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An LMI based model predictive control scheme with guaranteed $\mathcal{H}_\infty$ performance and its application to active suspension

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***Abstract*— In the framework of LMI optimization, we present a robust MPC scheme that is guaranteed to achieve closed-loop $\mathcal{H}_\infty$ performance for linear systems with control and output constraints. The main ingredient is the introduction of a dissipation constraint combined with on-line minimizing the $\mathcal{H}_\infty$ performance level. Simulation results for a realistic active vehicle suspension show that the proposed scheme has the capability of automatically relaxing the performance level in order to obey hard time-domain constraints, while enhancing it when sufficient 'reserves' in the dissipation constraint have been accumulated so as to improve performance.**

***Index Terms*— Model predictive control, $\mathcal{H}_\infty$ performance, hard constraints, dissipation theory, LMI optimization**

I. INTRODUCTION

With the rapid development of computing, model predictive control, also refer to as moving (or receding) horizon control, has become an attractive feedback strategy for controlling constrained systems, not only in traditional fields such as refining and petrochemicals where slow dynamics are dominant [1], but also in aerospace and defense (see [2], [3], [4] for some new reports). Over the last few years, also academic research of MPC has achieved significant progress. By introducing the so-called stability constraints (equality and inequality terminal constraints or contractive constraints) and appropriately computing the terminal penalty, the nominal stability issue of MPC is in general well addressed; for a complete survey on this issue we refer for example to [5], [6], [7].

For robust MPC, a general concept is to replace the minimization problem by a constrained minimax problem, where the maximization is performed over a set of uncertainties and/or disturbances. The first minimax formulation of MPC can be traced back to [8], where coefficients of an SISO FIR model were assumed to be uncertain and to vary within given bounds. A crucial issue of minimax MPC formulations is the difficulty concerning their implementation. In order to render them tractable, the original formulation used the l_∞-norm of the error signal as an objective, whereas subsequent approaches provide generalizations to the l_1-norm [9]. More recently, minimax formulations with quadratic criteria are addressed in the powerful framework of LMI (linear matrix inequality) optimization at the price of involving conservatism (e.g. [10], [11], [12], [13]) or by restricting uncertainties that only appear in the system's static gain (or the input matrix) (e.g. [14], [15]). Another important issue of minimax MPC formulations is to guarantee stability. It is shown in [16] that no stability guarantee can be given in the original minimax MPC formulation, and a remedy is suggested as well. For stable systems with uncertain input matrix, the use of an infinite prediction horizon guarantees robust stability [17], while the predicted control is set to zero for all times beyond the (finite) control horizon. For integral control, additional constraints are required to force the integrating modes to be zero at the end of the finite control horizon [17]. LMI based robust MPC formulations guarantee robust stability for both stable and unstable systems by relying on an infinite (prediction and control) horizon. As in the nominal case, the use of an infinite horizon plays a crucial role to achieve robust stability. The crucial difference of various robust MPC formulations lies in the way how to construct the predicted control such that the resulting optimization problem is tractable; either the use of a constant feedback law over the full horizon [18] or a 1-step (N-steps) prediction control concatenated by a constant feedback law [11], [19] have been proposed. A further difference is found in whether one uses open-loop or closed-loop prediction. Classical MPC schemes rely on open-loop prediction. However, open-loop prediction implies that the uncertain system is predictively controlled without feedback information. Through maximization, the effect of uncertainties and disturbances are overestimated which might easily result in infeasibility of the corresponding minimax optimization problems. The need for a feedback prediction paradigm in robust MPC schemes is clarified in [20], [21], [22]. In the context of receding horizon $\mathcal{H}_\infty$ control, robust MPC is investigated for two different purposes. One aims at a solution of time-varying or nonlinear $\mathcal{H}_\infty$ control problems (e.g. [23], [24], [25], [26], [27]), whereas the other is related to incorporating well-known robustness guarantees through $\mathcal{H}_\infty$ constraints into MPC schemes (e.g. [28]). For more detailed survey and discussion on robust MPC we refer e.g. to [7], [29].

Still, it is rarely addressed how to guarantee performance and robust performance in MPC, although some minimax formulations include additive bounded disturbances (e.g. [22], [30]). Most receding horizon $\mathcal{H}_\infty$ control schemes concentrate on possible solutions to nonlinear $\mathcal{H}_\infty$ control in which no hard time-domain constraints are taken

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into account [23], [24], [25], [26].

Preliminary attempts to overcome these deficiencies are presented in [31], in which we discuss an l_2-gain attenuation scheme for linear systems with actuator saturation that involves less conservatism than previous suggestions. Building on the work of [32], we include an additional dissipation constraint into the on-line optimization problem to guarantee closed-loop system dissipativity. This paper provides a continuation with the purpose of arriving at a robust MPC scheme that is tractable and guarantees nominal and robust $\mathcal{H}_\infty$ performance.

