selected as

$$W^{1/2} = \begin{bmatrix} 50000 & 5 \\ 5 & 10^7 \end{bmatrix} \text{ and } R = 1,$$

There is no feasible solution can be found by solving the LMIs for min-max optimization in [7].

Example 2. The example is to show the robust SCMPC for the input-unconstrained case (Algorithm I). We assume the uncertain model set (A_1, B_1) and (A_2, B_2) are the same as in Example 1. The actual process is assumed to be

$$(A, B) = \alpha_1(A_1, B_1) + \alpha_2(A_2, B_2) \text{ with } \alpha_1 = \alpha_2 = 0.5.$$

The model (A_1, B_1) is chosen as the nominal model for the robust SCMPC on-line optimization. The objective function is selected in the form of (13). The weighting on the state and control variables are $W = \text{diag}\{1,1\}$ and $R = 80$. The prediction (optimization) horizon $N=5$. The constant contraction parameters are $\beta_1 = \beta_2 = 0.1$. The initial condition is selected as $x_0 = [1 \ -1]^T$. Figures 1(a), 1(b), and 1(c) show the simulated state variable response and the control input when the robust SCMPC algorithm I and a static gain feedback control $u(k) = Kx(k)$ were applied separately. The constant control gain matrix K is solved from the first step of Algorithm I, and is solved as a most aggressive control as it should be when the control input is not constrained. However, the first-step computed static-gain feedback control is not necessarily the actual control. The actual applied control will depend on various factors, such as the form of the objective function, weighting inside the objective function, as well as the on-line robust stability constraint. The contraction parameter β in the stability constraint also plays an important role in terms of the control performance. It is noted that the tuning

rules identified in [5] also stand here. For instance, a longer prediction horizon N and a smaller contraction parameter β tend to produce a slower and smoother plant output response, and vice versa.

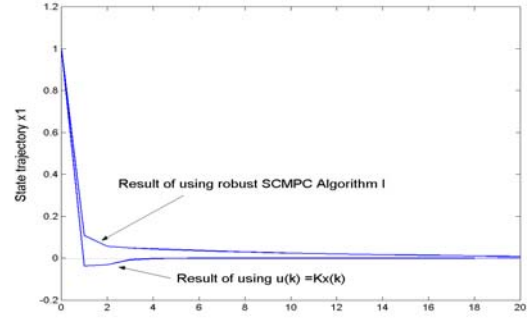


Fig.1(a) Input-unconstrained robust SCMPC (state x_1)

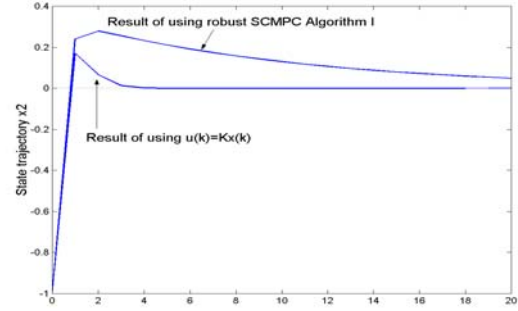


Fig.1(b) Input-unconstrained robust SCMPC (state x_2)

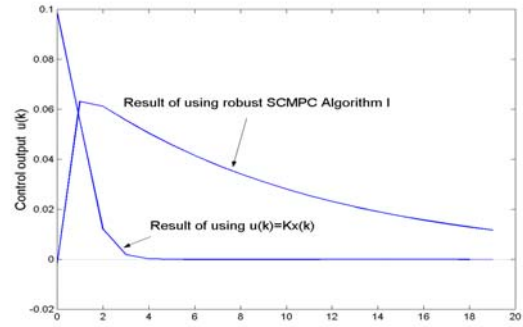


Fig.1(c) Input-unconstrained robust SCMPC (control u)

Example 3. This example shows the control input constrained case (Algorithm II). The actual process and nominal model are the same as in the previous example. The control input is assumed to be bounded by 1. The objective function is selected as (13). The predicted optimization horizon $N=5$. The weighting parameters are $W = \text{diag}\{1,1\}$ and $R = 0.1$. To force the possibly fastest response, contraction parameters are chosen as $\beta_1 = \beta_2 = 1$. The initial condition is selected as $x_0 = [2 \ 2]^T$. Figures 2(a), 2(b), and 2(c) show the simulated state variable response and the control input when the robust SCMPC algorithm II and a static gain feedback control $u(k) = Kx(k)$ were applied separately. Both control algorithms satisfy the control

constraint at the first step. However, the robust SCMPC algorithm is able to tighten the control and achieve a faster response as the state variables contract to smaller regions. This is not surprising since the constant gain feedback control solved at the first step naturally becomes a low-gain controller due to the initial condition and the limitation on the control input.

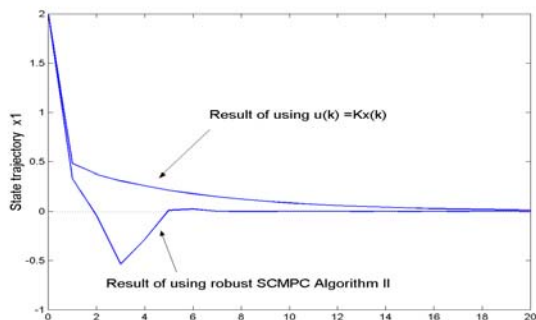


Fig.2(a) Input-constrained robust SCMPC (state x_1)

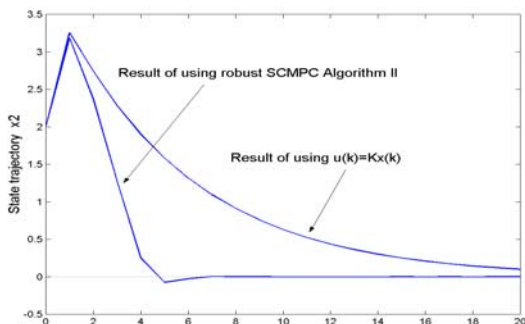


Fig.2(b) Input-constrained robust SCMPC (state x_2)

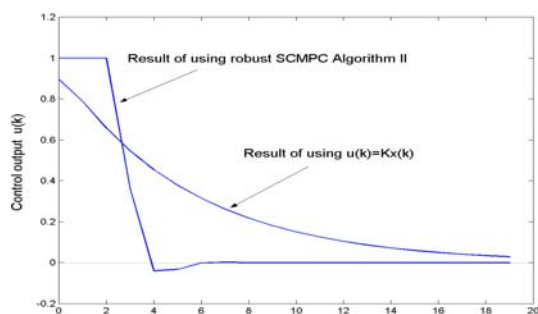


Fig.2(c) Input-constrained robust SCMPC (control u)

VI. CONCLUSIONS

In this paper we proposed a stability constrained model predictive control scheme that robustly stabilizes linear systems with polytopic model uncertainty descriptions. A Lyapunov equation type of stability constraint is explicitly imposed on the predicted state at each sample time. The objective function can be chosen as any type of convex function in any prediction horizon. The proposed approach guarantees closed-loop stability if the baseline robust stability constraint can be constructed through a feasible

linear matrix inequality solution. The control move solved from the on-line optimization further fine-tunes the actual implemented control action. Future work will address the situation when the initial state does not start from within the feasible region for the robust stability constraint.

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