Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

A Model Predictive Controller for Multirate Cascade Systems

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***Abstract*— Cascade control strategy is commonly employed in process control. Usually the inner loop is run at a higher sampling rate than the outer loop to achieve an effective cascade control. In this paper, we present a model predictive controller (MPC) that could handle this multirate cascade control strategy in a straightforward manner. There is only one controller to be designed, thus reducing the complexity of tuning the cascade control system. An illustrative example is presented to demonstrate the effectiveness of the proposed control design method.**

I. INTRODUCTION

As showed in Figure 1, a typical cascade control system consists of an inner loop and an outer loop. Cascade controllers are useful when the outer loop plant contains right-half plane zeros or a time delay or when the inner loop has significant disturbance and uncertainty[7].

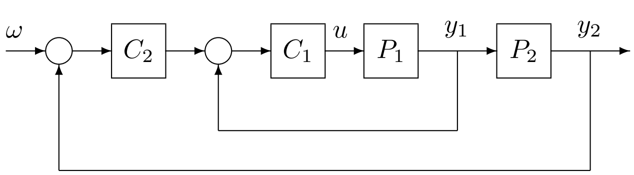

Fig. 1. A typical cascade system

The conventional approach to designing a cascade control system is that an appropriate inner loop controller is first determined to match the desired dynamics of the inner control loop. With the inner loop closed, the outer loop controller is next designed. Usually the inner loop has a higher bandwidth than the outer loop. In practical implementation, the inner loop usually operates at a faster sampling rate than the outer loop.

In cascade control, there is one manipulated variable and two or more measurement. Thus, in this paper, we proposed to take an alternative approach and design a predictive controller for the cascade control problem by sampling the plant outputs, y_1 and y_2 at different rates, and generates the control signal u at the faster rate. In other words, we treat the plant as a non-square multivariable plant with possibly multirate sampling and control requirements and design the required controller accordingly following the Model Predictive Control (MPC) approach. Using this approach, only one controller needs to be designed. In addition, different inner- and outer-loop sampling rate and loop interactions can be accounted for in the design process.

For single-rate cascade control systems, in which the input updating and the output sampling rates are the same, efforts have been made to get more efficient and convenient control strategies. In [10], an IMC-based design for cascade control using the state space technique was presented. A cascade predictive structure with an adaptive predictive controller for the inner loop and a PID controller for the outer loop was implemented in [13] for a distributed collector solar filed. To control an open-loop unstable Continuous Stirred Tank Reactor System, Nagrath *et al.*[8] developed a state estimation-based model predictive control approach which employs a single MPC strategy that incorporates both loops' measurements and manipulates the system input. Predictive cascade controllers for the control of the position, velocity and rotor flux of an induction machine were also reported in [2], [6].

The rest of this paper is organized as follows. In Section 2, we review a state space formulation of model predictive control for multirate systems. Section 3 shows the main results of this paper, that is, a predictive controller for multirate cascade systems. Section 4 gives a numerical example on a multirate cascade system which verifies the proposed design. Finally, a brief concluding remark is made in Section 5.

II. STATE SPACE FORMULATION OF A MULTIRATE MODEL PREDICTIVE CONTROLLER

One popular version of Model Predictive Control (MPC) is the Generalized Predictive Control (GPC). Some recent interests in extending MPC to multirate situation can be found in [1], [3], [4], [11], [12]. In particular, [12] used the lifting technique to develop a GPC scheme for non-uniformly sampled multirate systems. In this section, we recall the state space MPC formulation approach for multirate systems.

Unlike a conventional single-rate system, a multirate system has different rates for measurement sampling and control updating. Generally the sampling and updating pattern are periodic over a larger period T known as the frame period. For example, if the measurement sampling period is nh, and the control updating period is mh, where m and n are integers and h is the base period, then the frame period is $T = gh$, where g is the least common multiple of m and n.