The remaining paper is structured as follows: Section II gives the setup of the proposed robust MPC scheme in the framework of LMI optimization. Guaranteed $\mathcal{H}_\infty$ performance for the closed-loop system and the dissipation constraint are discussed in Section III. Based on a 2 DOF quarter-car model, we apply the proposed MPC scheme to provide a solution of an active suspension control problem. Simulation and comparison results are given in Section IV. In conclusive remarks, we highlight the potentials for guaranteeing robust $\mathcal{H}_\infty$ performance.

II. An LMI based MPC scheme

Consider systems to be controlled that are described as

$$x(k+1) = Ax(k) + B_1 w(k) + B_u u(k) \tag{1a}$$
$$z_1(k) = C_1 x(k) + D_1 w(k) + D_{1u} u(k) \tag{1b}$$
$$z_2(k) = C_2 x(k) + D_{2u} u(k) \tag{1c}$$

subject to the following time-domain constraints:

$$|z_{2i}(k)| \le z_{2i,max} \quad \forall k \ge 0,\ i = 1, 2, \ldots, p_2. \tag{2}$$

Here $x \in \mathbb{R}^n$ denotes the vector of states, $w \in \mathbb{R}^{m_1}$ the vector of external inputs, $u \in \mathbb{R}^{m_2}$ the vector of control inputs, $z_1 \in \mathbb{R}^{p_1}$ the vector of performance outputs and $z_2 \in \mathbb{R}^{p_2}$ the vector of constrained outputs. Note that control constraints can be described in (1c) and (2) with $C_2 = 0$ and $C_{2u} = I$. We assume that (A, B_u) is stabilizable and (C_1, A) is detectable.

The basis of model predictive control is the on-line solution of an optimization problem with control and output constraints, updated by the actual measurement at each sampling time [6], [7]. The obtained control action is injected into the system until the next sampling time.

In this paper, we suggest to repeatedly solve a constrained $\mathcal{H}_\infty$ control problem on-line. At time $k \ge 0$ with the actual state $x(k)$, the optimization problem is formulated as follows:

$$\min_{\gamma, Q=Q^T, Y, Z=Z^T} \gamma \tag{3}$$

subject to the following matrix inequalities

$$\begin{pmatrix} Q & * & * & * \\ 0 & \gamma I & * & * \\ AQ + B_u Y & B_1 & Q & * \\ C_1 Q + D_{1u} Y & D_1 & 0 & \gamma I \end{pmatrix} > 0, \tag{4a}$$

$$\begin{pmatrix} r & x(k)^T \\ x(k) & Q \end{pmatrix} \ge 0, \tag{4b}$$

$$\begin{pmatrix} \frac{1}{r} Z & C_2 Q + D_{2u} Y \\ (C_2 Q + D_{2u} Y)^T & Q \end{pmatrix} \ge 0, \tag{4c}$$
$$Z_{jj} \le z_{2j,max}^2,$$

$$\begin{pmatrix} p_0 - p_{k-1} + x(k)^T P_{k-1} x(k) & x(k)^T \\ x(k) & Q \end{pmatrix} \ge 0, \tag{4d}$$

where $p_0 = x(0)^T P_0 x(0)$ and p_k is recursively updated to

$$p_k := p_{k-1} - \left[x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k) \right]. \tag{5}$$

Assume that the above optimization problem admits a (close to optimal) solution denoted as $(\gamma_k, Q_k, Y_k, Z_k)$. Then, the feedback control is defined as follows:

$$u(i) = K_k x(i) \quad \forall i \ge k \tag{6}$$

with $K_k = Y_k P_k$ and $P_k = Q_k^{-1}$. According to the principle of MPC, only the first control action (*i.e.*, for $i = k$) is injected into the system until the next sampling time. Updated by the actual closed-loop state, the above optimization problem is solved again. We stress that the actual state $x(k)$ does indeed appear in the matrix inequalities (4b) and (4d). Hence, any solution of the optimization problem and, in turn, the actual applied feedback gain K_k depend on the actual state $x(k)$. Throughout this paper we drop this dependence for notational simplicity.

The implementation of this MPC scheme is possible since P_{k-1} and p_{k-1} have been determined at the previous time instant $k-1$ and are held fixed, which implies that (4d) is affine in Q. Moreover, (4a) also defines an LMI constraint in γ, Q and Y. Finally, for some fixed r, (4c) is an LMI in Q, Y and Z. Hence, for a fixed r, (3) is an LMI optimization problem that can numerically solved on-line. Note that (4d) can be dropped for time $k = 0$ as it will become clear in the following discussion.

