Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Minimum-Time Swing-up of A Rotary Inverted Pendulum by Iterative Impulsive Control

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***Abstract*— The minimum time swing up of a rotary inverted pendulum is considered. For our rotary inverted pendulum, a DC motor rotates a stiff arm at one end in the horizontal plane. The opposite end of the arm is instrumented with a joint whose axis is along the radial direction of the motor. A pendulum is suspended at the joint. The task is to design a controller that swings up the pendulum and maintains it upright and maintains the arm position. From practical tuning point of view, a PID controller plus an impulse controller is proposed for the swing up control. An iterative tuning of the impulsive control actions is applied to achieve the minimum-time swing-up. To make the overall control strategy more robust, a new mode switching control method is also proposed. Compared to the existing dual mode nonlinear controller provided by the manufacturer, the swing up time is significantly reduced as demonstrated by extensive experimental results.**

***Index Terms*— Rotary inverted pendulum; swing up control; PID control; impulsive control; energy based mode control; minimum-time control.**

I. Introduction

Inverted pendulum has been widely used in both linear and nonlinear control education with applications to other under-actuated mechanical systems, involving nonlinear dynamics, robotics and aerospace vehicles testing [1], [2], [3]. In this paper, a rotary type inverted pendulum, also known as the Furuta pendulum, is considered. The objective is to swing up the pendulum and make it stable at the "upright" position with two different but appealing control problems. The first is to balance and stabilize the pendulum at its upright position. The popular method is to linearize along the desired equilibrium point and apply the linear quadratic regulator (LQR) or pole placement technique [4]. The second is to swing up the pendulum from its hanging position to the upright position which is a more challenging control problem due to its nonlinear under-actuated mechanical nature. If practical factors such as the actuator saturation and the component friction are taken into account, the swing up control problem will be more complicated. To make the control process globally stable, mode switching between these two controllers, i.e., the swing up controller and the upright position regulator, should be carefully considered.

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This work was demonstrated during the *First International Summer School of Iterative Learning Control* (http://www.csois.usu.edu/ilc/summerschool03). The movie clips before and after applying the impulse step control scheme for the swing up process are viewable from http://www.csois.usu.edu/people/yqchen/misc/ILC_impulsive_before.mpg and http://www.csois.usu.edu/people/yqchen/misc/ILC_impulsive_after.mpg.

Different control algorithms have been proposed for swing up control [5], [6], [7], [8], [9], [10]. Most of the methods dealt with the simplified second order model of the rotary inverted pendulum. The rotation of the arm was not taken into full consideration, nor the disturbance of friction as in the partial feedback linearization approach [5], [6], [7] where the trajectory of the pendulum is actually pre-specified. Another technique, namely energy based control [8], [9], [10], neglects the reaction torques from the pendulum to the arm, so that the energy control method can be studied without considering the position and the velocity of the arm. In [8], the stability property of the energy based control is analyzed. Recently, a fourth order model of the Furuta system is presented in [4], where a speed-gradient algorithm is used for swing up for the order 4 nonlinear system where the arm momentum is considered. For non-linear swing up process, neural network and fuzzy logic algorithms were tried in [11], [12], [13] while in [14], [15], under-actuated pendulum control method was investigated by using dynamic programming or reinforcement learning method.

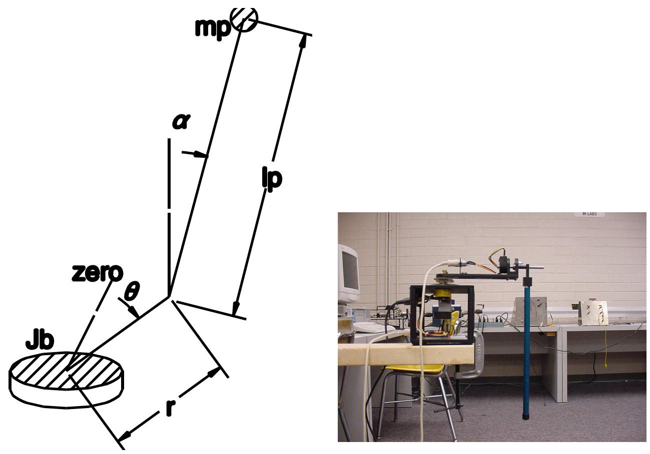
Since the Furuta pendulum has practical implications for industry applications, it is desirable to develop control algorithms that can deal with the model uncertainty and measurement noise with simple controller structure. Therefore, in this paper, we seek to apply PID controller for swing up control. To achieve minimum time swing up, we propose to add an impulsive or pulse-step feedforward controller which is to be tuned through several iterative tuning experiments. During our experiments, we also applied a new simple and robust mode switch control.

