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Sliding Control Approach to Underactuated Multibody Systems

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Abstract— **A robust control algorithm is proposed for stabilization and tracking control of underactuated multibody mechanical systems governed by nonlinear equations of motion. Sliding, or variable structure, control is a simple but robust nonlinear control approach that is capable of handling both disturbances and parameter uncertainties. We formulate the sliding control approach for general underactuated multibody systems, and define first order sliding surfaces, one per actuated degrees of freedom, as a linear combination of the tracking position and velocity errors of both actuated and unactuated coordinates. The controllers are then determined based on a Lyapunov function construction. The Lyapunov stability analysis, along with the bounds defined for parameter uncertainties and disturbances, guarantee that all system trajectories reach and remain on the sliding surfaces. The sliding surfaces are proved to be asymptotically or marginally stable, depending on the presence or absence of potential energy terms in the equations of motion. A 3-dimentional multibody model of a complex satellite system is presented and its equations of motion are derived. The proposed sliding control approach is developed for the satellite system and applied to maneuver the satellite using its appendages when it has lost all three rotational degrees of freedom.**

I. INTRODUCTION

Underactuated mechanical systems are those that have fewer actuators than total degrees-of-freedom (DOF). There are several types of underactuated systems, including multi-rigid body systems such as robots and satellite systems [1 – 5], deformable multibody systems such flexible link robots and surface vessels, systems actuator with coupled actuator and rigid body dynamics, and underactuated spacecrafts with or without shape change actuators [6 – 15]. Research in control of underactuated systems has surged in the past decade. Several classifications have been presented and stability and controllability issues have been discussed [16 – 17]. Some of the control approaches include feedback linearization [9 – 10, 18 – 19], controlled Lagrangians and matching conditions [20 – 21], sliding mode control [3, 7, 22], and backstepping approaches [23].

In this paper we apply the sliding control approach [24] to underactuated nonlinear multibody systems. First, we introduce simple formulation of the sliding control approach to underactuated mechanical systems by defining the sliding surfaces as a linear combination of the actuated and unactuated position and velocity tracking errors. Lyapunov theory is used to derive a control law, which guarantees all system trajectories will reach and stay on the sliding surfaces. The sliding surfaces are shown to be asymptotically stable when potential energy is present but only marginally stable in its absence. A multibody model of the complex satellite system is presented and its equations of motion are derived. The proposed sliding control algorithm is applied to the satellite system to reorient the it using its appendages such as antenna, rods, and solar arrays, assuming it has lost all three rotational DOF.

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II. SLIDING CONTROL FOR UNDERACTUATED SYSTEMS

The goal of sliding control approach is to define an asymptotically stable surface such all system trajectories converge to this surface and slide along it until they reach their desired destination [24].

A. Sliding Control Formulation for Underactuated Multibody Systems

Consider a multibody mechanical system with n DOF represented by the generalized coordinate vector $\boldsymbol{q}$. The equations of motion of the system may be written as

$$M(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{u} + \mathbf{d}, \qquad (1)$$

where M is the $n \times n$ mass matrix, $\mathbf{u}$ is the control vector, $\mathbf{d}$ is the disturbance vector, and $\mathbf{f}$ is the vector of Coriolis and centrifugal forces as well as conservative and nonconservative forces. If the system in Eq. (1) is underactuated, then the control vector has a size $m < n$, and the equations of motion may be partitioned into m actuated and $r = n - m$ unactuated equations as

$$\begin{bmatrix} M_{aa} & M_{au} \\ M_{au}^T & M_{uu} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_u \end{bmatrix} = \begin{bmatrix} \mathbf{f}_a + \mathbf{u} + \mathbf{d}_a \\ \mathbf{f}_u + \mathbf{d}_u \end{bmatrix}, \quad (2)$$

where M_{aa} $(m \times m)$, M_{uu} $(r \times r)$, and M_{au} $(m \times r)$ are the mass sub-matrices corresponding to the actuated, unactuated, and coupling terms, respectively. Similarly, the vectors in Eq. (1) are partitioned as $\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{\mathbf{q}}_a^T & \ddot{\mathbf{q}}_u^T \end{bmatrix}^T$, $\mathbf{f} = \begin{bmatrix} \mathbf{f}_a^T & \mathbf{f}_u^T \end{bmatrix}^T$, and $\mathbf{d} = \begin{bmatrix} \mathbf{d}_a^T & \mathbf{d}_u^T \end{bmatrix}^T$. Equation (2) may be solved for the accelerations, yielding