It is easy to show that (4a) arises from a standard LMI based solution of the unconstrained $\mathcal{H}_\infty$ control problem. With a Lyapunov-type function $V(x) := x^T P x$, $P = Q^{-1}$, the feasibility of (4a) leads to the dissipation inequality

$$V(x(i+1)) + \|z_1(i)\|^2 - \gamma^2 \|w(i)\|^2 \le V(x(i)) \tag{7}$$

for any $i \ge k$. By taking the Schur complement, (4b) is equivalent to $r - x(k)^T P x(k) \ge 0$. Hence, LMI (4b) forces the actual state $x(k)$ to be contained in the ellipsoid

$$\mathcal{E}(P, r) := \{x \in \mathbb{R}^n : V(x) \le r\}. \tag{8}$$

Remark 1: If $x(0) \in \mathcal{E}(P, r)$ and if the system were controlled with the feedback law (6) for all future times, then one can easily show with (7) that the energy of the performance outputs would be bounded as $\sum_{i=0}^{\infty}\|z(i)\|^2 \leq r + \gamma^2\alpha$, if the disturbance energy were bounded as $\sum_{i=0}^{\infty}\|w(i)\|^2 \leq \alpha$. This can serve as an indication for choosing the tuning parameter r. However, r should not be chosen too small in order to avoid that the LMI (4b) is infeasible.

The dissipation constraint (4d), firstly appearing in [32], is introduced in order to guarantee $\mathcal{H}_\infty$ performance for the closed-loop moving horizon system, as discussed in Section III. By minimizing the $\mathcal{H}_\infty$ performance index γ, the presented scheme is able to shape solutions in terms of the actual state so as to manage automatically the trade-off between requiring high performance and respecting hard constraints.

The properties of the suggested MPC scheme are as follows:

- a constrained $\mathcal{H}_\infty$ control problem is solved on-line, updated with the actual state, unlike standard MPC schemes where an open-loop optimal control problem is involved, and unlike [10] that solves a constrained (robust) LQR problem;
- similarly to [10], a state feedback gain (and not an open-loop control sequence) is computed by solving the optimization problem;
- the objective functional to be minimized is the $\mathcal{H}_\infty$ norm from the external input w to the performance output z_1, with the purpose to manage automatically the trade-off between requiring high performance and respecting hard constraints;
- an additional dissipation constraint is introduced to guarantee dissipativity and hence $\mathcal{H}_\infty$ performance for the closed-loop moving horizon system.

III. Closed-loop $\mathcal{H}_\infty$ performance

In this section, we show that with the help of the additional dissipation constraint (4d) the closed-loop moving horizon system is dissipative, along the line of [32]. Taking the Schur complements in (4d) implies

$$p_0 - p_{k-1} + x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k) \geq 0\,. \tag{9}$$

This inequality can be combined with the recursion (5) in order to conclude that the dissipation constraint enforces

$$\sum_{i=1}^{k}[x(i)^T P_{i-1} x(i) - x(i)^T P_i x(i)] \geq 0. \tag{10}$$

Assume that at each time $k \geq 0$, the LMI optimization problem (3) admits an (almost) optimal solution $(\gamma_k, Q_k, Y_k, Z_k)$ and define $K_k = Y_k Q_k^{-1}$ as well as $P_k = Q_k^{-1}$. Then the feasibility of (4a) implies that the dissipation inequality (7) is satisfied with $\gamma = \gamma_k$ and $V(x) = x^T P_k x$ for each $k \geq 0$. We stress that this does not imply dissipativity of the closed-loop moving horizon system. Indeed, assume that, up to time $l > 0$, the feedback control sequence computed by $u(k) = K_k x(k)$ for $k = 0, 1, \ldots, l$ has been used to control the system. Exploiting (7) for $k = 0, 1, \ldots, l$ leads to

$$\begin{aligned}
\|z_1(0)\|^2 - \gamma_0^2\|w(0)\|^2 &\leq x(0)^T P_0 x(0) - x(1)^T P_0 x(1)\\
\|z_1(1)\|^2 - \gamma_1^2\|w(1)\|^2 &\leq x(1)^T P_1 x(1) - x(2)^T P_1 x(2)\\
&\vdots\\
\|z_1(l)\|^2 - \gamma_l^2\|w(l)\|^2 &\leq x(l)^T P_l x(l) - x(l{+}1)^T P_l x(l{+}1)
\end{aligned}$$

and hence

$$\begin{aligned}
\sum_{k=0}^{l}\|z_1(k)\|^2 - \gamma_k^2\|w(k)\|^2 &\leq x(0)^T P_0 x(0) -\\
&- \sum_{k=1}^{l}\left[x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k))\right] -\\
&- x(l+1)^T P_l x(l+1).
\end{aligned} \tag{11}$$

By (10), we infer

$$\begin{aligned}
\sum_{k=0}^{l}\|z_1(k)\|^2 - \max\{\gamma_0, \gamma_1, \ldots, \gamma_l\}^2\|w(k)\|^2 &\leq\\
\leq x(0)^T P_0 x(0) - x(l+1)^T P_l x(l+1),&
\end{aligned} \tag{12}$$

which implies dissipation with level $\max\{\gamma_0, \gamma_1, \ldots, \gamma_l\}$. We are now in the position to state the following result.

