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Adaptive Rejection of Periodic & Non-periodic Disturbances for a Class of Nonlinearly Parametrized Systems

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***Abstract*— In this paper, Lyapunov-based adaptive control is presented for a class of nonlinearly parametrized systems in the presence of periodic disturbances. Through the use of an integral Lyapunov function, the controller singularity problem is elegantly solved as it avoids the nonlinear parametrization from entering into the adaptive control and repetitive control. Both global stability of the adaptive system and asymptotic convergence of the tracking error are established, and tracking error bounds are provided to quantify the control performance analytically.**

I. INTRODUCTION

Adaptive control has been extensively studied in the literature for nonlinear systems that are linear-in-the-parameters. However, only few results available for nonlinear systems that are nonlinearly parametrized owing to its dif£culty in analysis and design though nonlinear parametrization is common in many control applications such as fermentation processes [1], bioreactor processes [2], and friction dynamics [3]. Adaptive control for nonlinearly parametrized systems has been an important and challenging area. In [4], a globally stable output-feedback controller was developed using high-gain adaptation for nonlinearly parametrized systems with known and constant relative degree. Via Lyapunov synthesis, an interesting control design was provided for a class of £rst-order nonlinearly parametrized plants similar to those arising in fermentation processes [1]. In [5], based on a min-max optimization strategy, a novel control scheme was investigated for nonlinear systems with convex/concave parametrization. Recently, a family of integral Lyapunov functions is used to avoid the control singularity problem in feedback linearization-based designs, and to design the derect adaptive controller for a class of nonlinearly parametrized systems [6].

Many practical systems such as batch processes perform repeatable tasks and are commonly subject to periodic disturbances [7]. Perfect tracking for such tasks may not be achieved by the aforementioned adaptive control designs. Learning control and repetitive control are the alternative methods to address this problem. Both methods exploit the repetitive features for improving system performance. Learning control is formulated as that a single £nite horizon tracking task is repeatedly performed, and for each operation cycle the system is returned to the same initial condition. Repetitive control is for the periodic reference trajectory and disturbance with a known period, and there is no initial repositioning between successive periods. For more details, refer to [8], [9], [10] and references therein. In the literature, learning control without initial repositioning is fundamentally the same as repetitive control. Thus, we are not going to distinguish them, but take them as the same, and call this approach, repetitive control in this paper with the above understanding. There have been attempts made to develop such schemes [11], [12], [13], [14]. Lyapunov-based saturated learning approaches were presented for robotic manipulators in [11] and for general nonlinear error dynamics in [13]. Learning algorithms were systematically developed in [12], based on kernel and in¤uence functions. In [14], non-linear iterative learning was developed elegantly by using Lyapunov adaptive techniques, and the dead-zone modi£cation for robust parameter adaptation. Adaptive iterative learning control has been studied for a class of nonlinear systems in strict feedback form [24].

The key in the Lyapunov method is the choice of Lyapunov function. The resulting controller is not unique and the control performance varies with the choice of the function [15]. In this paper, the integral Lyapunov function, proposed in [6], [15] and discussed further in [16], [17], is utilized for the repetitive control design. The developed adaptive repetitive controller is applicable to a class of nonlinearly parametrized systems. Note that efforts have been made for the balanced incorporation of adaptive control and learning control. For robotic manipulators, different update laws were introduced for parameter adaptation, e.g., in iteration domain [18] and in time domain [19]. In [20], the learning rules were proposed for PID gains estimation. The research results for nonlinear systems can be found in [14]. However, these works are all for systems that are linear-in-the-parameters. Compared with the previous works, the main contributions of the paper lie in:

(i) the use of the integral Lyapunov function in avoiding the nonlinear parametrization from entering into the repetitive controller, consequently avoiding the possible singularity problem,
(ii) the combination of repetitive control and adaptive control leading to the globally stable adaptive system, and
(iii) the performance analysis providing the explicit bounds on the tracking error.

The rest of the paper is organized as follows. The problem formulation is given in Section 2. In Section 3, non-adaptive repetitive control is presented for nonlinearly parameterized systems. The adaptive one is developed by using general

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σ-modi£cation parameter estimator and the performance analysis is presented in Section 4 followed by Section 5, which concludes the work.

