Sensorless Speed Control of Induction Motors*

Hassan K. Khalil and Elias G. Strangas†

Abstract

We consider field-oriented speed control of induction motors without rotor position sensors. We augment the traditional approach with flux and speed observers and derive a sixth-order nonlinear model that describes the motor in field-oriented coordinates. The model takes into consideration the error in flux estimation. The flux regulation problem is a simple one and we follow the traditional approach of using PI controllers. For the speed regulation problem, we simplify the model by assuming that flux regulation takes place relatively fast and by using a (high-gain) PI controller to regulate the q-axis current to its command. This results in a third-order nonlinear model in which the speed and two flux estimation errors are the state variables, the q-axis current is the control input and a speed estimate (provided by the high-gain observer) is the measured output. This nonlinear model is the main contribution of this paper because it enables us to perform rigorous analysis of the closed-loop system under different controllers. In the current paper, we limit our analysis to the design of PI controllers via linearization. The linearized model is used to study when a PI controller can stabilize the nonlinear third-order model at the desired equilibrium point. The analysis reveals an important role played by the steady-state product of the flux frequency and the q-axis current in determining the control properties of the system.

1 Introduction

During the last decade, there has been a considerable interest in electric drives without mechanical sensors (e.g., optical encoders, tacho generators, resolvers, etc.) because of their low cost and high reliability. The main idea is to use intrinsic motor electro-mechanical properties to estimate the rotor speed or position. Several methods have been proposed in the literature; see, for example, [3], [8], [9], [10], and [12].

There are two basic approaches for speed and position estimation in induction motors. The first approach uses the fundamental machine model to design model reference adaptive systems, nonlinear observers, extended Kalman filters, or adaptive observers. It has long been recognized that the challenging part in this approach is keeping a load stationary at (or near) zero flux frequency. The second approach uses secondary phenomena or the parasitic effects of the machine to develop methods that will be effective at low frequency. This paper belongs to the first approach.

Examination of the literature on the first approach shows the following drawbacks:

- Analysis is limited to local linear models;
- Model uncertainty is usually ignored;
- No analysis of the closed-loop system.

The goal of this paper is to address the foregoing drawbacks. We study a traditional field-oriented control [7, 8], where a flux observer is used to estimate the rotor flux. We concentrate on the speed control problem, where the motor speed is required to track a given speed command in the presence of unknown load. In earlier work [11], we studied the torque control problem where the motor torque is required to track a given torque while the speed is treated as a given input. The two key elements of our approach are:

- To keep track of the error in estimating the rotor flux, field orientation is performed using the estimated flux angle and two additional state variables are added by projecting the flux estimation error into the field-oriented coordinates.
- A high-gain observer is used to estimate the speed from current measurements.

We derive a sixth-order nonlinear model that describes the motor in field-oriented coordinates and formulate the flux and speed regulation problems. For the flux regulation problem we follow the traditional approach of using PI controllers. For the speed regulation problem we simplify the model by assuming that flux regulation takes place relatively fast and by using a (high-gain) PI controller to regulate the q-axis current to its command. This results in a third-order nonlinear model in which the speed and two flux estimation errors are the state variables, the q-axis current is the control input and a speed estimate (provided by the high-gain

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†The authors are with the Department of Electrical and Computer Engineering, Michigan State University, East Lansing, MI 48824-1226.

observer) is the measured output. This nonlinear model is the main contribution of this paper because it enables us to analyze the closed-loop system under different controllers. We limit our analysis to the design of PI controllers via linearization. The linearized model is used to study when a PI controller can be designed to stabilize the nonlinear third-order model at the desired equilibrium point. The analysis reveals an important role played by the steady-state product of the flux frequency and the q-axis current in determining the control properties of the system. When this product is zero, it is impossible to stabilize the system by a PI controller; in fact, it is impossible to robustly stabilize the system by any controller that uses integral action. When it is positive, the system is minimum phase and a PI controller can be designed to achieve good performance and robustness properties. Finally, when it is negative, the system is non-minimum phase and a PI controller cannot stabilize the system; one has to resort to a more complex controller. We present simulation results that confirm our analysis findings.