Theorem 1: Suppose that

- (A, B_u) is stabilizable and (C_1, A) is detectable;
- at each time k, there exists an r such that the optimization problem (3) with the actual state $x(k)$ as initial condition admits an (almost) optimal solution $(\gamma_k, Q_k, Y_k, Z_k)$.

If controlling the system with $u(k) = K_k x(k)$ with the feedback gain $K_k = Y_k Q_k^{-1}$, the closed-loop moving horizon system has the following properties:

(i) for vanishing disturbances it is asymptotically stable;
(ii) the hard constraints (2) are respected;
(iii) the dissipation inequality

$$\sum_{i=0}^{k}\|z_1(i)\|^2 - \gamma^2\|w(i)\|^2 \leq x(0)^T P_0 x(0) \tag{13}$$

is guaranteed for $k \geq 0$, with $\gamma := \max\{\gamma_0, \ldots, \gamma_k\}$ (which is finite due to feasibility);
(iv) the discrete-time l_2-gain from w to z_1 is guaranteed to be not larger than γ.

Proof: Define $\gamma := \max\{\gamma_0, \gamma_1, \ldots, \gamma_k\}$ which is finite due to the assumption of feasibility. Then (12) implies property (iii) due to $P > 0$ and hence property (iv) for $x(0) = 0$. Moreover, the stability property (i) is proved by showing that $\sum_{i=0}^{\infty}\|z_1(i)\|^2$ is bounded and exploiting detectability of (C_1, A). At each time k, the feasibility of (4b) forces the actual state $x(k)$ to be contained in the ellipsoid $\mathcal{E}(P_k, r)$. Hence, the time-domain constraints (2) are respected since

$$|z_{2i}(k)|^2 \le \max_{x\in\mathcal{E}(P_k,r)} \left|\left(C_2 + D_{2u}YQ^{-1}\right)_i x\right|^2 \le \\ \le r \left\|\left((C_2Q + D_{2u}Y)Q^{-\frac{1}{2}}\right)_i\right\|_2^2 \le z_{2i,max}^2, \quad (14)$$

follows from (4c). □

Remark 2: In the above, we prove the property (ii) without restricting the disturbance energy. On the other hand, if the disturbances satisfy $\|w(k)\|^2 \le \frac{r_k - x(k)^T P_k x(k)}{\gamma_k}$, it can be shown that the feasibility of the optimization problem (3) at time $k = 0$ implies its feasibility at any $k > 0$.

Discussion of dissipation constraint

Let us now highlight the dissipation constraint (4d). It follows from (10) that for $k = 1$, the dissipation constraint enforces $x(1)^T P_0 x(1) - x(1)^T P_1 x(1) \ge 0$. Hence, switching the control from the feedback gain K_0 with P_0 to a new K_1 with P_1 is only allowed if $x(1)^T P_1 x(1)$ does not exceed $x(1)^T P_0 x(1)$ (which is implied by $P \le P_0$).

For $k = 2$, however, the dissipation constraint requires

$$\left[x(1)^T P_0 x(1) - x(1)^T P_1 x(1)\right] + \\ + \left[x(2)^T P_1 x(2) - x(2)^T P_2 x(2)\right] \ge 0. \quad (15)$$

If $\left[x(1)^T P_0 x(1) - x(1)^T P_1 x(1)\right] > 0$, our scheme permits that $\left[x(2)^T P_1 x(2) - x(2)^T P_2 x(2)\right]$ becomes negative. In general, $\left[x(k)^T P_{k-1} x(k) - x(k)^T P_k x(k)\right] < 0$ is allowed for $k \ge 2$ if sufficient 'reserves' have been accumulated in the value of

$$\sum_{i=1}^{k-1} \left[x(i)^T P_{i-1} x(i) - x(i)^T P_i x(i)\right]$$

(which is nonnegative by (10)). This contrasts with the much stronger constraint $P \le P_{k-1}$ as imposed in [24], thus leaving more freedom in the optimization to achieve smaller values of γ and hence better performance. In other words, with the dissipation constraint we capture how to save 'energy' from the actual decrease of the dissipation level, in order to render the requirement of non-increase less stringent for the subsequent time-instant, while even allowing for accumulating such 'reserves' so as to enhance performance. From this point of view, the introduced dissipation constraint is less conservative than requiring the value function to be non-increasing, as in [33], in order to achieve robust stability.