II. Problem Formulation

Consider a class of single-input single-output (SISO) nonlinear systems described by

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_n = \frac{1}{b(x)}[f(x) + g(x)(u + \delta(t))] \\ y \ = x_1 \end{cases} \tag{1}$$

where $x = [x_1, x_2, ..., x_n]^T = [y, \dot{y}, \cdots, y^{n-1}]^T \in R^n$, $u \in R$ and $y \in R$ are the state variables, system input and output, respectively; $\delta(t) \in R$ is the disturbance to the system; $g(x)$ is a known continuous function; functions $f(x), b(x) \in C^1$ can be expressed as

$$f(x) = \theta^T w_f(x) + f_0(x), \; b(x) = \theta^T w_b(x) + b_0(x)$$

where $\theta \in R^p$ is a vector of unknown constant parameters, $w_f(x) \in R^p$ and $w_b(x) \in R^p$ are known regressor vectors, and $f_0(x), b_0(x) \in C^1$ are known functions. The parameter θ enters into the parameterized system (1) nonlinearly due to the parametrization of $b(x)$. Denote $x_d(t) = [y_d(t), \dot{y}_d(t), \cdots, y_d^{n-1}(t)]^T$, where $y_d(t)$ is the desired output to be tracked.

Assumption 1: The desired trajectory $x_d(t)$ is of periodicity with known period T and $\delta(t)$ is also periodic with the same period, i.e., $x_d(t) = x_d(t-T)$ and $\delta(t) = \delta(t-T)$.

Remark 1: Assumption 1 implies that the desired output $y_d(t)$ and its derivatives up to $(n-1)$th order as well as the disturbance $\delta(t)$ are of the periodicity.

The control objective of this paper is described as follows: Given the desired trajectory $x_d(t)$ for nonlinearly parameterized system (1) in the presence of the disturbance $\delta(t)$, design an adaptive repetitive controller such that the system follows the desired trajectory, while all the states and the control remain bounded.

In Section III, the non-adaptive controller is £rst developed for system (1) in the presence of the periodic disturbance, and then non-adaptive control is considered for systems with non-periodic uncertainty. For simplicity of presentation, in Section IV, the adaptive control design is presented only for system (1). The same design procedure can be taken for the systems with non-periodic uncertainty.

III. Non-Adaptive Repetitive Control

In this section, the control design is given by assuming that θ is known. De£ne the £ltered error e_f as

$$e_f = [\Lambda^T \; 1]e(t), \; e = [e_1, e_2, \cdots, e_n] = x - x_d$$

where $\Lambda = [\lambda_1, \lambda_2, \cdots, \lambda_{n-1}]^T$ is chosen such that the polynomial $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz. For the control design, the following scalar function is chosen [15]

$$V_f = \int_0^{e_f} \sigma b_\alpha(\bar{x}_{n-1}, \sigma + \nu_1) d\sigma$$

with

$$\bar{x}_{n-1} = [x_1, x_2, \cdots, x_{n-1}]^T,$$
$$\nu_1 = y_d^{(n-1)} - [\Lambda^T, \; 0]e$$

and

$$b_\alpha(x) = b(x)\alpha(x).$$

Function V_f is referred to as the integral Lyapunov function and $\alpha(x)$ is the smooth weighting function. The integral Lyapunov function is used to remove the possible singularity problem and offers a design tool to retain some system's nonlinearites. In fact, there is no necessity to cancel all the nonlinearities for a stable closed-loop. In many cases some nonlinearities of the system may be helpful for achieving control objective. Examples for choosing V_f are given in [15]. In this paper, both $b(x)$ and $g(x)$ are assumed to be strictly positive. Without losing generality, the following assumption is made.

Assumption 2: $\alpha(x) > 0$ is chosen such that there exist positive constants $g_{\alpha,0}$, $b_{\alpha,1}$ and $b_{\alpha,0}$ satisfying that $g_\alpha(x) \geq g_{\alpha,0} > 0$ and $b_{\alpha,1} \geq b_\alpha(x) \geq b_{\alpha,0} > 0$, for all $x \in R^n$.