2 Controller design

The induction motor is represented in the stator frame of reference by the equations [7]

$$\frac{d}{dt}\lambda_r = \left(-\frac{R_r}{L_r}I + p\omega J\right)\lambda_r + \frac{R_r}{L_r}L_m i_s \quad (1)$$

$$\sigma L_s \frac{d}{dt}i_s = -\frac{L_m}{L_r}\left(-\frac{R_r}{L_r}I + p\omega J\right)\lambda_r - \left(R_s + \frac{L_m^2 R_r}{L_r^2}\right)i_s + v_s \quad (2)$$

$$m\frac{d\omega}{dt} = -\frac{3pL_m}{2L_r}\lambda_r^T J i_s - b_1\omega - \frac{1}{m}T_L \quad (3)$$

where $\lambda_r \in R^2$ is the rotor flux, $i_s \in R^2$ is the stator current, $v_s \in R^2$ is the stator voltage, and ω is the rotor speed. The parameters L_r, L_s, and L_m denote the rotor, stator, and mutual inductances, $\sigma = 1 - L_m^2/L_sL_r$ is the leakage parameter, R_r and R_s are rotor and stator resistances, m is the rotor's moment of inertia, b_1 is a friction coefficient, p is the number of pole pairs, I is the 2×2 identity matrix, and $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The resistances R_s and R_r, the moment of inertia m, and the friction coefficient b_1 will be treated as uncertain parameters with $\hat{R}_s$, $\hat{R}_r$, $\hat{m}$, and $\hat{b}_1$ as their nominal values, respectively. The load torque T_L will be treated as a bounded time-varying disturbance. Our goal is to design a feedback controller for the stator voltage v_s that uses only measurements of the stator current i_s such that the speed ω asymptotically tracks a bounded time-varying reference speed ω_{ref}.

Flux Observer

We would like to design the controller using field orientation along the rotor flux λ_r [7, 8]. Since λ_r is not measured, we use the open-loop observer [13]

$$\dot{\hat{\lambda}}_r = \left(-\frac{\hat{R}_r}{L_r}I + p\omega_{ref}J\right)\hat{\lambda}_r + \frac{\hat{R}_r}{L_r}L_m i_s \quad (4)$$

to estimate λ_r. The observer duplicates the flux equation (1), with the unavailable speed ω replaced by its reference ω_{ref}. Orienting the vectors $\hat{\lambda}_r$, i_s, v_s, and $e = \hat{\lambda}_r - \lambda_r$ along the vector $\hat{\lambda}_r$, and denoting the direct-axis components by λ_d, i_d, v_d, and e_d, respectively, and the quadrature-axis components by $\lambda_q(=0)$, i_q, v_q, and e_q, respectively, we represent the motor by the equations

$$\frac{d\lambda_d}{dt} = -\hat{\alpha}_r\lambda_d + \hat{\alpha}_r L_m i_d \quad (5)$$

$$\frac{di_d}{dt} = \alpha_r\beta\lambda_d - (\alpha_s\eta + \alpha_r\beta L_m)i_d + p\omega_{ref}i_q + \hat{\alpha}_r L_m i_q^2/\lambda_d + \gamma v_d - \alpha_r\beta e_d - \beta p\omega e_q \quad (6)$$

$$\frac{di_q}{dt} = -\beta p\omega\lambda_d - p\omega_{ref}i_d - (\alpha_s\eta + \alpha_r\beta L_m)i_q - \hat{\alpha}_r L_m i_d i_q/\lambda_d + \gamma v_q + \beta p\omega e_d - \alpha_r\beta e_q \quad (7)$$

$$\frac{d\omega}{dt} = \mu[i_q(\lambda_d - e_d) + i_d e_q] - b\omega - T_L/m \quad (8)$$

$$\frac{de_d}{dt} = -\alpha_r e_d + (p\omega_{ref} - p\omega + \hat{\alpha}_r L_m i_q/\lambda_d)e_q + (\hat{\alpha}_r - \alpha_r)(L_m i_d - \lambda_d) \quad (9)$$