The remaining part of this paper is organized as follows. In Sec. II, the mathematical model of the Furuta system is presented. The PID algorithm for swing up process is described in Sec. III. Sec. IV is devoted to the proposed pulse-step control method with details on how the control signal can be constructed. For completeness, the mode control and the balance control are discussed together in Sec. V. Experiment results are presented in Sec. VI. Finally, Sec. VII concludes this paper with some remarks on further investigations.

II. Model of the Furuta Pendulum

A. A Description of the Furuta System

The Furuta pendulum system consists of a rotary servo motor system which drives an independent output gear [16]. The rotary pendulum arm is mounted to the output gear and the pendulum is attached to the hinge. Clearly, this is an under-actuated mechanical system. A schematic representation of the system is shown in Fig. 1 [16] where α denotes the angle of the pendulum to the upright position and θ denotes the angle of the rotor arm. The control purpose is to design a controller that starts with the pendulum in the "down" hanging position, swings it up and maintains it upright.



(a) An illustrative configuration (swinging up)

(b) Laboratory Setup (down hanging position)

Fig. 1. The Rotary Inverted Pendulum System (Furuta Pendulum)

B. The Dynamic Model

To derive a dynamic system model, the coordinate frame systems shown in Fig. 1 are introduced. With some standard assumptions such as no friction, rigid objects etc., the dynamic model are given as follows [16]:

$$(m_pr^2+J_b)\ddot{\theta}+m_pr\ddot{\alpha}I_p\cos(\alpha)-m_pr\dot{\alpha}^2I_p\sin(\alpha)=T \quad (1)$$

$$\begin{aligned} m_pI_p\cos(\alpha)\ddot{\theta}r - m_pI_p\sin(\alpha)\dot{\alpha}\dot{\theta}r \\ +m_p\ddot{\alpha}I_p^2 - m_pgI_p\sin(\alpha)=0 \end{aligned} \quad (2)$$

where T is the input torque from the DC motor; m_p is the mass of the pendulum; I_p is the length from the center of gravity of the pendulum w.r.t. the motor axis; J_b is the moment of inertia of the arm and the gears; θ is the deflection of the arm from the zero position; α is the deflection of the pendulum from the vertical upright position; and r is the length of the arm.

III. Swing Up Using A Simple PID Positive Feedback Controller

As stated above, the goal of the Furuta controller is to swing up the pendulum from stable "down" position to the unstable equilibrium "up" position and be balanced there. The overall controller can actually be divided into three parts: 1) the swing up controller, 2) the mode switching controller and 3) the balancing controller/regulator.

Many different control algorithms can be used to perform the swing up control. Here, a positive feedback PID controller is proposed because of its simple structure, effectiveness and easy tuning. For the balancing control, full-state feedback LQR is applied. The mode switching controller determines when to switch between the two controllers (swing up controller and balancing controller). In particular, when the pendulum is under balancing control and receives some disturbance, it may need to switch back to swing up controller. A good mode switching controller can switch the controller between these two controllers smoothly and make the whole control process globally robust. A robust energy-based mode switching controller is presented in Sec. V.