Lemma 1: Under Assumption 2, V_f satis£es the following inequality:

$$\frac{1}{2} b_{\alpha,0} e_f^2 \leq V_f \leq \frac{1}{2} b_{\alpha,1} e_f^2$$

Proof: The proof is straitforward by the de£nition of V_f and Assumption 2. ∎

From (1), the time derivative of e_f can be written as

$$\dot{e}_f = \frac{1}{b(x)}[f(x) + g(x)(u + \delta(t))] + \nu \tag{2}$$

where $\nu = -y_d^{(n)} + [0 \; \Lambda^T]e$.

Differentiating V_f along (2) yields

$$\begin{aligned} \dot{V}_f &= \frac{\partial V_f}{\partial e_f}\dot{e}_f + \frac{\partial V_f}{\partial \bar{x}_{n-1}}\dot{\bar{x}}_{n-1} + \frac{\partial V_f}{\partial \nu_1}\dot{\nu}_1 \\ &= \alpha(x) e_f[\theta^T w(z) + g(x)(u + \delta(t)) + h(z)] \end{aligned} \tag{3}$$

where

$$z = [x^T, x_d^T, y_d^n]^T,$$

and

$$\begin{aligned} w(z) &= w_f(x) \\ &\quad + \frac{1}{e_f \alpha(x)} \int_0^{e_f} \left[\sigma \sum_{i=0}^{n-1} \frac{\partial w_{b,\alpha}(\bar{x}_{n-1}, \sigma + \nu_1)}{\partial x_i} x_{i+1} \right. \\ &\quad \left. + \nu w_{b,\alpha}(\bar{x}_{n-1}, \sigma + \nu_1) \right] d\sigma \\ h(z) &= f_0(x) \\ &\quad + \frac{1}{e_f \alpha(x)} \int_0^{e_f} \left[\sigma \sum_{i=0}^{n-1} \frac{\partial b_{0,\alpha}(\bar{x}_{n-1}, \sigma + \nu_1)}{\partial x_i} x_{i+1} \right. \\ &\quad \left. + \nu b_{0,\alpha}(\bar{x}_{n-1}, \sigma + \nu_1) \right] d\sigma \end{aligned} \tag{4}$$

Following the design in [15], u is chosen as

$$u = \frac{1}{g(x)}\left[-\kappa\frac{e_f}{\alpha(x)} - \theta^T w(z) - h(z)\right],\ \kappa > 0 \qquad (5)$$

which renders (3) to

$$\dot{V}_f = -\kappa e_f^2 + g_\alpha(x) e_f \delta(t) \qquad (6)$$

Remark 2: Owing to the presence of $\delta(t)$, it is clear that the asymptotic stability cannot be guaranteed even if $\delta(t)$ is periodic. Therefore, model-based controller (5) cannot solve the complete disturbance rejection problem.

Repetitive control is now incorporated in the feedback control in order to solve the problem. Let us consider the controller

$$u = u_r + u_f \qquad (7)$$

where u_f is the feedback control given by (5) and u_r is the repetitive control given by

$$u_r(t) = -\hat{\delta}(t) \qquad (8)$$

The disturbance estimation is updated based on the following learning law

$$\hat{\delta}(t) = \begin{cases} \mathrm{sat}(\hat{\delta}_e(t)) + \gamma g_\alpha(x(t)) e_f(t), & t > 0 \\ 0, & t \in [-T, 0] \end{cases} \qquad (9)$$

where $\gamma > 0, \gamma g_\alpha(x)$ is the learning gain, $\hat{\delta}_e(t) = \hat{\delta}(t-T)$ and $\mathrm{sat} : R \to R$ is the saturation function de£ned by

$$\mathrm{sat}(\hat{\delta}_e(t)) = \begin{cases} \hat{\delta}_e(t), & |\hat{\delta}_e(t)| \le \bar{\delta} \\ (\hat{\delta}_e(t)/|\hat{\delta}_e(t)|)\bar{\delta}, & |\hat{\delta}_e(t)| > \bar{\delta} \end{cases}$$

where $\bar{\delta}$ is the saturation bound satisfying that $\bar{\delta} \ge \delta_0$ and $\delta_0 = \sup_{t\in[-T,0]} \delta(t)$.