IV. Application to active suspension

In this section, we apply the proposed robust MPC scheme to provide a solution of the active suspension control problem. As an example, we consider a 2 DOF quarter-car model shown in Fig. 1, where (k_s, c_s) consist of the so-called passive suspension; k_u stands for the tire stiffness; m_s and m_u represent sprung and unsprung masses, respectively. Moreover, $x_s - x_u$ is the suspension stroke, $x_u - x_o$ the tire deflection and x_o the vertical ground displacement caused by road unevenness; u_f is the scalar active force generated by a hydraulic actuator and can be considered as control input.

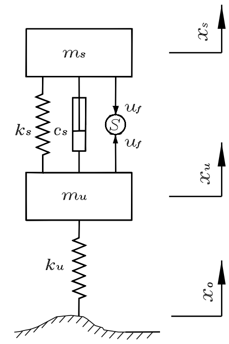

Fig. 1. 2 DOF quarter-car model with an active suspension

Performance requirements for advanced vehicle suspensions include isolating passengers from vibration and shock arising from road roughness (ride comfort), suppressing the hop of the wheels so as to maintain firm, uninterrupted contact of wheels to road (good handling or good road holding) and keeping suspension strokes within an allowable maximum (e.g. [34]). In fact, the active suspension control problem can be formulated as a constrained disturbance attenuation problem. To quantify ride comfort, the body acceleration is chosen as controlled performance output, i.e., $z_1 = \ddot{x}_s$. In order to ensure a firm uninterrupted contact of wheels to road, the dynamic tire load cannot exceed the static ones [35], i.e.,

$$k_u\left(x_u(t) - x_o(t)\right) < (m_s + m_u)g \quad \forall t \ge 0. \quad (16)$$

Moreover, the suspension stroke limitation in the form of

$$|x_s(t) - x_u(t)| \le x_{max} \quad \forall t \ge 0 \quad (17)$$

has to be taken into account to prevent excessive suspension bottoming, which can lead to rapid deterioration of ride comfort and possible structural damage. Due to actuator saturation, it is in addition assumed that the active force is bounded as

$$|u_f(t)| \le u_{max} \quad \forall t \ge 0. \quad (18)$$

Clearly, (16) – (18) are hard time-domain constraints. Hence, we choose the normalized active force, the normalized suspension stroke and the relative dynamic tire load as constrained outputs, i.e., $z_2 = \left(\frac{u_f}{u_{max}}, \frac{x_s - x_u}{x_{max}}, \frac{k_u(x_u - x_o)}{(m_s + m_u)g}\right)^T$ with $z_{2i,max} = 1, i = 1, 2, 3$.

Define $x = (x_s - x_u,\ \dot{x}_s,\ x_u - x_o,\ \dot{x}_u)^T$ as state variables and consider the ground velocity as disturbance input. Then, by discretizing the ideal dynamics equations of the quarter-car model with a sampling time of $\delta = 0.02s$, we obtain a system in the form of (1). For the simulation we have chosen the following nominal values for the physical parameters (cf [35]):

$$m_s = 320kg,\ m_u = 40kg,\ k_s = 18\frac{kN}{m},\ c_s = 1\frac{kNs}{m}$$

$$k_u = 200\frac{kN}{m},\ x_{max} = 0.08m,\ u_{max} = 1.5kN.$$

In the context of vehicle ride and handling, road disturbances can in general be classified as vibration and shock [34]. Vibrations are consistent and typically specified as random process. Shocks are discrete events of relatively short duration and high intensity, caused for example by a pronounced bump or pothole on an otherwise "smooth" road and can be viewed as energy-bounded signals. An excessive suspension bottoming or wheel-hop may happen in this case. Hence, we consider the case of an isolated bump in an otherwise smooth road surface. The corresponding disturbance (ground velocity) is given by

$$w(t) = \begin{cases} \frac{\pi V A}{L} \sin \frac{2\pi V}{L} t & 0 \le t \le \frac{L}{V}, \\ 0 & t > \frac{L}{V}, \end{cases}$$

