Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Robust Input-output Decoupling Control for Induction Motors

Huangang Wang , Wenli Xu, Teilong Shen and Geng Yang

***Abstract*— A robust input-output decoupling control strategy for stator flux and torque of induction motors (IM) is proposed. In order to avoid full state measurements, robust decoupling controllers of stator flux and torque are developed. Experimental results are presented.**

I. INTRODUCTION

In the past two decades, several feedback control approaches based on input-output decoupling and linearization have been proposed for the induction motors (IM) (see [1] and references therein). As is well-known, the classical field oriented control (FOC) has been improved by achieving exact input-output decoupling and linearization via state feedback with the change of coordinates [2], [3]. Meanwhile, the attention has been focused on the direct torque control (DTC) problem for IM [4], [5]. Different from FOC, the control objective of DTC is expressed in terms of torque and stator flux regulation. Besides its simplicity, it is claimed that the achieved performance of DTC is (in some instances) superior to FOC for the robustness with respect to the parameter variation [6]. This paper proposes a robust input-output decoupling control strategy for torque and stator flux of IM. The experimental results are presented.

II. DYNAMICAL MODEL AND PROBLEM FORMULATION

The dynamics of IM in the fixed stator reference frame (α, β) can be described by

$$\begin{aligned} \dot{i}_s &= \frac{R_r\phi_s}{\sigma L_s L_r} - \mu i_s + \omega_r \mathcal{J}(i_s - \frac{\phi_s}{\sigma L_s}) + \frac{1}{\sigma L_s} u_s \\ \dot{\phi}_s &= -R_s i_s + u_s \\ \dot{\omega}_r &= (T_e - T_L)/D_m, \quad T_e = i_s^T \mathcal{J} \phi_s \end{aligned} \tag{1}$$

where the subscripts s and r stand for stator and rotor quantities, ϕ, i and u_s denote flux linkage, current and stator voltage vectors, respectively. R, L and M denote the resistance of the self and mutual inductance. ω_r is the rotor speed, D_m is the moment of inertia, T_e is the electromagnetic torque and T_L is the load torque, and

$$\begin{aligned} &\sigma = 1 - M^2/L_s L_r \\ &\mu = (R_s L_r + R_r L_s)/(L_s L_r - M^2) \\ &\mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

This work was supported by the Doctoral Dissertation Founding of Tsinghua University (9809)

Huangang Wang, Wenli Xu and Geng Yang are with the Department of Automation, Tsinghua University, Beijing, 100084, China. wanghuangang99@mails.tsinghua.edu.cn

Teilong Shen is with the Department of Mechanical Engineering, Sophia University, Tokyo, 102-8554, Japan. tetu-sin@sophia.ac.jp

Define $\psi_s = |\phi_s|$ and $\psi_r = |\phi_r|$. Then with pre-feedback

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = \begin{bmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{bmatrix} \begin{bmatrix} u_\psi \\ u_T \end{bmatrix} \tag{2}$$

the system (1) can represented as

$$\begin{aligned} &\dot{\psi}_s = -aR_s L_r \psi_s + aR_s M \psi_r \cos\gamma + u_\psi \\ &\psi_s \dot{\theta}_s = -aR_s M \psi_r \sin\gamma + u_T \\ &\dot{\psi}_r = -aR_r L_s \psi_r + aR_r M \psi_s \cos\gamma \\ &\psi_r \dot{\theta}_r = aR_r M \psi_s \sin\gamma + \psi_r \omega_r \end{aligned} \tag{3}$$

where $a = (L_s L_r - M^2)^{-1}$, θ_s, θ_r denote the angle of stator and rotor flux, respectively, and $\gamma = \theta_s - \theta_r$.

Note that the torque $T_e = aM\psi_s\psi_r \sin\gamma$. Thus we define the output of the system as $y = [\psi_s, T_e]^T$. Then, the control problem considered is as follows: for given y^*, find a feedback controller $u = \alpha(y, y^*)$ with the measurements of the T_e and ψ_s such that $y \to y^*$ holds for any given initial condition of $y(0) \in \mathcal{D}$, where $\mathcal{D}$ is a given range including y^*.

III. CONTROLLER DESIGN

It is easy to show that the system (3) has relative degree $\{1, 1\}$ under the assumption $\psi_s \neq 0$ and $\psi_r \neq 0$, which is based on a physical consideration. Our first result shows that the zero dynamics is asymptotically stable, if the initial conditions and the reference values satisfy the condition described in the following proposition.