Lemma 2: Under Assumptions 1 and 2, all the signals in the closed-loop system consisting of plant (1) and repetitive controller (7) are globally uniformly bounded, and the error between the actual and the desired trajectories converges to zero asymptotically as $t \to \infty$, i.e., $\lim_{t\to\infty} e(t) = 0$.

Proof: Combining (1) and (7), the time derivative of e_f can be written as

$$\dot{e}_f = \frac{1}{b(x)}[f(x) + g(x)(u_f + \tilde{\delta}(t))] + \nu \qquad (10)$$

with

$$\tilde{\delta}(t) = \delta(t) - \hat{\delta}(t),$$

which renders (6) to

$$\dot{V}_f = -\kappa e_f^2 + g_\alpha(x) e_f \tilde{\delta}(t) \qquad (11)$$

De£ne the Lyapunov function candidate

$$V_1(t) = V_r(t) + V_f(t)$$

with

$$V_r(t) = \frac{1}{2\gamma}\int_{t-T}^{t} \tilde{\delta}^2(\tau)d\tau.$$

The derivative of V_1 with respect to time can be computed as, keeping in mind of (11),

$$\begin{aligned}
\dot{V}_1(t) &= \dot{V}_r(t) + \dot{V}_f(t) \\
&= \frac{1}{2\gamma}\{[\delta(t) - \hat{\delta}(t)]^2 - [\delta(t-T) - \hat{\delta}(t-T)]^2\} \\
&\quad + \dot{V}_f(t) \\
&\le \frac{1}{2\gamma}\{[\delta(t) - \hat{\delta}(t)]^2 - [\delta(t) - \mathrm{sat}(\hat{\delta}(t-T))]^2\} \\
&\quad + \dot{V}_f(t) \\
&= -\frac{1}{2\gamma}\{[\hat{\delta}(t) - \mathrm{sat}(\hat{\delta}(t-T))]^2 \\
&\quad + 2\tilde{\delta}(t)[\hat{\delta}(t) - \mathrm{sat}(\hat{\delta}(t-T))]\} + \dot{V}_f(t) \\
&= -\frac{1}{2\gamma}[\gamma^2 g_\alpha^2(x(t)) e_f^2(t) + 2\gamma g_\alpha(x(t)) e_f(t)\tilde{\delta}(t)] \\
&\quad - \kappa e_f^2(t) + g_\alpha(x(t)) e_f(t)\tilde{\delta}(t) \\
&= -\left(\frac{\gamma}{2} g_\alpha^2(x(t)) + \kappa\right) e_f^2(t)
\end{aligned} \qquad (12)$$

Because $b_\alpha(x) \in C^1$, V_f is a C^1 function of x and x_d. This ensures that $V_f(0) \in L_\infty$ for any bounded initial values $x(0)$ and $x_d(0)$.

From (12), $\dot{V}_1 \le 0$, which implies $V_1(t) \le V_1(0)$. Thus, $V_1 \in L_\infty$ and $V_f \in L_\infty$ as well. By Assumption 2, integrating (12) leads to

$$\int_0^t e_f^2(\tau)d\tau \le \frac{2}{\gamma g_{\alpha,0} + 2\kappa}(V_1(0) - V_1(t)),$$

which implies $e_f \in L_2$. In addition, by Lemma 1, $e_f \in L_\infty$. It follows that $e \in L_\infty$ from the de£nition of e_f, and $x \in L_\infty$ from the boundedness of x_d. It is easy to check that $u_f \in L_\infty$ from (5), $\tilde{\delta}(t) \in L_\infty$ from (9), and $\dot{e}_f \in L_\infty$ from (10). Therefore, by Barbalat's lemma, $\lim_{t\to\infty} e_f(t) = 0$, which implies $\lim_{t\to\infty} e(t) = 0$. ■