$$\frac{de_q}{dt} = -(p\omega_{ref} - p\omega + \hat{\alpha}_r L_m i_q/\lambda_d)e_d - \alpha_r e_q + (\hat{\alpha}_r - \alpha_r)L_m i_q + p(\omega_{ref} - \omega)\lambda_d \quad (10)$$

where $\alpha_r = \frac{R_r}{L_r}$, $\hat{\alpha}_r = \frac{\hat{R}_r}{L_r}$, $\alpha_s = \frac{R_s}{L_s}$, $b = \frac{b_1}{m}$, $\beta = \frac{1-\sigma}{\sigma L_m}$, $\gamma = \frac{1}{\sigma L_s}$, $\eta = \frac{1}{\sigma}$, and $\mu = \frac{3pL_m}{2mL_r}$. For later use, define $\hat{\alpha}_s = \frac{\hat{R}_s}{L_s}$, $\hat{\mu} = \frac{3pL_m}{2\hat{m}L_r}$, and $\hat{b} = \frac{\hat{b}_1}{\hat{m}}$. The variables λ_d, i_d and i_q are available on line, since they can be calculated from i_s and $\hat{\lambda}$, while ω, e_d and e_q are not available.

Flux Regulation

In field orientation, the flux λ_d is regulated to a reference flux $\lambda_{ref} > 0$, which is taken here as a constant. We follow the traditional approach of using two PI controllers [8]. First, we view i_d as a control input to equation (5) and design the PI controller

$$I_d^* = \frac{(K_{fp}s + K_{fi})}{s}[\lambda_{ref} - \lambda_d]$$

Then, we design the PI controller

$$V_d = \frac{(K_{dp}s + K_{di})}{s}[I_d^* - I_d]$$

With tight feedback loops, we can ensure the regulation of λ_d to λ_{ref} for a wide range of variation of the term $(p\omega_{ref}i_q + \hat{\alpha}_r L_m i_q^2/\lambda_d - \alpha_r\beta e_d - \beta p\omega e_q)$, which acts as an input on the right-hand side of (6). The design should ensure that λ_d starts at a positive value and approaches λ_{ref} monotonically so that λ_d is always positive. The initial condition of λ_d is determined by the initial condition of the observer (4), which is at our disposal.

Speed Observer

With the flux regulated, we turn now to the design of the speed controller. Since we do not measure the speed ω, we need to use an observer to estimate it. Towards that end, rewrite equations (7) and (8) as

$$\frac{di_q}{dt} = -\beta p\lambda_d\omega - f_1 + \gamma v_q + \delta_1 \tag{11}$$

$$\frac{d\omega}{dt} = \hat{\mu} i_q\lambda_d - \hat{b}\omega + \delta_2 \tag{12}$$

where $f_1 = p\omega_{ref} i_d + (\hat{\alpha}_s\eta + \hat{\alpha}_r\beta L_m)i_q + \hat{\alpha}_r L_m i_d i_q/\lambda_d$ is available on line, and δ_1 and δ_2 are uncertain terms,

$$\delta_1 = [(\hat{\alpha}_s - \alpha_s)\eta + (\hat{\alpha}_r - \alpha_r)\beta L_m]i_q + \beta p\omega e_d - \alpha_r\beta e_q$$

$$\delta_2 = (\mu - \hat{\mu})i_q\lambda_d + \mu(-i_q e_d + i_d e_q) - (b - \hat{b})\omega - \frac{T_L}{m}$$

The change of variables

$$\begin{aligned}\Omega &= \omega - \frac{\delta_1}{\beta p\lambda_d} \qquad (13)\\ &= \left(\frac{\lambda_d - e_d}{\lambda_d}\right)\omega - \frac{1}{\beta p\lambda_d}\{[(\hat{\alpha}_s - \alpha_s)\eta \\ &\quad + (\hat{\alpha}_r - \alpha_r)\beta L_m]i_q - \alpha_r\beta e_q\}\end{aligned}$$

brings equations (11) and (12) into the form

$$\frac{di_q}{dt} = -\beta p\lambda_d\Omega - f_1 + \gamma v_q \tag{14}$$

$$\frac{d\Omega}{dt} = \hat{\mu} i_q\lambda_d - \hat{b}\Omega + \delta_3 \tag{15}$$