$$\begin{aligned} \ddot{\mathbf{q}}_a &= M_{aa}'^{\,-1} (\mathbf{f}_a' + \mathbf{d}_a' + \mathbf{u}), \\ \ddot{\mathbf{q}}_u &= M_{uu}'^{\,-1} (\mathbf{f}_u' + \mathbf{d}_u' - M_{au}^T M_{aa}^{-1} \mathbf{u}), \end{aligned} \quad (3)$$

where

$$\begin{aligned} M_{aa}' &= M_{aa} - M_{au} M_{uu}^{-1} M_{au}^T, \\ \mathbf{f}_a' &= \mathbf{f}_a - M_{au} M_{uu}^{-1} \mathbf{f}_u, \\ \mathbf{d}_a' &= \mathbf{d}_a - M_{au} M_{uu}^{-1} \mathbf{d}_u, \\ M_{uu}' &= M_{uu} - M_{au}^T M_{aa}^{-1} M_{au}, \\ \mathbf{f}_u' &= \mathbf{f}_u - M_{au}^T M_{aa}^{-1} \mathbf{f}_u, \\ \mathbf{d}_u' &= \mathbf{d}_u - M_{au}^T M_{aa}^{-1} \mathbf{d}_u. \end{aligned}$$

B. Surface Definition for Underactuated Multibody Systems

The sliding surfaces are defined as a weighted combination of position tracking error, $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}^d$, and velocity tracking error, $\dot{\tilde{\mathbf{q}}} = \dot{\mathbf{q}} - \dot{\mathbf{q}}^d$ [24], where the superscript "*d*" denotes the desired values. The m surfaces for the underactuated case are defined by appending the weighted position and velocity errors of the unactuated coordinates to the actuated ones,

$$\mathbf{s} = \alpha_a \dot{\tilde{\mathbf{q}}}_a + \lambda_a \tilde{\mathbf{q}}_a + \alpha_u \dot{\tilde{\mathbf{q}}}_u + \lambda_u \tilde{\mathbf{q}}_u, \quad (4)$$

where $\alpha_a, \lambda_a \in \Re^{m \times m}$ are diagonal, positive-definite matrices and $\alpha_u, \lambda_u \in \Re^{m \times r}$. The time derivative of the surface is defined as, using Eq. (3),

$$\begin{aligned} \dot{\mathbf{s}} &= M_s \mathbf{u} + \mathbf{f}_s + \mathbf{d}_s + \dot{\mathbf{s}}_r, \\ M_s &= \alpha_a M_{aa}'^{-1} - \alpha_u M_{uu}'^{-1} M_{au}^T M_{aa}^{-1}, \\ \mathbf{f}_s &= \alpha_a M_{aa}'^{-1} \mathbf{f}_a' + \alpha_u M_{uu}'^{-1} \mathbf{f}_u', \\ \mathbf{d}_s &= \alpha_a M_{aa}'^{-1} \mathbf{d}_a' + \alpha_u M_{uu}'^{-1} \mathbf{d}_u', \\ \dot{\mathbf{s}}_r &= -\alpha_a \ddot{\mathbf{q}}_a^d - \alpha_u \ddot{\mathbf{q}}_u^d + \lambda_a \dot{\tilde{\mathbf{q}}}_a + \lambda_u \dot{\tilde{\mathbf{q}}}_u, \end{aligned} \quad (5)$$

Assuming that our model of M_s is exact (i.e. $M_s = \hat{M}_s$) and $\hat{M}_s^{-1}$ exists, then the control vector can be calculated by setting $\dot{s} = 0$, yielding

$$\mathbf{u} = -\hat{M}_s^{-1} \left[\hat{\mathbf{f}}_s + \hat{\mathbf{d}}_s + \dot{\mathbf{s}}_r + \mathbf{k}\, sat(\mathbf{s}/\boldsymbol{\varepsilon}) \right], \quad (6)$$

where $\mathbf{k}\, sat(\mathbf{s}/\boldsymbol{\varepsilon}) = [k_1\, sat(s_1/\varepsilon_1) \ \ \ldots \ \ k_m\, sat(s_m/\varepsilon_m)]^T$ and the saturation function "*sat*" is a continuous approximation of the *sign* function. The reaching condition can be derived by defining $\frac{1}{2}s^2$ as the Lyapunov function such that $\frac{1}{2}\frac{d}{dt}s^2 = s\dot{s} \le -\eta|s|, \eta > 0$. Hence, vector $\boldsymbol{k}$ is selected such that the reaching conditions are satisfied for all surfaces:

$$\mathbf{k} = \mathbf{F}_s + \mathbf{D}_s + \boldsymbol{\eta}, \left| \mathbf{f}_s - \hat{\mathbf{f}}_s \right| \le \mathbf{F}_s, \left| \mathbf{d}_s - \hat{\mathbf{d}}_s \right| \le \mathbf{D}_s \quad (7)$$

C. Stability Conditions

In the case of fully actuated systems (with full state feedback), it has been well established that sliding control method can guarantee the system response reaching the sliding surface and that the surface is asymptotically stable. Such claims cannot be made, however, for underactuated systems. The controller of Eq. (6) does guarantee that all system trajectories will reach the surface by selecting α_a and α_u such that $\hat{M}_s^{-1}$ exists and proper selection of the estimation error functions in Eq. (7). However, there is no guarantee that, once on the surface, the trajectory will lead to the desired origin. Stability of the surface can be established if the combination of the m surface linear equations and the r unactuated acceleration equations of Eq. (3) are proved to be stable. Thus, let's consider the case where the trajectory has reached the surface, $\mathbf{s} = 0$. Assume that there are no disturbances or parameter uncertainties and substitute for $\boldsymbol{u}$ in Eq. (3). The linear surface equations and the unactuated part of Eq. (3) form a complete set of n equations:

$$\alpha_a \dot{\tilde{\mathbf{q}}}_a + \lambda_a \tilde{\mathbf{q}}_a + \alpha_u \dot{\tilde{\mathbf{q}}}_u + \lambda_u \tilde{\mathbf{q}}_u = 0 \quad (8a)$$

$$\ddot{\mathbf{q}}_u - M_{uu}'^{-1} (\mathbf{f}_u' + M_{au}^T M_{aa}^{-1} M_s^{-1} (\mathbf{f}_s + \dot{\mathbf{s}}_r)) = 0 \quad (8b)$$

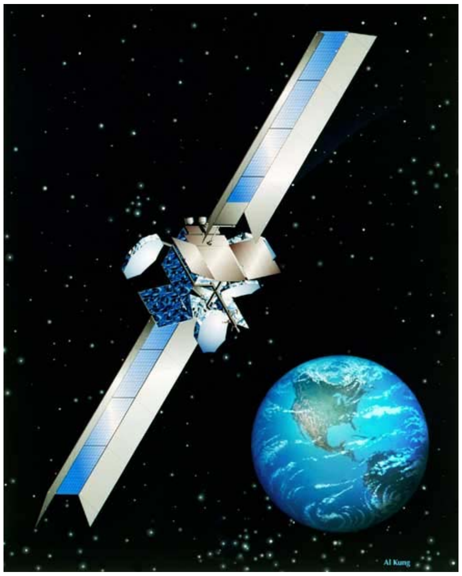

Fig. 2: The Boeing 702 Satellite system courtesy of Space Technology

If we linearize Eq. (8a) equations about the equilibrium point $(\mathbf{q}_e, 0)$, the local stability criteria reduce to conditions under which the linearized state matrix in Eq. (8a) is Hurwitz. These conditions general lead to selection of the surface parameters α_a, λ_a, α_u, and λ_u.

For nonlinear multibody mechanical systems, the sources of nonlinearity are normally velocity squared terms and geometric. Hence, linearization leads to the conclusion that if there is potential energy present, then the system will be locally asymptotically stable near the equilibrium point. As an example, consider the inverted pendulum on a cart problem. The system has two DOF but only one controller (the force applied to move the cart). The goal is to stabilize the pendulum arm to, or make it track an oscillatory motion about its unstable equilibrium point (vertically upward arm) while stabilizing the cart at some desired location. The only conditions of stability for this problem are that 1) the initial position of the arm must be above the horizontal and 2) the rate of exponential convergence of the arm angle, while on the surface, must be faster than the cart position.