Now we start to derive the PID control law for our Furuta system. Here a positive feedback loop is used to swing up the pendulum. It actually consists of two loops as shown in Fig. 2. The outer loop specifies the trajectory for the arm

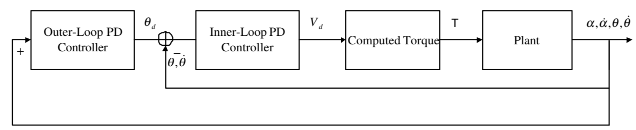


Fig. 2. Swing up using positive feedback PID controller alone

angles and at the same time excites the internal dynamics to swing pendulum to the balancing position. By moving the arm back and forth, one can eventually bring up the pendulum. It is fairly intuitive to design the outer loop as follows:

$$\theta_d = P\alpha + D\dot{\alpha} \quad (3)$$

where θ_d is the given trajectory of the the arm and α is the pendulum angle deviated from the down hanging position which is positive in the clockwise direction and negative in the counterclockwise direction. Note that α is limited within $\pm 180°$ (wrapped around).

The values of the two parameters P and D play a key role in bringing up the pendulum smoothly. To prevent the pendulum from colliding with the other components, we need to limit θ within $\pm 90°$. Initially, P can be chosen as 0.5. To properly choose D, a compromise should be made between increasing the reaction time and decreasing the noise amplification. In our system, D is set to be 0.001 (sec.) at first. P and D can be tuned to adjust the "positive damping" in the system and meet the experiment criterions.

The inner loop performs the position control of the arm. For the servo arm to track the desired position, a feedback PD controller is designed as follows:

$$V_d = K_p(\theta_d - \theta) + K_d\dot{\theta}. \quad (4)$$

where K_p and K_d is the parameters to be tuned.

The first thing to do is to find out the closed-loop transfer function between the input and the output of the arm angle. For the Furuta system, the model of the motor is given by

$$V = I_mR_m + K_mK_g\dot{\theta}, \quad (5)$$

where

V(Volts):	Voltage applied to motor;
I_m(Amp):	Current in motor;
K_m(V/deg./sec.):	Back EMF constant;
K_g:	Gear ratio in motor; gearbox and external gears
θ(deg.):	Arm angular position.

The torque generated by the motor is then given by

$$T = K_mK_gI_m = J_s\ddot{\theta}, \quad (6)$$

where J_s is the total moment of inertia of the arm, the gears and the pendulum w.r.t. the motor axis.

By some mathematical manipulations, the closed-loop transfer function is obtained as follows:

$$\frac{\theta}{\theta_d} = \frac{K_p + K_d s}{\frac{J_s R_m}{K_m K_g} s^2 + (K_m K_g + K_d)s + K_p}. \quad (7)$$

So, the closed-loop system has a second-order characteristic polynomial:

$$s^2 + 2\zeta\omega_0 s + \omega_0^2, \quad (8)$$

where ζ can be set to about 0.707 and the natural frequency of the control system should be much larger than the natural frequency of the pendulum. In this way, the closed-loop response of the arm could be considerably faster than that of the pendulum and a better compromise between overshoot and transient time can be achieved. To prevent the arm from moving too much to over-damp the pendulum, a saturation block is applied between the inner loop and outer loop. Again, to limit the motor input voltage V_{motor} within ± 5 (volt), a saturator block can be added in front of the voltage input to the DC motor. At this point, we remark that based on our experimental experience, it is hard to achieve the minimum time swing up control by only tuning PID parameters. More advanced components should be added into the swing up controller. In this paper, we propose to apply the impulse or the pulse-step control which is explained in detail in the next section.

IV. Iterative Impulsive Control

Several different strategies can be combined to swing up the pendulum. To achieve a minimal time swing up, it follows from the Pontryagin's maximum principle that the minimal time strategy for swinging up the pendulum is of bang-bang type. The complexity of the minimal-time control strategy increases with the order of the system. For a second-order plant, a simple pulse-step control can be used to give fast set-point changes and sub-optimal results. This impulsive control is inspired by the optimal control theory, but also comes from our observations. That is, by applying a pulse step torque at one end of the pendulum, with the direction of the torque the same as the velocity of the pendulum, it can be expected to swing up the pendulum more aggressively.

Pulse-step control may give good results for the system using simple controllers as illustrated in [17]. The motivations of using impulsive control were well explained in [18], [19] with some applications demonstrated in [20], [21]. The analysis and design methods for impulsive control systems can be found in [22], [19], [23].

Note that, the pulse-step control method proposed in this paper is actually an open loop strategy. To make the control strategy more robust, a feedback-feedforward structure should be considered where the uncertainty in the system model and the disturbance can be compensated by the feedback controller.