where V is the vehicle forward velocity, A and L are the height and the length of the bump, respectively. Fig. 2 shows bump responses, where $A = 0.1m$, $L = 5m$ and $V = 60\frac{km}{h}$. For the model predictive controller with guaranteed $\mathcal{H}_\infty$ performance, we choose $r = 5.0$. The optimization problem is feasible at each time, hence, it is guaranteed to respect control and output constraints. For reasons of comparison, we designed two constrained $\mathcal{H}_\infty$ controllers by minimizing (3) subject to (4a) and (4c), for $r = 5.0$ and $r = 15.0$ respectively, and fix the obtained feedback gains. According to [31], the fixed controllers are guaranteed to respect the constraints if the disturbance energy is less than $0.09\frac{m^2}{s^2}$ and $0.11\frac{m^2}{s^2}$, respectively. Moreover, the disturbance attenuation level γ takes the values 7.5 and 11.7, respectively. Note that the energy of the disturbance in Fig. 2 takes the value of $8.2\frac{m^2}{s^2}$, which is much larger than the allowable value for the two fixed controllers such that there is no guarantee that they respect the constraints. The results are plotted in Fig. 2 as dashed ($r = 5.0$) and dash-dotted ($r = 15.0$) lines, respectively. It can be clearly seen that the MPC with $\mathcal{H}_\infty$ performance respects constraints by relaxing on-line the performance level (see the bottom picture in Fig. 2) and achieves much better performance (ride comfort) than the fixed controller with $r = 15.0$ which in turn violates the control constraint slightly. The fixed controller with $r = 5.0$ achieves the best performance among all designs, but it strongly violates both the control and the output constraints.

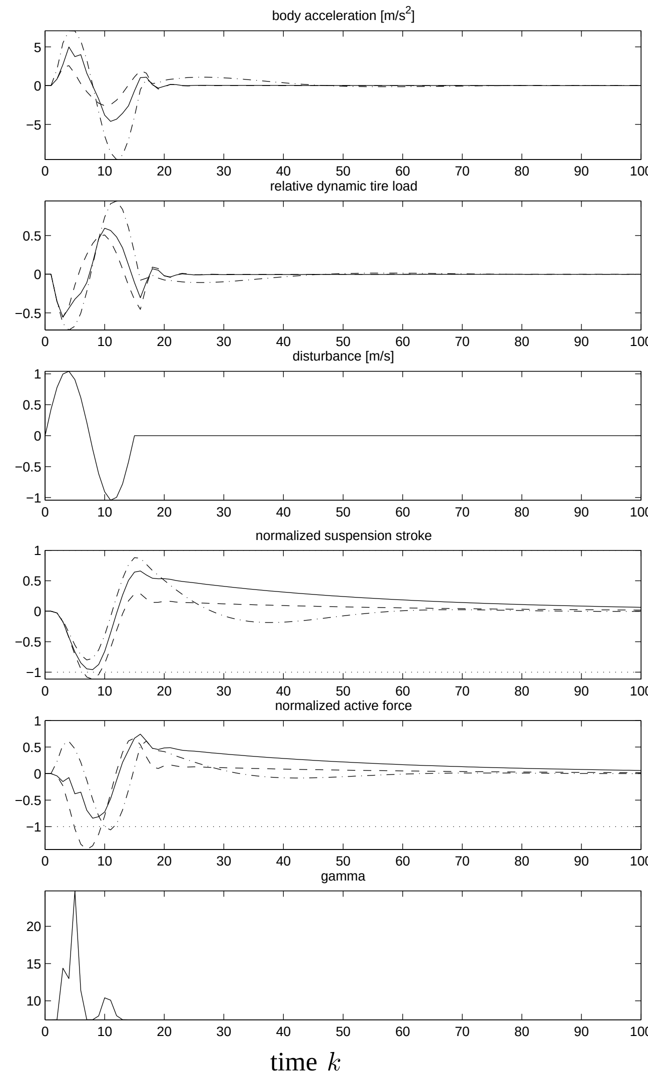

Fig. 2. Bump responses: MPC with $\mathcal{H}_\infty$ performance (——), fixed $\mathcal{H}_\infty$ controllers for $r = 5.0$ (– –) and $r = 15.0$ (– · –)

V. Conclusive remarks on robust $\mathcal{H}_\infty$ performance

For linear systems with control and output constraints we have proposed a robust MPC scheme in which a constrained $\mathcal{H}_\infty$ control problem is solved on-line within the framework of LMI optimization. A dissipation constraint that is less conservative than requiring monotonicity of the value function is introduced to guarantee closed-loop system dissipativity and, hence, $\mathcal{H}_\infty$ performance. Moreover, the proposed MPC scheme is able to automatically relax the performance level in order to obey hard constraints and enhance it when sufficient 'reserves' have been accumulated in the system, measured by the dissipation constraint.