Now let us consider a controllable and observable, single-input single-output linear multirate system, supposing its base period state space model in discrete-time form is represented by

$$\begin{cases} x_{kT+h} &= Ax_{kT} + B\Delta u_{kT} \\ y_{kT} &= Cx_{kT}, \end{cases} \tag{1}$$

then the basic idea of the state space MPC for multirate systems can be described as follows:

Firstly, a prediction model is constructed based on the base period model. For example, at the time instant $kT + imh(i = 0, \cdots, n)$, the prediction model is

$$\hat{Y} = \Phi x(kT + imh) + \Psi \Delta U, \tag{2}$$

where

$$\hat{Y} = \begin{bmatrix} \hat{y}(kT+imh+h) \\ \hat{y}(kT+imh+2h) \\ \vdots \\ \hat{y}(kT+imh+N_p h) \end{bmatrix}, \quad \Phi = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix},$$

$$\Delta U = \begin{bmatrix} \Delta u(kT+imh) \\ \Delta u(kT+imh+h) \\ \vdots \\ \Delta u(kT+imh+(N_u-1)h) \end{bmatrix}, \tag{3}$$

$$\Psi = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_u}B \end{bmatrix}. \tag{4}$$

Then the state feedback law can be obtained by minimizing the cost function

$$\begin{aligned} J &= \sum_{j=1}^{N_p} \|\omega(kT+imh+jh) - \hat{y}(kT+imh+jh)\|^2 \\ &+ \lambda \sum_{j=1}^{N_u} \|\Delta u(kT+imh+(j-1)h)\|^2, \end{aligned} \tag{5}$$

subject to the following constraints:

$$\Delta u(kT+imh+jh) = 0, \quad j \neq 0, m, 2m, \cdots. \tag{6}$$

The parameter N_p is known as the upper prediction horizon, and for simplicity, the lower prediction horizon is chosen to be one in the above formulations. The prediction horizons define the interval over which the tracking error is minimized (ω is the setpoint for y). The control horizon, N_u, defines the degree of freedom available for the minimization. The control weighting, λ, can be used to penalize excessive control activity, but in practice, it is more commonly used to ensure a numerically well-conditioned algorithm.

Substituting (2) into (5) and solving the minimization problem, we have the following control law:

$$\Delta U = K_1 \omega(kT+imh) + K_2 x(kT+imh), \tag{7}$$

where

$$\begin{aligned} K_1 &= (\Psi^T\Psi + \lambda I)^{-1}\Psi^T, \\ K_2 &= -(\Psi^T\Psi + \lambda I)^{-1}\Psi^T\Phi. \end{aligned} \tag{8}$$

Using the so-called receding horizon strategy, only the first element of the control vector obtained from Equation (7), is applied to the plant. The control calculation is then repeated at the next control updating instance. Since the prediction model is the same at every control updating instance, it can be seen that the controller gains K_1 and K_2 are time invariant.

If the state variables are not directly measurable, and in particular, in multirate situation, because control and measurement occur at different rate, the state variable may not be available at certain $kT+imh$. In this case the control law is modified to

$$\Delta U = K_1 \omega(kT+imh) + K_2 \hat{x}(kT+imh|kT+jnh), \tag{9}$$

where $\hat{x}(kT+imh|kT+jnh)$ denotes the estimation of the state at $kT+imh$ based on the most up-to-date input and output measurements. One possible approach to designing a state estimator for the multirate system is via the so-called receding or moving horizon state estimator[5], [9].

III. A PREDICTIVE CONTROLLER FOR MULTIRATE CASCADE SYSTEMS

In this section, we present our main results, that is, developing a predictive controller for a multirate cascade control system. Such a predictive controller can control not only the inner loop but also the outer loop at the same time. The basic idea for this approach arises from the fact that a multirate two-loop cascade control system can also be treated as an equivalent multirate single-input two-output control system. We hence can design a predictive controller for this single-input two-output system with the technique introduced in Section 2. As it is showed in Figure 2, this predictive controller is simpler and more compact than the conventional design with two predictive controllers for the cascade control and thus it is more convenient to design and tune.