Remark 3: The choice of the learning laws is not unique in the repetitive control design. For instance, the following learning law can be obtained by saturating the entire right-hand side of (9) as:

$$\hat{\delta}(t) = \begin{cases} \mathrm{sat}(\hat{\delta}_e(t)), & t > 0 \\ 0, & t \in [-T, 0] \end{cases} \qquad (13)$$

where $\hat{\delta}_e(t) = \hat{\delta}(t-T) + \gamma g_\alpha(x(t)) e_f(t)$, which keeps $\hat{\delta}(t)$ within the saturation bound for all time. Computing the derivative of V_r gives rise to

$$\begin{aligned}
\dot{V}_r(t) &= \frac{1}{2\gamma}[\tilde{\delta}^2(t) - \tilde{\delta}^2(t-T)] \\
&= -\frac{1}{2\gamma}[\hat{\delta}(t) - \hat{\delta}(t-T)][2\delta(t) \\
&\quad - \hat{\delta}(t) - \hat{\delta}(t-T)] \\
&= -\frac{1}{2\gamma}[\hat{\delta}(t) - \hat{\delta}(t-T)][2\tilde{\delta}(t) \\
&\quad + \hat{\delta}(t) - \hat{\delta}(t-T)]
\end{aligned} \qquad (14)$$

Cancelling the term $-\frac{1}{2\gamma}[\hat{\delta}(t)-\hat{\delta}(t-T)]^2$ results in

$$\dot{V}_r(t) \quad \leq \quad -\frac{1}{\gamma}[\hat{\delta}(t)-\hat{\delta}(t-T)]\tilde{\delta}(t)$$

According to the de£nition of saturation function, it can be seen that

$$[\hat{\delta}(t)-\hat{\delta}(t-T)]\tilde{\delta}(t) \geq \gamma g_\alpha(x(t))e_f(t)\tilde{\delta}(t) \tag{15}$$

Thus,

$$\dot{V}_r(t) \leq -g_\alpha(x(t))e_f(t)\tilde{\delta}(t),$$

which leads to

$$\dot{V}_1(t) \leq -\kappa e_f^2(t) \tag{16}$$

Therefore, Lemma 2 still holds if one replaces (9) with (13).

Remark 4: Comparing equations (12) and (16), both the equations have the same control parameter $\kappa > 0$ to guarantee the stability of the closed-loop systems, and (12) has an additional parameter for adjusting the bound on $\dot{V}_1(t)$ by γ. The larger the γ is, the less the bound is. Increasing γ may improve the convergence rate of learning law (9). On the other hand, (16) does not confer such a bene£t through the only change made is the learning law, changed from (9) to (13). This small change leads to different treatment and different performance. This is the case actually due to the cancellation of the term in (14) and the use of inequality (15).

In this paper, we give a general method for adaptive repetitive control for such a class of systems, and both the different learning laws can guarantee the convergence of the tracking errors. However, the two different learning laws are by no means exclusive. Other forms exist, and modi£cations are possible. We are not going to elaborate further as it is not the purpose of the paper.

Now we consider the control design for the following class of nonlinear uncertain systems

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad i=1,2,\cdots,n-1 \\ \dot{x}_n = \frac{1}{b(x)}[f(x)+g(x)(u+\Delta(\varsigma,x,t)+\delta(t))] \\ y \ = x_1 \end{cases} \tag{17}$$

where $\Delta(\varsigma,x,t)$ represents the matching non-periodic uncertainty and ς is the uncertain variable on a pre-speci£ed compact set.

Assume that $\|\Delta(\varsigma,x,t)\| \leq \rho(x,t)$, for all $(x,t) \in R^n \times R^+$, and the bound function $\rho(x,t)$ is known. We construct the following controller, by adding u_s to (7),

$$u = u_r + u_f + u_s, \tag{18}$$

$$u_s = -\rho \mathrm{sign}(e_f) \tag{19}$$

which renders (10) to

$$\begin{aligned} \dot{e}_f \quad = \quad & \frac{1}{b(x)}[f(x)) \\ & +g(x)(u_f+u_s+\Delta(\varsigma,x,t)+\tilde{\delta}(t))]+\nu \end{aligned} \tag{20}$$

This in turn results in

$$\dot{V}_f \leq -\kappa e_f^2 + g_\alpha(x)e_f\tilde{\delta}(t) \tag{21}$$

To complete the stability and convergence analysis, we can follow the lines similar to those after (11) in the proof of Lemma 2.