In the absence of potential energy, the system is only marginally stable since there are $2r$ poles of the linearized system exactly equal to zero independent of our choice of surface parameters. In such cases, multiple surfaces must be constructed to bring all the states to their desired equilibrium point. In other words, we can only position m DOF at a time, so we need to construct additional surfaces by changing the surface parameters and position the remaining states at a later stage. Such discontinuous sliding mode control approach has been proposed for the nonholonomic integrator [25].

III. Boeing 702 Satellite System

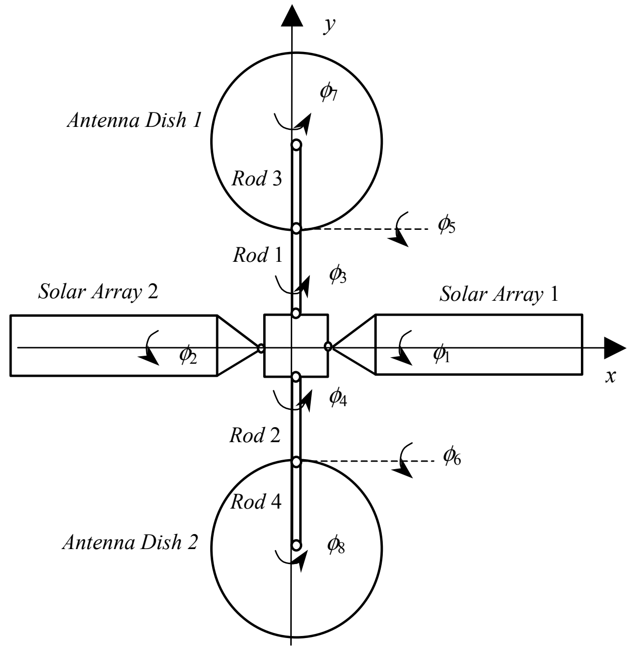

Fig. 1: Top view of the Boeing 702 Satellite system multibody model

Consider the Boeing 702 Satellite system, shown in Fig. 1. The objective is attitude control of the main vehicle through its appendages when the vehicle has lost one or all three of its reaction wheels.

A multibody model of the satellite consists of a main vehicle (length l_0, width w_0) and its appendages consisting of two solar arrays (length l_s, mass m_s), two dish antennas (radius l_r, mass m_d) and four rods (length l_r, mass m_r), as shown in Fig. 2.

There are a total of 14 DOF, 6 for the main vehicle (body 0) and 8 articulated DOF for the 8 appendages (bodies 1 – 8). The articulated generalized coordinates are collected into a vector as $\boldsymbol{\phi} = [\phi_1 \quad \phi_2 \quad \ldots \quad \phi_8]^T$. The vehicle motion is represented by 3 linear motion variables $\mathbf{r} = [x \quad y \quad z]^T$ which are actuated by thrusters and 3 Euler angles $\boldsymbol{\theta} = [\theta_x \quad \theta_y \quad \theta_z]^T$.

A. Equations of Motion

The Lagrangian of our satellite model can be written as:

$$L = \tfrac{1}{2} m_0 \dot{\mathbf{r}}^T \dot{\mathbf{r}} + \tfrac{1}{2} \boldsymbol{\omega}_0^T J_0 \boldsymbol{\omega}_0 + \sum_{i=1}^{8} \tfrac{1}{2} m_i \dot{\mathbf{r}}_i^T \dot{\mathbf{r}}_i + \tfrac{1}{2} \boldsymbol{\omega}_i^T J_i \boldsymbol{\omega}_i \qquad (9)$$

where m_0 and J_0 are the mass and inertia matrix of the main vehicle, and m_i and J_i are the masses and inertia matrix of the appendage body i. Assuming there are control inputs (actuators) for the 8 articulated coordinates, u_i, and the 3 translational coordinates of the main vehicle, $\mathbf{u}_r$, the Lagrangian equations of motion are derived and written in matrix form as

$$\begin{bmatrix} M_{\phi\phi} & M_{\phi r} & M_{\phi\theta} \\ M_{\phi r}^T & M_{rr} & M_{r\theta} \\ M_{\phi\theta}^T & M_{r\theta}^T & M_{\theta\theta} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\phi}} \\ \ddot{\mathbf{r}} \\ \ddot{\boldsymbol{\theta}} \end{bmatrix} = -\begin{bmatrix} \mathbf{f}_\phi \\ \mathbf{f}_\mathbf{r} \\ \mathbf{f}_\theta \end{bmatrix} + \begin{bmatrix} \mathbf{u}_\phi \\ \mathbf{u}_r \\ \mathbf{0} \end{bmatrix}, \qquad (10)$$