where $\delta_3 = \delta_2 - \hat{b}\delta_1/(\beta p\lambda_d) - \frac{d}{dt}[\delta_1/(\beta p\lambda_d)]$ is a continuous function of $(\lambda_d, i_d, i_q, \omega, e_d, e_q, T_L, \omega_{ref})$. The change of variables (13) is invertible provided $\lambda_d - e_d \neq 0$. We use the high-gain observer

$$\frac{d\hat{i}_q}{dt} = -\beta p\lambda_d\hat{\Omega} - f_1 + \gamma v_q + \left(\frac{\alpha_1}{\varepsilon}\right)(i_q - \hat{i}_q) \tag{16}$$

$$\frac{d\hat{\Omega}}{dt} = \hat{\mu} i_q\lambda_d - \hat{b}\hat{\Omega} - \left(\frac{\alpha_2}{\varepsilon^2\beta p\lambda_d}\right)(i_q - \hat{i}_q) \tag{17}$$

where ε is a small positive parameter and α_1 and α_2 are positive constants that assign the roots of $s^2 + \alpha_1 s + \alpha_2 = 0$ at desired locations in the left-half plane. The scaled estimation errors $\eta_1 = (i_q - \hat{i}_q)/\varepsilon$ and $\eta_2 = \Omega - \hat{\Omega}$ satisfy

$$\varepsilon\dot{\eta}_1 = \alpha_1\eta_1 - \beta p\lambda_d\eta_2 \tag{18}$$

$$\varepsilon\dot{\eta}_2 = -\left(\frac{\alpha_2}{\beta p\lambda_d}\right)\eta_1 - \varepsilon\hat{b}\eta_2 - \varepsilon\delta_3 \tag{19}$$

For small ε, the closed-loop system will be a singularly perturbed one [6], with η_1 and η_2 as the fast variables. The stability of the fast dynamics is determined by the matrix $\begin{bmatrix} -\alpha_1 & -\beta p\lambda_d \\ \frac{\alpha_2}{\beta p\lambda_d} & 0 \end{bmatrix}$, in which $\lambda_d > 0$ is treated as a constant. The characteristic equation $s^2 + \alpha_1 s + \alpha_2 = 0$ is Hurwitz. From the high-gain observer theory [1], we know that if the control input v_s is bounded uniformly in ε, then the estimation error $\Omega - \hat{\Omega}$ will be $O(\varepsilon)$ after a short transient period $[0, T(\varepsilon)]$, where $\lim_{\varepsilon\to 0} T(\varepsilon) = 0$. Moreover, the closed-loop system with feedback from $\hat{\Omega}$ recovers the performance of the closed-loop system with feedback from Ω as ε tends to zero. Hence, we design the speed controller as if Ω was available for feedback. The boundedness of v_s uniformly in ε is ensured by limiting v_s to the rated voltage, imposed by the limitation of DC voltage in the inverter.

Speed Controller

The design can be simplified by reducing the order of the system. First, assuming that the flux regulator acts fast enough to regulate λ_d to λ_{ref}, we take $\lambda_d = \lambda_{ref}$ and $i_d = \lambda_{ref}/L_m$ and drop equations (5) and (6). Second, we note from equation (7) that for any current command i_q^*, we can design v_q as the PI controller

$$V_q = \frac{(K_{qp}s + K_{qi})}{s}[I_q^* - I_q]$$

with sufficiently large gains to regulate i_q to i_q^*. This allows us to view i_q as the control input. Thus, the speed controller can be designed using the third-order model

$$\frac{de_d}{dt} = -\alpha_r e_d + \left(p\omega_{ref} - p\omega + \frac{\hat{\alpha}_r L_m i_q}{\lambda_{ref}}\right)e_q \tag{20}$$

$$\frac{de_q}{dt} = -\left(p\omega_{ref} - p\omega + \frac{\hat{\alpha}_r L_m i_q}{\lambda_{ref}}\right)e_d - \alpha_r e_q + (\hat{\alpha}_r - \alpha_r)L_m i_q + p(\omega_{ref} - \omega)\lambda_{ref} \tag{21}$$

$$\frac{d\omega}{dt} = \mu\left[i_q(\lambda_{ref} - e_d) + \frac{e_q\lambda_{ref}}{L_m}\right] - b\omega - \frac{T_L}{m} \tag{22}$$

$$\Omega = \left(\frac{\lambda_{ref} - e_d}{\lambda_{ref}}\right)\omega + \frac{\alpha_r e_q}{p\lambda_{ref}} - a i_q \tag{23}$$

where Ω is viewed as the measured output, $a = [(\hat{\alpha}_s - \alpha_s)\eta + (\hat{\alpha}_r - \alpha_r)\beta L_m]/(\beta p\lambda_{ref})$ and the goal is to have Ω track ω_{ref}.