Remark 5: In the above, we present a way to deal with the uncertain systems in the spirit of robust control. One may only apply robust control to solve the problem, for example, by choosing

$$u_s = -(\rho+\delta_0)\mathrm{sign}(e_f)$$

and

$$u_r = 0.$$

Asymptotic tracking can still be achieved, but at the price of high control chattering, comparing with our design using u_r. Thus the introduction of u_r is helpful for reducing chattering. Robust control is one well-known deterministic approach to treat the uncertainties in a dynamic system. Upper bounds on the uncertainties play an important role in the control design. In the published literature, the design techniques are available for developing continuous robust controllers. Following these techniques, we can develop a continuous controller to replace the discontinuous one so that the resulting closed-loop system is uniformly ultimately bounded. However, asymptotic tracking cannot be achieved even in the presence of the periodic disturbance.

IV. ADAPTIVE REPETITIVE CONTROL

In the case of unknown θ, the repetitive controller given by (7) is not realisable. We propose the following adaptive repetitive controller

$$u \quad = \quad u_r + \hat{u}_f \tag{22}$$

$$\hat{u}_f \quad = \quad \frac{1}{g(x)}\left[-\kappa\frac{e_f}{\alpha(x)} - \hat{\theta}^T w(z) - h(z)\right] \tag{23}$$

where u_r is given as (8). Denote $\hat{\theta}$ the estimate of θ and $M > \|\theta\|$ an upper bound on $\|\theta\|$. The following parameter update law is used

$$\dot{\hat{\theta}} = -\Gamma[\alpha(x)e_f w(z) + \sigma(\|\hat{\theta}\|)\hat{\theta}] \tag{24}$$

with the general switching σ-modi£cation

$$\sigma(\|\hat{\theta}\|) \quad = \quad \begin{cases} 0, & \text{if } \|\hat{\theta}\| \leq \hat{M} \\ \sigma_m(\|\hat{\theta}\|), & \text{if } \hat{M} < \|\hat{\theta}\| < 2\hat{M} \\ \sigma_0, & \text{if } \|\hat{\theta}\| \geq 2\hat{M} \end{cases} \tag{25}$$

where

$$\sigma_m(\|\hat{\theta}\|) \geq 0,$$

as well as

$$\sigma_m(\hat{M}) = 0 \text{ and } \sigma_m(2\hat{M}) = \sigma_0.$$

$\hat{M}$ is given by the update law

$$\dot{\hat{M}} = \beta\sigma(\|\hat{\theta}\|)\hat{\theta} \tag{26}$$

with $\hat{M}(0) = 0$ and $\beta > 0$.

Typical σ-modi£cation estimation schemes can be found in [21]. Update law (26) was proposed in [22] and is used in the general σ-modi£cation algorithm.

Theorem 1: Under Assumptions 1 and 2, all the signals in the closed-loop adaptive system consisting of plant (1), repetitive controller (22) and adaptive law (24) are globally uniformly bounded, and the error between the actual and the desired trajectories converges to zero asymptotically as $t \to \infty$, i.e., $\lim_{t\to\infty} e(t) = 0$.

Proof: Consider the Lyapunov candidate

$$V_2 = V_1 + \frac{1}{2}\left(\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{\beta}\tilde{M}^2\right)$$

with

$$\tilde{\theta} = \theta - \hat{\theta}$$

and

$$\tilde{M} = \hat{M} - M.$$

The derivative $\dot{V}_1$ can be computed as

$$\dot{V}_1 = -\left(\frac{\gamma}{2}g_\alpha^2(x) + \kappa\right)e_f^2 + \alpha(x)e_f\tilde{\theta}^T w \qquad (27)$$