A. An Equivalent Multivariable System

We first give a multirate multivariable system which is equivalent to the multirate two-loop cascade system.

Consider a cascade system with the following inner loop system P_1 and outer loop system P_2

For this multirate double-loop cascade system, we assume that both P_1 and P_2 are controllable and observable. The continuous-time plant can be represented by

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} P_1 \\ P_1 P_2 \end{bmatrix} u(s) \tag{10}$$

Without loss of any generality, we assume that its updating and sampling rates are periodic over a frame period T. Let us suppose that the updating period of u is mh, sampling period of y_1 is n_1h and n_2h is the sampling period of y_2. Here m, n_1 and n_2 are integers and h is the base period. Furthermore, we assume that T is the smallest common multiple of mn_1h and mn_2h. The discrete-time base period model is

$$\begin{cases} \tilde{x}_{k+1} &= \tilde{A}\tilde{x}_k + \tilde{B}u_k \\ y_k &= \tilde{C}\tilde{x}_k \end{cases} \quad (11)$$

Next add an integrator into the model

$$\begin{bmatrix} \Delta\tilde{x}_{k+1} \\ y_k \end{bmatrix} = \begin{bmatrix} \tilde{A} & O \\ \tilde{C} & I \end{bmatrix} \begin{bmatrix} \Delta\tilde{x}_k \\ y_{k-1} \end{bmatrix} + \begin{bmatrix} \tilde{B} \\ O \end{bmatrix} \Delta u_k$$

$$y_k = \begin{bmatrix} \tilde{C} & I \end{bmatrix} \begin{bmatrix} \Delta\tilde{x}_k \\ y_{k-1} \end{bmatrix}$$

Now we can formulate the cascade control system as a multirate controller design for a single-input two-output system using the following compact form

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{cases} \quad (12)$$

We thus transfer the original cascade system to an equivalent multivariable system based on which a predictive controller will be derived.

B. Design of the Predictive Controller

For the multirate single-input two-output system (12),we can find a predictive controller through the technique described in Section 2.

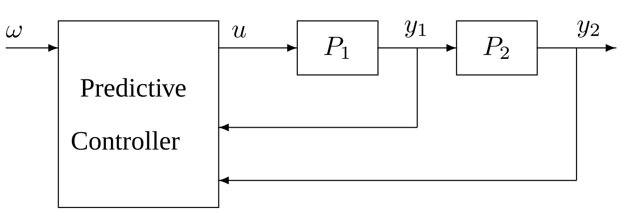

Fig. 2. A controller for a two-loop cascade system

For the whole system, there is only one setpoint ω which is for the output y_2. Hence we can write the cost function as follows,

$$\begin{aligned} J &= \sum_{j=1}^{N_p} \|\omega(kT+imh+jh) - \hat{y}_2(kT+imh+jh)\|^2 \\ &+ \lambda \sum_{j=1}^{N_u} \|\Delta u(kT+imh+(j-1)h)\|^2, \end{aligned} \quad (13)$$

subjecting to the following constraints:

$$\Delta u(kT+imh+jh) = 0, \quad j \neq 0, m, 2m, \cdots. \quad (14)$$

Now we can compute the predictive control law as follows:

$$\hat{Y}_2 = \bar{\Phi}x(kT+imh) + \bar{\Psi}\Delta U, \quad (15)$$

where

$$\hat{Y}_2 = \begin{bmatrix} \hat{y}_2(kT+imh+h) \\ \hat{y}_2(kT+imh+2h) \\ \vdots \\ \hat{y}_2(kT+imh+N_ph) \end{bmatrix}, \quad \bar{\Phi} = \begin{bmatrix} \Gamma A \\ \Gamma A^2 \\ \vdots \\ \Gamma A^{N_p} \end{bmatrix}, \quad (16)$$