It is natural to use integral control to ensure zero steady-state error when ω_{ref} and T_L are constant [4]. With $\Omega = \omega_{ref}$, the equilibrium equations are

$$0 = -\alpha_r\bar{e}_d + \left(p\omega_{ref} - p\bar{\omega} + \frac{\hat{\alpha}_r L_m\bar{i}_q}{\lambda_{ref}}\right)\bar{e}_q \tag{24}$$

$$0 = -\left(p\omega_{ref} - p\bar{\omega} + \frac{\hat{\alpha}_r L_m\bar{i}_q}{\lambda_{ref}}\right)\bar{e}_d - \alpha_r\bar{e}_q + (\hat{\alpha}_r - \alpha_r)L_m\bar{i}_q + p(\omega_{ref} - \bar{\omega})\lambda_{ref} \tag{25}$$

$$0 = \mu\left[\bar{i}_q(\lambda_{ref} - \bar{e}_d) + \frac{\bar{e}_q\lambda_{ref}}{L_m}\right] - b\bar{\omega} - \frac{T_L}{m} \tag{26}$$

$$\omega_{ref} = \left(\frac{\lambda_{ref} - \bar{e}_d}{\lambda_{ref}}\right)\bar{\omega} + \frac{\alpha_r\bar{e}_q}{p\lambda_{ref}} - a\bar{i}_q \tag{27}$$

Solving (24) and (25) for $\bar{e}_d$ and $\bar{e}_q$ in terms of $\bar{i}_q$ and $\tilde{\omega} \stackrel{\text{def}}{=} \bar{\omega} - \omega_{ref}$ and substituting in (27), we obtain

$$\left(-p\tilde{\omega} + \frac{\hat{\alpha}_r L_m \bar{i}_q}{\lambda_{ref}}\right)\left(-p\tilde{\omega} + \frac{(\hat{\alpha}_r - \alpha_r) L_m \bar{i}_q}{\lambda_{ref}}\right)\omega_c = - \frac{(\hat{\alpha}_s - \alpha_s)\eta\Delta\bar{i}_q}{\beta\lambda_{ref}} \qquad (28)$$

where $\omega_c = p\omega_{ref} + \hat{\alpha}_r L_m \bar{i}_q/\lambda_{ref}$ and $\Delta = \alpha_r^2 + (-p\tilde{\omega} + \hat{\alpha}_r L_m \bar{i}_q/\lambda_{ref})^2$. To gain insight into the problem, let us consider first the case when $\hat{\alpha}_s = \alpha_s$, for which (28) reduces to

$$\left(-p\tilde{\omega} + \frac{\hat{\alpha}_r L_m \bar{i}_q}{\lambda_{ref}}\right)\left(-p\tilde{\omega} + \frac{(\hat{\alpha}_r - \alpha_r) L_m \bar{i}_q}{\lambda_{ref}}\right)\omega_c = 0$$

Assuming that $\omega_c \neq 0$, the equation has two solutions:

$$\tilde{\omega} = \frac{(\hat{\alpha}_r - \alpha_r) L_m \bar{i}_q}{p\lambda_{ref}} \quad \text{or} \quad \tilde{\omega} = \frac{\hat{\alpha}_r L_m \bar{i}_q}{p\lambda_{ref}}$$