where

$$M_{\phi\phi} = \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & m_{35} & 0 & m_{37} & 0 \\ 0 & 0 & 0 & m_{44} & 0 & m_{46} & 0 & m_{48} \\ 0 & 0 & m_{35} & 0 & m_{55} & 0 & m_{57} & 0 \\ 0 & 0 & 0 & m_{46} & 0 & m_{66} & 0 & m_{68} \\ 0 & 0 & m_{37} & 0 & m_{57} & 0 & m_{77} & 0 \\ 0 & 0 & 0 & m_{48} & 0 & m_{68} & 0 & m_{88} \end{bmatrix},$$

and

$$M_{\phi r} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ M_{3\mathbf{r}} \\ M_{4\mathbf{r}} \\ M_{5\mathbf{r}} \\ M_{6\mathbf{r}} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, M_{\phi\theta} = \begin{bmatrix} M_{1\boldsymbol{\theta}} \\ M_{2\boldsymbol{\theta}} \\ M_{3\boldsymbol{\theta}} \\ M_{4\boldsymbol{\theta}} \\ M_{5\boldsymbol{\theta}} \\ M_{6\boldsymbol{\theta}} \\ M_{7\boldsymbol{\theta}} \\ M_{8\boldsymbol{\theta}} \end{bmatrix}, \mathbf{f}_\phi = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{bmatrix}, \mathbf{u}_\phi = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix}$$

The elements of the mass matrix and the Coriolis and centrifugal force vector will be furnished by the corresponding author upon request.

B. Surface Definition

This system has 14 DOF (n = 14) and 11 controllers (m = 11, r = 3). The mass matrix M and force vector f partitions according to Eq. (10) are defined as

$$m_{aa} = \begin{bmatrix} M_{\phi\phi} & M_{\phi r} \\ M_{\phi r}^T & M_{rr} \end{bmatrix}, m_{au} = \begin{bmatrix} M_{\phi\theta} \\ M_{r\theta} \end{bmatrix}, m_{uu} = M_{\theta\theta},$$

$$\mathbf{f}_a = \begin{bmatrix} \mathbf{f}_\phi^T & \mathbf{f}_r^T \end{bmatrix}^{\mathrm{T}}, \ \mathbf{f}_u = \mathbf{f}_\theta.$$

Since we are only concerned with attitude control, we will define 3 independent surfaces to maintain current position of the satellite in our control algorithm. In other words, we define exponentially stable surfaces for vehicle translational motion as:

$$s_\mathbf{r} = \alpha_\mathbf{r}\dot{\tilde{\mathbf{r}}} + \lambda_\mathbf{r}\tilde{\mathbf{r}} \qquad (11)$$

The main vehicle rotation is coupled to the appendage rotations through the equations of motion and by defining the remaining 8 surfaces according to Eq. (4) as:

$$\mathbf{s} = \alpha_\phi\dot{\tilde{\boldsymbol{\phi}}} + \lambda_\phi\tilde{\boldsymbol{\phi}} + \alpha_\theta\dot{\tilde{\boldsymbol{\theta}}} + \lambda_\theta\tilde{\boldsymbol{\theta}} \qquad (12)$$

If we set up the 11 surface equations (Eqs. 11 and 12) and the 3 unactuated equations of motion (Eq. 10) corresponding to $\boldsymbol{\theta}$ according to Eq. (8), and linearize about any equilibrium point, the resulting linear system will have $2r$ poles equal to zero and is only marginally stable. Therefore, we have to let some of the appendages stabilize at points other than our desired equilibrium to stabilize $\boldsymbol{\theta}$ at values the desired point. This problem can be overcome by changing $\boldsymbol{\phi}$ at a later time. This could change $\boldsymbol{\theta}$ again to an undesired value. We can, however, take advantage of system's physical properties and achieve the desired equilibrium for both $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ through a repetitive process. Details of such a procedure are not in the scope of this report and should be the subject of further research.

C. Simulations

The relevant geometric data are: $l_0 = w_0 = 1.8\,\mathrm{m}$, $l_s = 9\,\mathrm{m}$, $l_r = 0.9\,\mathrm{m}$. We have assumed the main vehicle to be a hollow cube with thickness .15m and density of 2700 kg/m^3. The solar arrays are assumed to be rectangular with width 9m, thickness: .0375m and density of 247 kg/m^3. The rods are have a radius .075m and density 1819 kg/m^3. The antenna dishes have a disk shape with radius l_r, thickness .05 m, and density 1819 kg/m^3.