$$\Delta U = \begin{bmatrix} \Delta u(kT+imh) \\ \Delta u(kT+imh+h) \\ \vdots \\ \Delta u(kT+imh+(N_u-1)h) \end{bmatrix}, \quad (17)$$

$$\bar{\Psi} = \begin{bmatrix} \Gamma B & 0 & \cdots & 0 \\ \Gamma AB & \Gamma B & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ \Gamma A^{N_p-1}B & \Gamma A^{N_p-1}B & \cdots & \Gamma A^{N_p-N_u}B \end{bmatrix}, \quad (18)$$

and

$$\Gamma = \text{the second row of } C$$

Then from Section 2, we have the following predictive control law,

$$\Delta U = K_1\omega(kT+imh) + K_2x(kT+imh) \quad (19)$$

with

$$\begin{aligned} K_1 &= (\bar{\Psi}^T\bar{\Psi} + \lambda I)^{-1}\bar{\Psi}^T, \\ K_2 &= -(\bar{\Psi}^T\bar{\Psi} + \lambda I)^{-1}\bar{\Psi}^T\bar{\Phi}. \end{aligned} \quad (20)$$

When the state variables x are not available at time $kT+imh$, the control law in (19) is replaced by

$$\Delta U = K_1\omega(kT+imh) + K_2\hat{x}(kT+imh \mid kT+jnh), \quad (21)$$

where $\hat{x}(kT+imh \mid kT+jnh)$ denotes estimation of the state variables at time $kT+imh$ based on the most up-to-date available state variables. The process of state estimation will be described next.

C. State Estimation

In practice, we usually can only measure the outputs and the internal state variables are not available. A state estimator is thus needed to reconstruct the state variables according to the available input and output information. This is feasible if the system is observable.

The design of state estimators can be implemented in various frameworks, such as the Kalman filter, which gives the optimal state estimate in the mean square error sense but need a noise model which may be difficult to obtain. This paper employs the receding or moving horizon state estimator to estimate the states that are not available in the prediction.

The basic strategy of moving horizon estimation (MHE) is to estimate the state vector based on a finite number of past measurement samples. The oldest measurement sample is discarded when a new sample becomes available. The memory length of the state estimator is thus fixed.

For simplicity, we adopt here the scheme proposed by [4] in which the design of the state estimator is treated in the same framework as that in the controller design, that is, by minimizing the following cost function,

$$J_e = \sum_{r=0}^{N_e-1} \mu(t_j - rh) \|y(t_j - rh) - C\hat{x}(t_j - rh)\|^2, \quad (22)$$

with respect to the following constraints,

$$\hat{x}(t_j - rh + h) = A\hat{x}(t_j - rh) + Bv(t_j - rh)\Delta u(t_j - rh), \quad (23)$$

where N_e, the estimation horizon, is the main tuning parameter for the state estimator. It determines the number of the past output measurement employed by the state estimator. The controller makes control signal changes at every mh intervals, that is , at time $0, mh, 2mh, \ldots$. But the plant output is only available at time $0, n_1h, n_2h, 2n_1h, 2n_2h, \ldots$. So it is easy to see that

$$v(t_j - rh) = \begin{cases} 1 & \text{if } t_j - rh = 0, mh, 2mh, \ldots \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

$$\mu(t_j - rh) = \begin{cases} 1 & \text{if } t_j - rh = 0, n_1h, n_2h, 2n_1h, 2n_2h, \ldots \\ 0 & \text{otherwise} \end{cases} \quad (25)$$