It is clear that the first solution is the one we should be interested in because it yields zero steady-state speed error in the nominal case $\hat{\alpha}_r = \alpha_r$. The equilibrium point corresponding to this solution is

$$\bar{e}_d = \bar{e}_q = 0, \quad \bar{i}_q = \frac{b\omega_{ref} + T_L/m}{\mu\lambda_{ref} - \frac{b(\hat{\alpha}_r - \alpha_r)L_m}{p\lambda_{ref}}},$$
$$\bar{\omega} = \omega_{ref} + \frac{(\hat{\alpha}_r - \alpha_r) L_m \bar{i}_q}{p\lambda_{ref}} \qquad (29)$$

Can we stabilize this equilibrium point using a PI controller? To answer this question, we linearize equations (20)–(23) at the equilibrium point (29), to obtain the linear model

$$\dot{x} = Ax + B(i_q - \bar{i}_q), \qquad \Omega - \omega_{ref} = Cx + D(i_q - \bar{i}_q)$$

$$\text{where} \quad A = \begin{bmatrix} -\alpha_r & \frac{\alpha_r L_m \bar{i}_q}{\lambda_{ref}} & 0 \\ -\frac{\alpha_r L_m \bar{i}_q}{\lambda_{ref}} & -\alpha_r & -p\lambda_{ref} \\ -\mu\bar{i}_q & \frac{\mu\lambda_{ref}}{L_m} & -b \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & (\hat{\alpha}_r - \alpha_r)L_m & \mu\lambda_{ref} \end{bmatrix}$$

$$C = \begin{bmatrix} -\frac{\bar{\omega}}{\lambda_{ref}} & \frac{\alpha_r}{p\lambda_{ref}} & 1 \end{bmatrix}, \; D = -\frac{(\hat{\alpha}_r - \alpha_r)L_m}{p\lambda_{ref}}$$

with the transfer function $G(s) = n(s)/d(s)$, in which

$$\begin{aligned} n(s) &= \mu\lambda_{ref}\left[s^2 + \alpha_r s + \frac{\omega_c \alpha_r L_m \bar{i}_q}{\lambda_{ref}}\right] \\ &\quad \times \left[1 - \frac{(\hat{\alpha}_r - \alpha_r)L_m}{\mu p \lambda_{ref}^2}(s+b)\right] \\ d(s) &= (s+b)\left[(s+\alpha_r)^2 + \left(\frac{\alpha_r L_m \bar{i}_q}{\lambda_{ref}}\right)^2\right] \\ &\quad + \frac{p\mu\lambda_{ref}^2}{L_m}\left(s + \alpha_r - \frac{\alpha_r L_m^2 \bar{i}_q^2}{\lambda_{ref}^2}\right) \end{aligned}$$

Let us note the important role played by $\omega_c \bar{i}_q$ in the control design. When $\omega_c \bar{i}_q = 0$, $G(s)$ has a zero at the origin. Hence, it is impossible to design any controller with integral action. This follows from the well-known theory of servomechanisms [2]. When $\omega_c \bar{i}_q < 0$, $G(s)$ has a real zero in the right-half plane; hence, it is nonminimum phase. It is possible to design a controller with integral action to stabilize the system, but such a controller cannot be a PI controller. This fact can be seen by root locus analysis. This leaves us with the case when $\omega_c \bar{i}_q$ is positive. In this case the transfer function $G(s)$ is minimum phase and we can design a PI controller with high-gain feedback to stabilize the closed-loop system and achieve good tracking properties. Such PI controller takes the form

$$I_q = \frac{(K_{wp}s + K_{wi})}{s}[\omega_{ref} - \Omega]$$

The condition $\omega_c \bar{i}_q = \bar{i}_q(p\omega_{ref} + \hat{\alpha}_r L_m \bar{i}_q/\lambda_{ref}) > 0$ is satisfied when the motor is operated in the motoring or braking modes, but not in the generating mode.

The condition $\omega_c \bar{i}_q = 0$ is satisfied if $\bar{i}_q = 0$ or $\omega_c = 0$. The case $\omega_c = 0$ indicates operation at zero frequency, that is, in a braking mode corresponding to certain speed and torque. It is well known in the induction motor literature on sensorless control that operating the motor at zero (or low) frequency is challenging, and that a design for such case will have to exploit secondary phenomena of the machine, which are not conveyed in the model (1)–(3). The case $\bar{i}_q = 0$, regardless of speed, indicates that the power into the machine is negative.