Proposition 1: If the output references and the initial conditions of the system (3) satisfy

$$|T_e^*| \leq T_{em} = \frac{M^2\psi_s^{*2}}{2\sigma L_s^2 L_r} \tag{4}$$

$$\cos\gamma > 0 \tag{5}$$

$$\psi_r^2(0) \geq \frac{M^2}{2L_s^2}\psi_s^{*2} - \sigma L_r\sqrt{T_{em}^2 - T_e^{*2}} \tag{6}$$

then, the zero dynamics are asymptotically stable.

Proof of proposition 1: Fixed the outputs of the system (3) at the constant values y^*, then the zero dynamics is as

$$\begin{aligned} \dot{\psi}_r &= -aR_r L_s \psi_r + aR_r M \psi_s^* \cos\gamma \\ \dot{\gamma} &= -R_r \psi_s^*/\psi_r^2 + aR_r L_s \tan\gamma \end{aligned} \tag{7}$$

Define $\chi = \psi_r^2 + \cos^2\gamma$, the time derivatives of χ along the zero dynamics is

$$\dot{\chi} = 2aR_r\left(1 + \sin^2\gamma/\psi_r^2\right)\left(M\psi_s^*\psi_r\cos\gamma - L_s\psi_r^2\right) \tag{8}$$

With (5), we can get that the equilibrium points are

$$\psi_r^{*2} = \frac{M^2}{2L_s^2}\psi_s^{*2} \pm \sigma L_r\sqrt{T_{em}^2 - T_e^{*2}},\ \gamma^* = \arccos\left(\frac{L_s\psi_r^*}{M\psi_s^*}\right)$$

which exist if and only if (4) is satisfied. From the phase diagram of χ and ψ_r we can see that only the larger one (marked as ψ^*_{r+}) is stable. From the initial conditions satisying (6), we have that $\psi_r \to \psi^*_{r+}$ and $\gamma \to \arccos(L_s\psi^*_{r+}/(M\psi^*_s))$.

In order to avoid the exact measurements of the full states, a domination feedback design can be used to develop a robust controller.

Proposition 2: The stator flux tracking error $e_\psi = \psi^*_s - \psi_s$ will converge to zero asymptotically, if u_ψ is given by

$$u_\psi = k_\psi e_\psi + \delta_\psi \text{sign} e_\psi \quad (9)$$

where $k_\psi \geq 0$, δ_ψ is a positive constant and satisfies

$$\delta_\psi \geq 2aR_sL_r\psi^*_s \quad (10)$$

Proof of proposition 2: Define $\Psi = \frac{1}{2}\left(\psi_s^2/R_s + \psi_r^2/R_r\right)$, the time derivative of Ψ is

$$\begin{aligned}\dot{\Psi} &= a(2M\psi_s\psi_r\cos\gamma - L_r\psi_s^2 - L_s\psi_r^2) + \psi_s u_\psi/R_s \\ &\leq -a\sqrt{L_r}(\psi_s - M\psi_r/L_r)^2 - \psi_r^2/L_r + \psi_s u_\psi/R_s\end{aligned}$$

When $\psi_s > \psi^*_s$, $\dot{\Psi} < 0$ holds, therefor, we consider $e_\psi > 0$ only. From the system (3), we have $\dot{\psi}_r < 0$ for all $\psi_r > M\psi_s/L_s$. Notice that $\psi_s < \psi^*_s$, ψ_r will decrease and converge into the subspace $\Omega = \{\psi_r | \psi_r < M\psi^*_s/L_s\}$ for any initial condition $\psi_r(0) > M\psi_s/L_s$. Therefore, in subspace Ω, the time derivative of the Lyapunov function $V_\psi = e_\psi^2/2$ is

$$\dot{V}_\psi < -(\delta_\psi - 2aR_sL_r\psi^*_s)|e_\psi| \leq 0 \quad (11)$$

The proof is completed.

Before the control of electrical torque, we assume that the magnitude of stator flux has converged to the reference value and remains constant, $\psi_s \equiv \psi^*_s$.