The control parameters were selected based on the stability criteria discussed earlier; i.e. precision attitude control while stabilizing the appendages at some equilibrium point. The control (and surface) parameters, which are not yet optimized, are $\varepsilon = .2, \eta = .01$, $\alpha_\mathbf{r} = 1$, $\lambda_\mathbf{r} = 2$, $\alpha_\phi = .001, \lambda_\phi = 0, \lambda_\theta = 2\alpha_\theta$, and

$$\alpha_\theta = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The system must start from rest and go back to rest at the desired position. The initial positions are $\mathbf{r}(0) = \boldsymbol{\theta}(0) = 0$ and $\phi(0) = [\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{6}, -\frac{\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{3}]^T$.

The desired position for the main vehicle in all cases is the origin. There are no desired positions for the articulated coordinates except the fact that they must reach equilibrium.

We have tried three individual cases where, in each case, a single main vehicle rotation is desired while keeping the other two rotation angles constant. The desired roll motion angles are $\theta_x^d = 5^\circ, 10^\circ, 15^\circ, -5^\circ$ & $\theta_y^d = \theta_z^d = 0$.

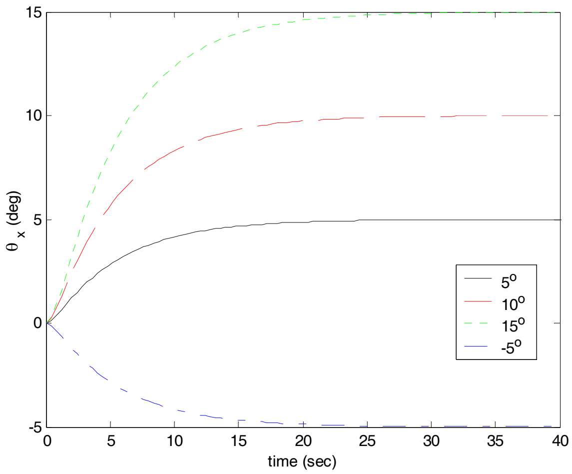

Fig. 3: Main vehicle roll angles for different desired

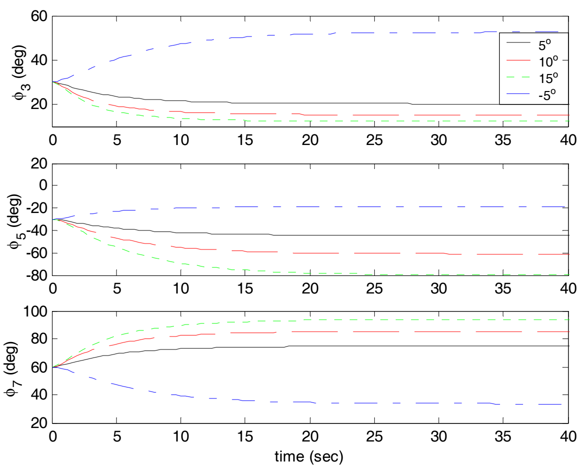

Fig. 4: Articulated angle time histories for different desired roll angles

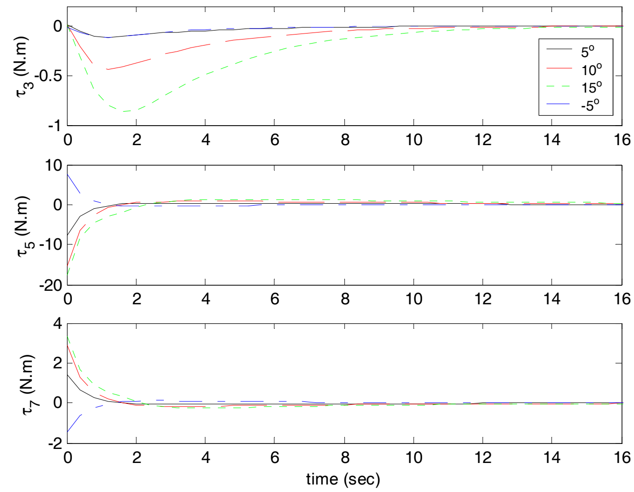

Fig. 5: Control torques of articulated angles for different desired roll angles

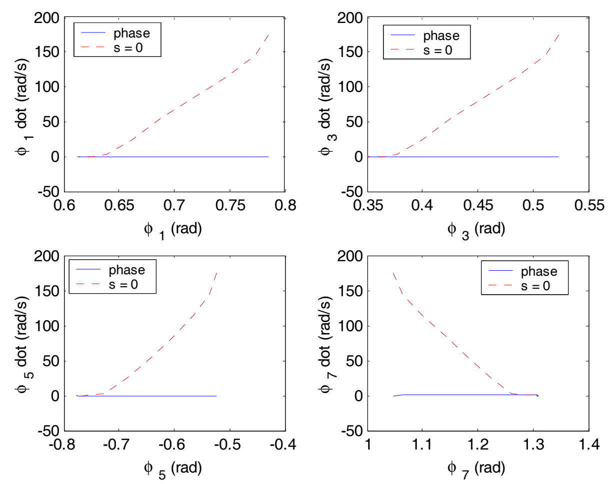

Fig. 6: Articulate angles phase plots for 5º roll angle

Figures 3 shows the time history of the main vehicle rotations for different desired roll angles. Figure 4 shows the time history of the 3 of the articulated angles during the roll motion.