In the foregoing discussion we assumed that $\hat{R}_s = R_s$. When $\hat{R}_s \neq R_s$, but $(\hat{\alpha}_s - \alpha_s)/\omega_c$ is relatively small, we view the right-hand side of (28) as a perturbation term. By perturbation analysis it can be verified that if $\bar{i}_q \neq 0$, so that the equilibrium point (29) is isolated, then equations (23)–(27) will have an equilibrium point that is $O((\hat{\alpha}_s - \alpha_s)/\omega_c)$ perturbation of (29).

To appreciate the usefulness of the nonlinear model (20)–(23), let us look at the naive alternative of assuming perfect field orientation ($\hat{\lambda} = \lambda$) and nominal parameters ($\hat{R}_s = R_s$ and $\hat{R}_r = R_r$). The model for this case can be obtained from (20)–(23) by setting $e_d = e_q = 0$ and $a = 0$, resulting in the linear model

$$\dot{\omega} = \mu\lambda_{ref} i_q - b\omega - T_L/m, \qquad \Omega = \omega$$

For constant ω_{ref} and T_L, there is a unique equilibrium point ($\bar{\omega} = \omega_{ref}, \bar{i}_q = \frac{b\omega_{ref} + T_L/m}{\mu\lambda_{ref}}$) and the transfer function at this point is $\mu\lambda_{ref}/(s+b)$. It is clear that this model does not convey the wealth of information we gained from the nonlinear model (20)–(23) and does not show the difficulties associated with designing PI controllers in the generating mode or at zero frequency.

3 Simulation Results

We demonstrate the utility of the model developed in the previous section by simulating an induction motor under PI controllers. We use an induction motor that has the following constants and rating (taken from [7, Example 5.6]): 200 V, 4 pole, 3 phase, 60 Hz, Y connected, $R_s = 0.183\ \Omega$, $R_r = 0.277\ \Omega$, $L_m = 0.0538$ H, $L_s = 0.0553$ H, $L_r = 0.056$ H, $m = 0.0165$ kg-m^2, and base power 5 hp. We added $b_1 = 0.01$ Kg-m^2/sec as a friction coefficient. We design the parameters of the controllers for the nominal case $\hat{R}_r = R_r$ and $\hat{R}_s = R_s$. The constants of the PI controllers of v_d, i_d, and v_q, chosen using linearized models, are $K_{dp} = 20$, $K_{di}/K_{dp} = 5$, $K_{fp} = 20$, $K_{fi}/K_{fp} = 5$, $K_{qp} = 300$, and $K_{qi}/K_{qp} = 1$. The flux observer (4) is initiated at $\hat{\lambda}(0) = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$ so that $\lambda_d(0) = 0.1$. The speed observer (16)–(17) is implemented with $\alpha_1 = \alpha_2 = 1$, $\varepsilon = 0.001$, $\hat{\mu} = \mu$, and $\hat{b} = b$. In the simulation, the components of v_s are limited to ± 200 V, but for all the reported results the control is not saturated, except for a short period during the speed observer transient. The speed reference is taken as a step input smoothed by the transfer function $1/(0.5s + 1)$.

Figure 1 shows simulation results for a speed command of 100 rad/sec applied at zero time with a load of 20 N.m applied between $t = 4$ s and $t = 8$ s. The simulation is for the nominal parameter case $\hat{R}_r = R_r$ and $\hat{R}_s = R_s$. According to (29), at the equilibrium point we have $\bar{e}_d = \bar{e}_q = \bar{\omega} - \omega_{ref} = 0$. The simulation confirms these equilibrium values.

Figure 2 repeats the simulation of Figure 1 for $R_r = 2\hat{R}_r$. According to (29), the equilibrium values are $\bar{e}_d = \bar{e}_q = 0$, and $\bar{\omega} - \omega_{ref} = \frac{(\hat{\alpha}_r - \alpha_r) L_m \bar{i}_q}{p \lambda_{ref}} = -0.4435 \bar{i}_q$, where

$$\bar{i}_q = \frac{b\omega_{ref} + T_L/m}{\mu\lambda_{ref} - \frac{b(\hat{\alpha}_r - \alpha_r)L_m}{p\lambda_{ref}}} = \frac{60.6061(1 + T_L)}{52.6714}$$

For $T_L = 20$, we obtain $\bar{i}_q = 24.164$ and $\bar{\omega} - \omega_{ref} = -10.716$. The simulation results confirm these calculations.