In our experiment, the pulse-step control signal is a step type function of the following form:

$$u_{ff}(t) = \begin{cases} 0 & : \quad \dot{\theta} = 0 \\ \bar{u} & : \quad \dot{\theta} < 0 \\ \underline{u} & : \quad \dot{\theta} > 0 \end{cases} \quad (9)$$

where $\bar{u}$ and $\underline{u}$ are the constant amplitudes to be further tuned. To make the control stable, the pulse step control signal is added before the inner-loop feedback, as shown in Fig. 3. By carefully setting the parameters, we can manage to achieve the optimal objective [24].

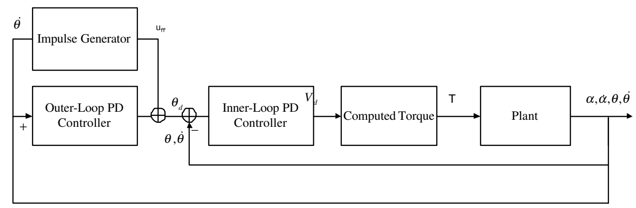


Fig. 3. Swing up using PID and Pulse Control

Pulse-step control is important to the minimal-time swing up, especially for the initial phase of swing up. At the beginning, the amplitude of the pendulum movement is small, so is the amplitude of the arm. Then, the energy of the pendulum increases slowly in the beginning. Applying a pulse control, that is, an additional torque independent of the initial states of the pendulum, the energy accumulating process will be speeded up and consequently the swing up time can be reduced.

In Sec. VI, we will show show to iteratively tune the pulse step parameters to achieve the minimum time swing up via a number of experiments.

V. Balance Control and Mode Switch Control

When the pendulum is almost upright, a state feedback controller should be implemented to maintain it upright and reject the possible external disturbance. The state feedback controller is designed using the well known linear quadratic regulator based on the linearized plant model. It may also be implemented by pole placement method or other linear control methods.

When the pendulum is at the upright position, it is easy to keep it up. However, keeping it upright nicely is not enough. It is required that the whole control process (swinging up, balancing, and the mode switching) must be robust. That is, when any disturbance is applied to the pendulum, the controller can switch properly between the swing up control and the balance control. The pendulum can swing up, approach the upright position and switch to the balance control smoothly. When it is disturbed from the balance state and deviated from the upright position to certain degrees larger than a threshold value, it should switch back to the swing up control and approach the upright position again. So, it is important to design a proper mode switching control to make the whole control system robust.

To design a mode control algorithm, one way is to find out the extent to which the balance control can still take effect. These can be some constraints on the angles and velocities of the pendulum and the arm. These constrains form a window with switching parameters obtainable by experiments. Note that mode control is a switching control and should avoids bouncing. So, the condition for switching from the swing up mode to the balance mode and the condition for switching from the balance mode to the swing up mode could be different.

As stated above, it is not trivial to make a practical control algorithm in the desirable way. Many parameters need to be estimated and tuned. Sometimes, the pendulum is disturbed from the upright position and enters the swinging up control mode. Under the effect of swing up control, the system may never satisfy the conditions to enter the balance control mode again.

There exists other control algorithms, such as dynamic programming and reinforcement learning control [14], [15], that can make the whole control process somewhat robust. But the dynamic programming may require impractical amount of computation effort and the reinforcement learning may need lots of trials before finding a desirable control law. The exploration phrase in the reinforcement learning may damage the actual hardware setup.

Here, we propose a simple but robust mode control logic to achieve the global stabilization. First, some observations for our experiment setup are presented.

- Assume that the system has zero energy at the down position. When the pendulum is approximately at the upright position, the velocities and angles of both pendulum and the arm are very small. So the kinetic energy of the system is very small. The energy the system holds is mostly the potential energy. Note that the potential energy varies little in the balance state.
- During the process of swinging up, the arm movement only has kinetic energy variation which is much smaller compared with the energy of the pendulum. So, the process can be viewed as the process of pumping energy into the pendulum.
- When the pendulum is approaching the upright position, the arm also approaches a standstill state. That is, the kinetic energy of the arm is pumped into the potential energy of the pendulum.
- Under the effect of the swing up control, when the pendulum approaches the upright position, the system energy is always increasing and arrives its local maximum in a single period.
- During the swing up process, the system energy goes to local minimal when it falls to the down position. This is due to the friction and reaction to the motor. Figure 4 shows a typical swing up process from one of our experiments.
- When the pendulum is disturbed from the upright position and goes beyond the control domain of the balance control, it will fall down. Then, the system energy is decreasing because of the electromagnetism effect and friction, as can be seen in Fig. 9.