Proposition 3: The torque tracking error $e_T = T^*_e - T_e$ will converge to zero asymptotically, if the initial conditions satisfy

$$\cos\gamma(t_0) > 0, \quad \psi_r(t_0) \geq \frac{M\psi^*_s}{\sqrt{2}L_s}, \quad |T_e(t_0)| \leq T_{em} \quad (12)$$

and the reference value satisfies $|T^*_e| < T_{em}$, and u_T is given by

$$u_T = R_sT^*_e/\psi^*_s + k_Te_T + \delta_T \text{sign} e_T \quad (13)$$

where $k_T \geq 0$, positive constant δ_T satisfies

$$\delta_T \geq |\omega_r|\psi^*_s + \left|\frac{2\sigma L_s^2 L_r}{M^2\psi^*_s}\left(\dot{T}^*_e + \frac{R_rT_{em}}{\sigma L_r}\right)\right| \quad (14)$$

Proof of proposition 3: From Proposition 1, if $\cos\gamma > 0$, then $\psi_r \to \psi^{*2}_{r+}$ and $\psi_r^2 \geq M\psi^*_s/(\sqrt{2}L_s)$, $\forall |T_e| \leq T_{em}$. In this case, we also have that $\cos\gamma(t) > 0$, $\forall t \geq t_0$, because

$$(a_T\psi^*_s\psi_r\cos\gamma)^2 \geq \frac{M^2\psi^{*2}_s}{\sqrt{2}\sigma L_s^2 L_r} - T_{em}^2 > 0 \quad (15)$$

When $e_T \neq 0$, we get

$$d|T_e|^2/dt = -K_1T_e^2 + z_eK_2e_TT_e \quad (16)$$

where $K_1 = 2R_r/(\sigma L_r)$, $K_2 \geq 2\left(\frac{R_s}{\psi_s^2} + \frac{k_T}{\psi^*_s}\right)$, $z_e = aM\psi_s\psi_r\cos\gamma$, moreover $e_TT_e \leq (|T^*_e| - |T_e|)|T_e|$. With the initial conditions (12), it can be proved that $\forall t > t_0$

$$\psi_r \geq M\psi^*_s/(\sqrt{2}L_s), \ \cos\gamma \geq 1/\sqrt{2}, \ |T_e| \leq T_{em} \quad (17)$$

Consider Lyapunov function $V_T = e_T^2/2$, then we can get that $\dot{V}_T < 0, \forall e_T \neq 0$.

IV. Experimental results

In order to make the experimental validation of the proposed control scheme, a TMS320F240 DSP based system has been used. The motor is a 3KW, 220V, 50Hz, 4-poles standard IM. In the experimentation, the function sign(x) is replaced by sat(x/ϵ), with small $\epsilon > 0$, to reduce chattering.

Fig. 1 shows the results in transient and in steady state (with $\omega_r = 450$rpm). The experimental results illustrated the fast response and smooth flux and torque operations compared with PID controller [5] and classical DTC [6] respectively.

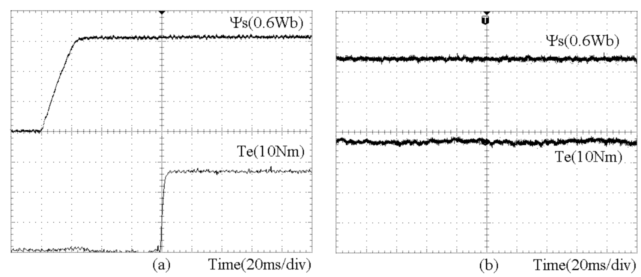

Fig. 1. Experimental results in transient (a) and steady state (b)

V. Conclusions

A robust input-output decoupling control strategy for induction motors is proposed based on the dynamic model in polar coordinates, and domination feedback is used in the robust controller to avoid full state measurements. The main drawback of the proposed strategy is the requirement of stator flux measurement, and the future research is to remove it by introducing a state observer.

References

[1] F. Khorrami, P. Krishnamurathy and H. Melkote, *Modeling and adaptive nonlinear control of electric motors*, Springer-Verlag, New York, 2003

[2] Z. Krzeminski, Nonlinear control of induction motor, *Preprints of 10th IFAC World Congress*, Munich, 1987, pp.349-354.

[3] R. Marino, S. Peresada and P. Tomei, Global adaptive output feedback control of induction motors with uncertain rotor resistance, *IEEE Trans. Automat. Contr.*, vol. 44, no. 5, 1999, pp. 967-983.

[4] M. Depenbrock, Direct self-control of inverter-fed machine, *IEEE Trans. Power Electron.*, vol. 3, no. 4, 1988, pp. 420-429.

[5] Y. Lai and J Chen, A new approach to direct torque control of induction motor drives for constant inverter switching frequency and torque ripple reduction, *IEEE Trans. Energy conversion.*, vol. 16, no.3, 2001, pp. 220-227.

[6] R. Ortega, N. Barabanov and G. Valderrama, Direct torque control of induction motors: stability analysis and performance improvement, *IEEE Trans. Automat. contr.*, vol. 46, no. 8, 2001, pp. 1209-1222.