and then the derivative $\dot{V}_2$ can be calculated as

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} + \frac{1}{\beta}\tilde{M}\dot{\tilde{M}} \\ &= \dot{V}_1 - [\alpha(x)e_f\tilde{\theta}^T w + \sigma(\|\hat{\theta}\|)\tilde{\theta}^T\theta] \qquad (28)\\ &\quad +\sigma(\|\hat{\theta}\|)\|\hat{\theta}\|\tilde{M} \\ &= \dot{V}_1 - \alpha(x)e_f\tilde{\theta}^T w - \sigma(\|\hat{\theta}\|)\|\hat{\theta}\|(\|\hat{\theta}\| - \hat{M}) \\ &\quad -\sigma(\|\hat{\theta}\|)(M\|\hat{\theta}\| - \theta^T\hat{\theta})\end{aligned}$$

From the de£nition of $\sigma(\|\hat{\theta}\|)$, it is seen that

$$\begin{aligned}&-\sigma(\|\hat{\theta}\|)(M\|\hat{\theta}\| - \theta^T\hat{\theta}) \\ &\quad\le -\sigma(\|\hat{\theta}\|)(M\|\hat{\theta}\| - \|\theta\|\|\hat{\theta}\|) \le 0 \\ &-\sigma(\|\hat{\theta}\|)\|\hat{\theta}\|(\|\hat{\theta}\| - \hat{M}) \le 0 \qquad (29)\end{aligned}$$

Keeping in mind of (27), we have

$$\dot{V}_2 \le \dot{V}_1 - \alpha(x)e_f\tilde{\theta}^T w \le -\left(\frac{\gamma}{2}g_\alpha^2(x) + \kappa\right)e_f^2 \qquad (30)$$

From (30), we know

$$\dot{V}_2 \le 0,$$

which implies

$$V_2(t) \le V_2(0).$$

Since $V_2(0) \in L_\infty$, then $V_2 \in L_\infty$. Thus, $V_1 \in L_\infty$, $\tilde{\theta} \in L_\infty$ and $\tilde{M} \in L_\infty$. By Assumption 2, integrating (30) leads to

$$\int_0^t e_f^2(\tau)d\tau \le \frac{2}{\gamma g_{\alpha,0} + 2\kappa}(V_2(0) - V_2(t)),$$

which implies $e_f \in L_2$.

In addition, by Lemma 1, $e_f \in L_\infty$. It follows from the de£nition of e_f that $e \in L_\infty$, and $x \in L_\infty$ from the boundedness of x_d. It is easy to check that $\hat{u}_f \in L_\infty$ and $\dot{e}_f \in L_\infty$. Therefore, by Barbalat's lemma, $\lim_{t\to\infty} e_f(t) = 0$, which implies that $\lim_{t\to\infty} e(t) = 0$. ■

Theorem 1 implies boundedness of all the signals in the closed-loop system and asymptotic convergence of the output error. The following theorem speci£es both the root-mean-square and the L_∞ bounds for the tracking error.

Theorem 2: If the closed-loop system (1), (22) and (24) satisfy Assumptions 1 and 2, then, for $n \ge 2$,

i) the root-mean-square tracking error bound is given by

$$\begin{aligned}\sqrt{\frac{1}{t}\int_0^t e_1^2(\tau)d\tau} &\le \frac{k_0}{\sqrt{2\lambda_0 t}}\|\zeta(0)\|\sqrt{1 - e^{-2\lambda_0 t}} \\ &\quad + \frac{k_0}{\sqrt{\lambda_0(\gamma g_{\alpha,0}^2 + 2\kappa)}}\sqrt{\bar{V}_2(0)} \qquad (31)\end{aligned}$$

where both $k_0 > 0$ and $\lambda_0 > 0$ are computable constants,

$$\zeta(0) = [e_1(0), e_2(0), \cdots, e_{n-1}(0)]^T$$

and

$$\begin{aligned}\bar{V}_2(0) &= \frac{1}{2}\Big(\frac{1}{\gamma}T\delta_0^2 + b_{\alpha,1}e_f^2(0) \qquad (32)\\ &\quad + \tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) + \frac{1}{\beta}M^2\Big),\end{aligned}$$

ii) L_∞ tracking error bound is given by

$$\begin{aligned}|e_1(t)| &\le k_0\|\zeta(0)\|e^{-\lambda_0 t} \\ &\quad + \frac{k_0}{\sqrt{\lambda_0(\gamma g_{\alpha,0}^2 + 2\kappa)}}\sqrt{\bar{V}_2(0)} \qquad (33)\end{aligned}$$