It is interesting to observe that the suggested dissipation constraint is quite general. For example, the setup of the optimization problem (3) can also be adapted for uncertain systems described by (1) with

$$\begin{pmatrix} A & B & B_u \\ C_1 & D & D_{1u} \\ C_2 & 0 & D_{2u} \end{pmatrix} \in \Omega, \tag{19}$$

where $\Omega \subseteq \mathbb{R}^{(n+p_1+p_2)\times(n+m_1+m_2)}$ denotes the uncertainty set. Similarly as discussed for nominal performance, we can guarantee robust $\mathcal{H}_\infty$ performance for a receding horizon implementation based on the following result.

Theorem 2: Suppose that the assumptions in Theorem 1 are satisfied for all system matrices satisfying (19). Then, by defining the feedback gain $K_k = Y_k Q_k^{-1}$ at each sampling time k, and by controlling the system with $u(k) = K_k x(k)$, the closed-loop moving horizon system achieves the properties (i) – (iv) in Theorem 1 robustly for all uncertain systems.

For rather general uncertainty sets Ω, the corresponding family of LMI problems might not be easily (or efficiently) solvable. For polytopic system uncertainty

$$\Omega = Co\left\{ \begin{pmatrix} A_i & B_{1,i} & B_{u,i} \\ C_{1,i} & D_{1,i} & D_{1u,i} \\ C_{2,i} & 0 & D_{2u,i} \end{pmatrix}, \ i = 1, 2, \ldots, l \right\} \quad (20)$$

with a moderate number l of generators we can proceed in a standard fashion, which just requires to replace (4a) and (4c) with

$$\begin{pmatrix} Q & * & * & * \\ 0 & \gamma I & * & * \\ A_i Q + B_{u,i} Y & B_{1,i} & Q & * \\ C_i Q + D_{u,i} Y & D_{1,i} & 0 & \gamma I \end{pmatrix} > 0, \quad (21a)$$

$$\begin{pmatrix} \frac{1}{r} Z & C_{2,i} Q + D_{2u,i} Y \\ (C_{2,i} Q + D_{2u,i} Y)^T & Q \end{pmatrix} \geq 0, \quad (21b)$$

$$Z_{jj} \leq z_{2j,max}^2, \ i = 1, 2, \ldots, l.$$

Hence our scheme is applicable to polytopic uncertain systems without any complication.

VI. Acknowledgment

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References

[1] S. Qin and T. Badgwell, "An overview of industrial model predictive control technology," in *Chemical Process Control – V*, J. Kantor, C. García, and B. Carnahan, Eds. AIChE, CACHE, 1997, pp. 232–256.

[2] R. Bhattacharya, G. J. Balas, M. A. Kaya, and A. Packard, "Nonlinear receding horizon control of an F-16 aircraft," *J. Guidance, Control, and Dynamics*, vol. 25, no. 5, pp. 924–931, 2002.

[3] W. B. Dunbar, M. B. Milam, R. Franz, and R. M. Murray, "Model predictive control of a thurst-vectored flight control experiment," in *Proc. IFAC World Congress*, 2002.

[4] A. Jadbabaie and J. Hauser, "Control of a thrust-vectored flying wing: a receding horizon–LPV approach," *Int. J. Robust Nonlinear Control*, vol. 12, pp. 869–896, 2002.

[5] H. Chen and F. Allgöwer, "Nonlinear model predictive control schemes with guaranteed stability," in *Nonlinear Model Based Process Control*, R. Berber and C. Kravaris, Eds. Dordrecht, The Netherlands: Kluwer Academic Publishers, 1998, pp. 465–494.

[6] F. Allgöwer, T. Badgwell, J. Qin, J. Rawlings, and S. Wright, "Nonlinear predictive control and moving horizon estimation - An introductory overview," in *Advances in Control, Highlights of ECC'99*, P. Frank, Ed. Springer Verlag, 1999, pp. 391–449.

[7] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, pp. 789–814, 2000.

[8] P. Campo and M. Morari, "Robust model predictive control," in *Proc. Amer. Contr. Conf.*, 1987, pp. 1021–1026.

[9] J. Allwright and G. Papavasiliou, "On linear programming and robust model predictive control using impulse responses," *Syst. Contr. Lett.*, vol. 18, pp. 159–164, 1992.

[10] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.

[11] Y. Lu and Y. Arkun, "Quasi-min-max MPC algorithms for LPV systems," *Automatica*, vol. 36, no. 4, pp. 527–540, 2000.

[12] A. Casavola, M. Giannelli, and E. Mosca, "Min-max predictive control strategies for input-saturated polytopic uncertain systems," *Automatica*, vol. 36, pp. 125–133, 2000.

[13] F. Cuzzola, J. C. Geromel, and M. Morari, "An improved approach for constrained robust model predictive control," *Automatica*, vol. 38, pp. 1183–1189, 2002.