Clearly, the controller is able rotate and stabilize the main vehicle in 30 seconds without requiring large appendage rotations. Figure 5 shows the required control torque for three of the appendages, which are reasonable and within the output torque range of commonly biaxial gimbals. Figure 6 shows four of the appendage trajectories reaching and sliding along the surfaces during the 5^o roll motion. Figures 7 and 8 verify the controller performance for 5^o pitch and yaw angle rotations, respectively.

IV. CONCLUSIONS

A general approach to sliding control of underactuated nonlinear multibody mechanical system was presented. The approach constructs as many first order surfaces as there are controllers from the tracking position and velocity errors of both actuated and unactuated coordinates. Lyapunov theory was used to develop the controller and guarantee all trajectories will reach and remain on the surface. Asymptotic stability of the surface was not established for the general case. However, the surface can be made locally asymptotically stable around the equilibrium points through proper selection of the surface parameters when potential energy is present. The surface can be made only marginally locally stable when there is no potential energy. The controller was successfully applied for attitude control of a complicated satellite system through its appendages. The system was only marginally stable which restricted us to only stabilize the appendages while controlling the exact orientation of the main vehicle. We are currently developing a multi-stage sliding controller to maintain the appendage position at desired values while reorienting the main vehicle.

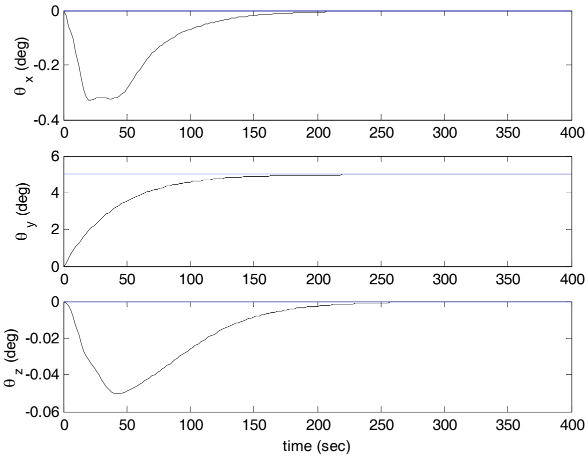

Fig. 7: Main vehicle Euler angles during pitch motion

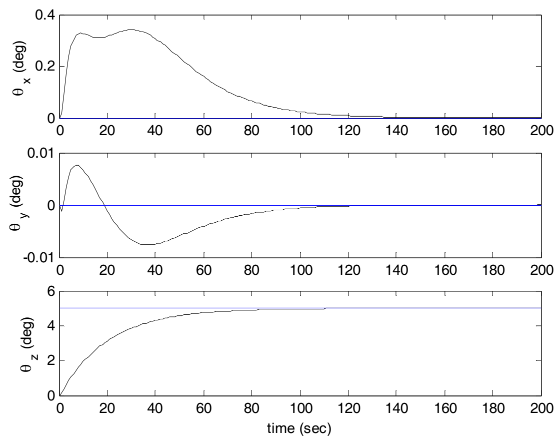

Fig. 8: Main vehicle Euler angles during yaw motion

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