Proof: Due to the space limitation, the complete prove is provided in [23]. ■

Remark 6: In Theorem 2, the error bounds are obtained for $n \ge 2$. In the case of $n = 1$, $e_f = e_1$. The L_2 bound for e_1 can be expressed as

$$\int_0^\infty e_1^2(\tau)d\tau \le \frac{2}{\gamma g_{\alpha,0}^2 + 2\kappa}\bar{V}_2(0) \qquad (34)$$

By Lemma 1, $V_f \ge \frac{1}{2}b_{\alpha,0}e_1^2$. Since

$$V_f(t) \le V_2(t) \le V_2(0) \le \bar{V}_2(0) \qquad (35)$$

then the L_∞ bound for e_1 can be given as

$$|e_1(t)| \le \frac{2}{b_{\alpha,0}}\bar{V}_2(0) \qquad (36)$$

Remark 7: From the de£nitions of ζ and e_f, $e = [\zeta^T,\ e_f - \Lambda^T\zeta]^T$, which implies $\|e\| \le (1 + \|\Lambda\|)\|\zeta\| + \|e_f\|$ [15]. The boundedness of states can be concluded by Lemma 1 and Theorem 2.

Furthermore, through suitably choosing design parameters, the bounds on states are adjustable and subsequently can be guaranteed within some compact subset of R^n. The assumption that $b_\alpha(x) \le b_{\alpha,1}$, for all $x \in R^n$, might be a strong restriction, but can be relaxed as argued due to the boundedness of states. For example, $b_\alpha(x) \le b_{\alpha,1}$ holds on the given compact set. We can also suppose that Assumption 2 holds on the same set, not on whole state space.

Remark 8: It is clear from (31) and (33) that the tracking error bounds depend on initial state error and design parameters γ and/or κ. Smaller steady-state tracking error bounds can be obtained by choosing larger γ and/or κ. However, careful adjustment should be made for achieving suitable transient response and control effort.

Remark 9: The update term $\gamma g_\alpha(x)e_f$ of (9) is independent of both $f(x)$ and $b(x)$. Thus the parametrization does not enter into the learning law (9). Owing to the use of the integral Lyapunov function. The situation may be different if other kind of Lyapunov functions is utilized,

$$V_f = \frac{1}{2}e_f^2.$$

The derivative of V_f along (10) can be calculated as

$$\begin{aligned} \dot{V}_f &= e_f \dot{e}_f \qquad (37) \\ &= e_f \left(\frac{1}{b(x)}[f(x) + g(x)(u_f + \tilde{\delta}(t))] + \nu \right) \end{aligned}$$

This suggests the choice of

$$u_f = \frac{1}{g(x)}(-\kappa b(x)e_f - f - b(x)\nu),$$

which leads to

$$\dot{V}_f = -\kappa e_f^2 + \frac{g(x)}{b(x)}e_f\tilde{\delta}(t) \qquad (38)$$

To cancel the second term on the right hand side of (38), we have to choose the update term as $\gamma\frac{g(x)}{b(x)}e_f$. The parametrization of $b(x)$ consequently enters into the learning law, which may result in a possible singularity problem due to the estimate $\hat{b}(x)(=\hat{\theta}^T w_b(x)+b_0(x))$ close to zero.

V. Conclusion

In this paper, Lyapunov based adaptive repetitive control has been proposed for systems with nonlinear parametrization. The controller singularity problem is solved as the use of the integral Lyapunov function avoids the nonlinear parametrization from entering into the adaptive control and repetitive control. Asymptotic convergence of the tracking error is established in the presence of periodic disturbances, while the global stability of the closed-loop system is guaranteed. Tracking error bounds have been provided to characterize the control performance.

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