[14] J. Löfberg, "Minimax MPC for systems with uncertain gain," in *Proc. 15th IFAC World Congress*, Barcelona, Spain, 2002.

[15] D. R. Ramirez and E. F. Camacho, "Charaterization of min-max MPC with bounded uncertainties and a quadrtic criterion," *Proc. Amer. Contr. Conf.*, pp. 358–363, 2002.

[16] A. Zheng and M. Morari, "Robust stability of constrained model predictive control," in *Proc. Amer. Contr. Conf.*, 1993, pp. 379–383.

[17] J. Lee and B. Cooley, "Min-max predictive control techniques for a linear state-space system with a bounded set of imput matrices," *Automatica*, vol. 36, no. 3, pp. 463–473, 2000.

[18] M. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," in *Proc. Amer. Contr. Conf.*, Baltimore, Maryland, 1994, pp. 440–444.

[19] A. Casavola, D. Famulare, and G. Franze, "A min-max predictive control algorithm for uncertain norm-bounded linear systems," in *Proc. 15th IFAC World Congress*, Barcelona, Spain, 2002.

[20] D. Q. Mayne, "Nonlinear model predictive control: An assessment," in *Chemical Process Control – V*, J. Kantor, C. García, and B. Carnahan, Eds. AIChE, CACHE, 1997, pp. 217–231.

[21] J. Lee and Z.-H. Yu, "Worst-case formulations of model predictive control for systems with bounded parameters," *Automatica*, vol. 33, no. 5, pp. 763–781, 1997.

[22] P. Scokaert and D. Q. Mayne, "Min-max feedback model predictive control for constrained linear systems," *IEEE Trans. Automat. Contr.*, vol. 43, no. 8, pp. 1136–1142, 1998.

[23] G. Tadmor, "Receding horizon revisited: An easy way to robustly stabilize an LTV system," *Syst. Contr. Lett.*, vol. 18, pp. 285–294, 1992.

[24] S. Lall and K. Glover, "A game theoretic approach to moving horizon control," in *Advances in Model-Based Predictive Control*, D. Clarke, Ed. Oxford University Press, 1994.

[25] R. Blauwkamp and T. Basar, "A receding-horizon approach to robust output feedback control for nonlinear systems," in *Proc. 38th IEEE Conf. Decision Contr.*, Phonex, AZ, 1999, pp. 4879–4884.

[26] L. Magni, H. Nijmeijer, and A. J. van der Schaft, "A receding horizon approach to the nonlinear H_∞ control problem," *Automatica*, vol. 37, pp. 429–435, 2001.

[27] Y. Kawai, T. Azuma, and M. Fujita, "Stabilizing receding horizon H_∞ control for LTV systems using extended terminal inequality conditions," in *Proc. 41th IEEE Conf. Decision Contr.*, Las Vegas, Nevada, 2002, pp. 3452–3457, dec. 2002.

[28] H. Chen, C. Scherer, and F. Allgöwer, "A game theoretic approach to nonlinear robust receding horizon control of constrained systems," in *Proc. Amer. Contr. Conf.*, Albuquerque, 1997, pp. 3073–3077.

[29] A. Bemporad and M. Morari, "Robust model predictive control: A survey," in *Robustness in Identification and Control, Lecture Notes in Control and Information Sciences, vol. 245*, A. V. A. Garulli, A. Tesi, Ed. Springer-Verlag, 1999, pp. 207–226.

[30] A. Bemporad, F. Borrelli, and M. Morari, "Robust model predictive control: piecewise linear explicit solution," in *Proc. European Control Conf.*, Porto, Portugal, 2001, pp. 939–944.

[31] H. Chen and C. W. Scherer, "Disturbance attenuation with actuator constraints by moving horizon H_∞ control," in *IFAC ADCHEM'03*, HongKong, China, 2003, pp. 447–452.

[32] C. W. Scherer, H. Chen, and F. Allgöwer, "Disturbance attenuation with actuator constraints by hybrid state-feedback control," in *Proc. 41th IEEE Conf. Decision Contr.*, Dec 2002, pp. 4134–4139.

[33] T. Badgwell, "A robust model predictive control algorithm for stable linear plants," in *Proc. Amer. Contr. Conf.*, Albuquerque, 1997, pp. 1618–1622.

[34] D. Hrovat, "Survey of advanced suspension developments and related optimal control applications," *Automatica*, vol. 33, no. 10, pp. 1781–1817, 1997.

[35] T. Gordon, C. Marsh, and M. Milsted, "A comparison of adaptive LQG and nonlinear controllers for vehicle suspension systems," *Vehicle System Dynamics*, vol. 20, pp. 321–340, 1991.