Minimization of Equation (22) with respect to $\hat{x}(t_j - N_eh + h)$ gives the following optimum state estimates,

$$\hat{x}(t_j - N_eh + h) = (MQM)^{-1}M^TQ(Y - P\Upsilon U) \quad (26)$$

where

$$M = \begin{bmatrix} CA^{N_e-1} \\ CA^{N_e-2} \\ \vdots \\ C \end{bmatrix},$$

$$P = \begin{bmatrix} CB & CAB & \cdots & \cdots & CA^{N_e-2}B \\ 0 & CB & \cdots & \cdots & CA^{N_e-3}B \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & CB \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \quad (27)$$

$$U = \begin{bmatrix} \Delta u(t_j - h) & \Delta u(t_j - 2h) & \cdots & \Delta u(t_j - N_eh + \end{bmatrix}$$
$$Y = \begin{bmatrix} y(t_j) & y(t_j - h) & \cdots & y(t_j - N_eh + h) \end{bmatrix}$$

and Q is a diagonal matrix which contains $\mu(t_j - rh)$, Υ is a diagonal matrix which contains $v(t_j - rh)$. The estimated state can be written as

$$\hat{x}(t_j) = E_yY + E_uU \quad (29)$$

where

$$E_y = A^{N_e-1}(M^TQM)^{-1}M^TQ$$
$$E_u = \begin{bmatrix} B & AB & \cdots & A^{N_e-2}B \end{bmatrix} \Upsilon - E_yP\Upsilon$$

IV. A NUMERICAL EXAMPLE

In this section, we verify the efficiency of the proposed MPC design in controlling the following multirate cascade system. The plant P_1 and P_2 has the following transfer function representations

$$P_1(s) = \frac{1}{10s+1} \qquad P_2(s) = \frac{e^{-5s}}{20s+1}$$

We choose P_2 to have a time delay because cascade controller is only useful if P_2 has RHP zeros or a time delay. Assume the sampling period of outer-loop is $T = 1s$, while the sampling period of inner-loop is T/m, where m is an integer. Using the techniques presented in Section 3 with tuning parameters chosen as: $N_p = m * 10$ (ensuring the same length of prediction horizon under different sampling patterns), $N_u = 2$ (choosing the same number of degree-of-freedom) and $\lambda = 0.01$, we can design the MPC controller for this multirate cascade system. For the estimator design, we choose the estimation horizon to be $N_e = 5$. For various values of $m = 2, 4$ and 8, the closed-loop response of the cascade control system is illustrated in Figure 3. The step signal is injected at $t = 1s$, and the inner-loop disturbance is added at $t = 100s$. It can be seen that the controller achieve zero steady-state error and give reasonable setpoint and disturbance responses. With the MPC framework, constraints on the intermediate measurement y_1 and the control signal u could be added.

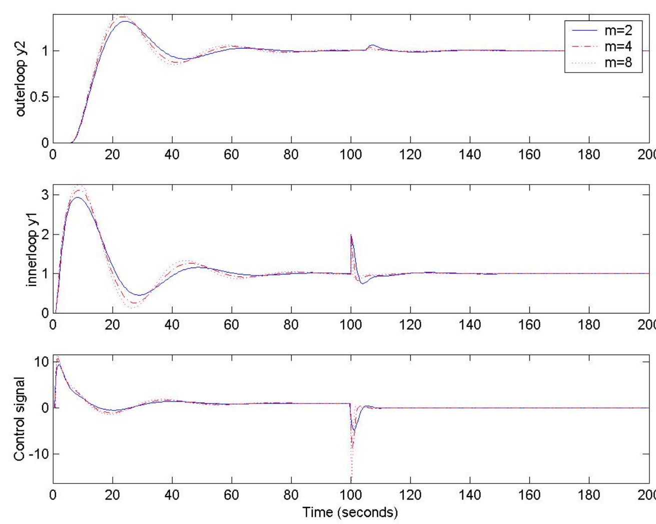

Fig. 3. Step and disturbance responses of the cascade control system

V. CONCLUSIONS

We present in this paper a MPC design for multirate cascade systems. The proposed predictive controller is simple to design and tune. Properties of the proposed control design, such as the effect of sampling ratio m and the estimation horizon N_e are currently under investigation.

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