From the above observations, it is clear that there is a correspondence between the energy level and the orbits for Hamiltonian systems [24]. It can be clearly concluded that the energy should be the right criterion for the mode switching control. Moreover, this energy-based mode switching control is simple to implement and can achieve a much better robust performance.

First, we can calculate E_{up}, the energy of the system when the pendulum is at its strictly up position. Then in the overall control process, the energy E (kinetic and potential) of the system is computed and compared with $E_c = E_{up} - \varepsilon$, where ε is a small positive value. When E is larger than E_c, the control law switches to the balance control. When E is lower than E_c, the control law switches to the swing up control. To avoid the switch bouncing (oscillation), the value of ε is, although important, easy to obtain during the experiments by trial-and-error method. Our experiments proved that this simple mode control helps achieve good global stabilization.

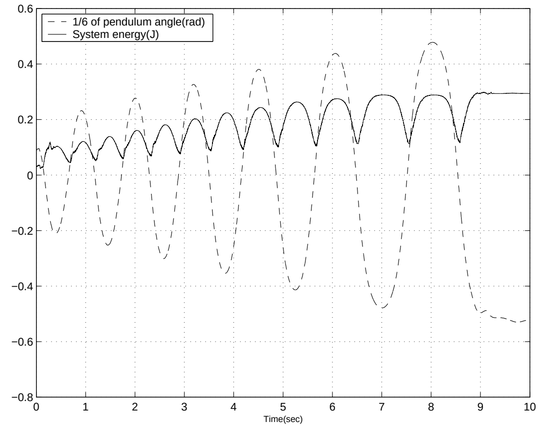


Fig. 4. Energy variation in a typical swing up process

VI. EXPERIMENTS

A. Hardware platform

The hardware platform of the Furuta system consists of a rotary servo motor system which drives an independent output gear. The rotary pendulum arm is mounted to the output gear. At the end of the pendulum arm is a hinge. The pendulum attaches to the hinge. Since we need to measure the states of the system, i.e., the angles and the velocities of the arm and the pendulum, two quadrature encodes are used with one for the arm and the other for the pendulum. See Fig. 1(b) for a photo of the system setup.

B. Balancing control and mode switching control

For balancing control, with the linearized model of the system, the following parameters are used

$$Q = \mathit{diag}([.25, 4, 0, 1])$$

$$R = 0.05$$

for the LQR controller design. For the mode switching control, the potential energy for the system at the upright position is computed as $E_{up} = 0.295$ (J). ε is set to be

$$\varepsilon = 0.01 E_{up} = 0.0295(\mathrm{J}).$$

C. PID controller for swing up

As stated in Sec. II, we first use the PID controller to perform the swing up control. The control law is shown in (3) and (4). For (3), P can be calculated theoretically to be 0.5 and D is set to 0.001 (sec.) at the first trial.

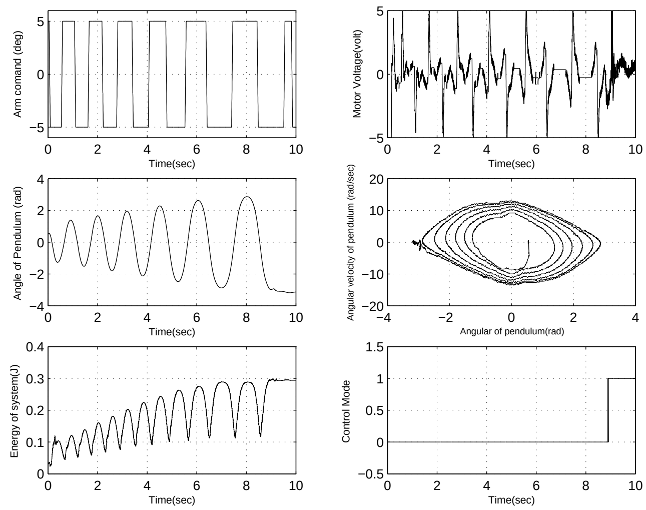


Fig. 5. Swing up process for the PID controller alone

To set the parameters in the control law (4), substituting (5), (6) and (7) into (4) gives the system characteristic polynomial as follows:

$$f(s) = s^2 + (14.5897 + 27.174K_d)s + 27.174K_p, \quad (10)$$

In (7), the damping ratio is set as $\zeta = 0.707$ and the natural frequency ω_0 of the inner loop control is set to be $6\omega_p$. $\omega_p = 6.46$ (rad./sec.) is the natural frequency of the pendulum for small oscillations. Finally, the initial parameters in (4) are given by $K_p = 1.045\text{V}/^\circ$ and $K_d = -0.0273(\text{V}/^\circ/\text{sec.})$. Using the Matlab/Simulink RTW (Real-Time Workshop) platform, the PID controller mentioned above can be implemented on our experiment setup. First, we want to achieve the minimum time swing up under the PID controller alone. To make the swing up faster and smoother, we further tuned the PID parameters. The output of the outer-loop controller is truncated to within $\pm 5^\circ$ to make the swing up more stable and efficient. P is tuned to be 0.7, so that the energy is pumped into pendulum more quickly. At the same time, the movement of the pendulum dose not conflict with our experimental setup. Similarly, $K_p = 1(\text{V}/^\circ)$ and $K_d = -0.02(\text{V}/^\circ/\text{sec.})$ is proved to be the optimal setting through experiments.

Figure 5 shows the dynamic process of the system for swing up. It can be seen that the swing up time is 8.742 sec. with the PID controller alone.

D. PID controller and impulse-step controller for swing up

To speed up the swing up, the proposed pulse-step control signal u_{ff} is added to the whole swing up process. It is expected to increase the control effort and pump energy to the pendulum more quickly. This open-loop feedforward control is quite simple. Referring to (9), here we use $\underline{u} = -\bar{u}$. The pulse-step control gives a constant speed feedforward control output signal, u_{ff}, as shown in Fig. 3. The optimal value of this control signal, $\bar{u}$, is decided by experiments in an iterative way. If $\bar{u}$ is too large, the arm may move too much to one direction and actually reduce the amplitude of the oscillations of the pendulum.

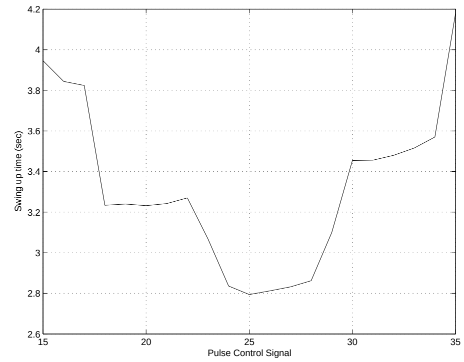


Fig. 6. Impulse signal amplitude $\bar{u}$ vs. the achieved swing up time

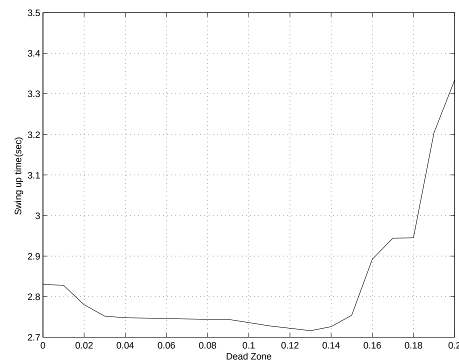


Fig. 7. Deadzone size t_z versus the swing up time at $\bar{u} = 24$ (deg.)

For our experiments, we start with $\bar{u} = 15^\circ$ and increase it incrementally. Figure 6 shows the curve of the $\bar{u}$ versus the achieved swing up time.

It can be seen that the optimal swing up time is $t = 2.794$ sec. at $\bar{u} = 24^\circ$, which is about one third of the swing up time achieved by the PID controller alone.

To further explore the possibility of reducing the swing up time, we first observe that there exists some interferences within the control signal when the velocity of the pendulum crosses zero. So, it is reasonable to add a deadzone block after the velocity signal of the pendulum's . When the output of the deadzone block is zero, no pulse control is activated. The arrange of the deadzone t_z is tuned in our experiments. Figure 7 shows t_z versus the swing up time at $\bar{u} = 24^\circ$. Clearly, there is an optimal choice of t_z which is found to be $t_z = 0.13$ sec.

To summarize, for the control strategy stated above, with $P = 0.7$, $D = 0.001$ (sec.), $K_p = 1(\text{V}/^\circ)$, $K_d = -0.02(\text{V}/^\circ/\text{sec})$, $\bar{u} = 24^\circ$ and $t_z = 0.13$ sec., the achieved minimal swing up time is that $t = 2.716$ sec. The dynamics of the swing up process is shown in Fig. 8.

Figure 9 shows the effectiveness of the energy-based mode switching control. The pendulum swings up to the balance position first and then a disturbance was applied to deviate the pendulum from the balancing position. In this case, the energy of the system is decreased. Therefore, the control mode is changed to the swing up control mode by which the energy of the system is increased again. Finally, the mode is switched back to the balance control mode again.

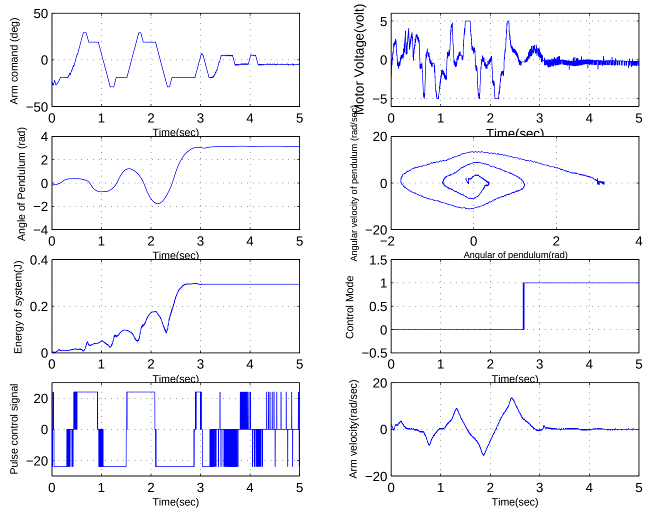


Fig. 8. Swing up using the PID feedback controller and the impulse feedforward controller

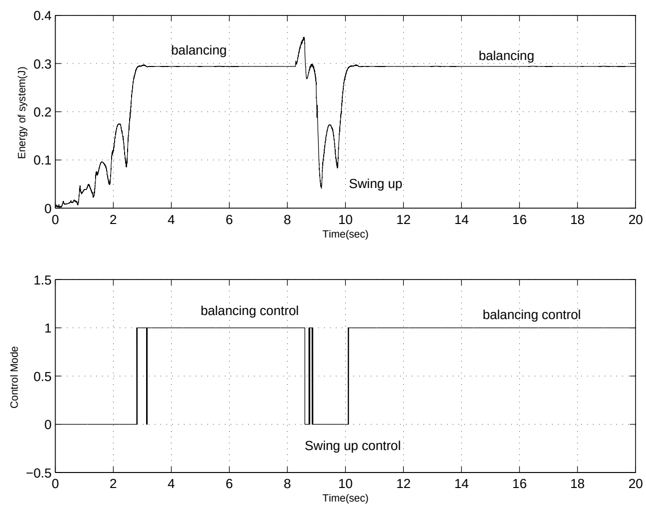


Fig. 9. Typical disturbance process and the control mode switching

VII. Conclusions

In this paper, a PID positive feedback controller and a feedforward impulse-step controller have been successfully applied to experimentally investigate the minimum-time swing up problem of a rotary inverted pendulum. The swing up time has been reduced approximately from being longer than 8 sec. to being less than 3 sec. To make the whole control process globally stable, an energy based mode switching control was also attempted. The control strategy experienced in this paper can also be applied to other under-actuated systems.

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