Figure 3 repeats the simulation of Figure 1 for $R_r = 2\hat{R}_r$ and $R_s = 2\hat{R}_s$. We expect the equilibrium point to be slightly perturbed from the one we obtained in Figure 2. Indeed, $\bar{e}_d$ and $\bar{e}_q$ are no longer zero.

Figure 4 shows speed reversal from 50 to -50 rad/s at no load, with nominal parameters. Notice that the system settles back at the equilibrium point following a transient during speed reversal. The important factor that limits the performance is control saturation. In the case of Figure 4, reversing speed from 100 to -100 rad/s would cause control saturation and the performance would degrade significantly; actually the variables oscillate during saturation.

Finally, in Figure 5 we demonstrate the importance of the condition $\omega_c \bar{i}_q > 0$. This condition is satisfied for all the cases in Figures 1 to 4. In the case of Figure 5, we apply a speed command of 10 rad/sec at zero time. Then at time $t = 4$ sec, we apply a load of -1 N.m. For these values, $\bar{i}_q = -1.1546$ and $\omega_c = 18.9758$; hence $\omega_c \bar{i}_q < 0$. It is clear from the simulation that the equilibrium has been destabilized after applying the load. The simulation is shown only until $t = 0.521$ s because beyond that time the control saturates.

4 Conclusions

The main contribution of this paper is the nonlinear model (20)–(23). To illustrate the usefulness of this model, we focused our attention on one issue; namely, to analyze PI controllers and derive conditions that could not have been obtained by the simple linear model that assumes perfect field orientation. For future work, a challenging problem would be to use this nonlinear model to design a nonlinear controller and estimate the region of attraction.

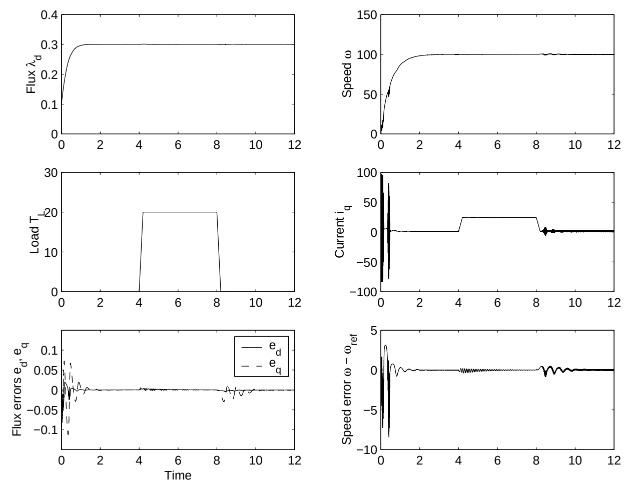

Figure 1: Simulation with nominal parameters.

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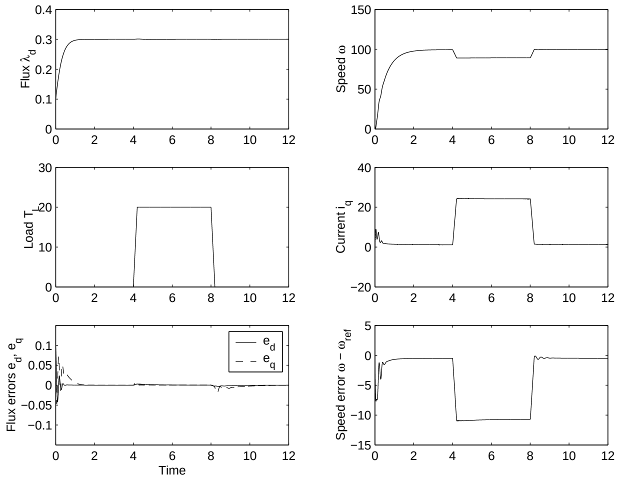

Figure 2: Simulation with 100% increase in R_r and nominal R_s.

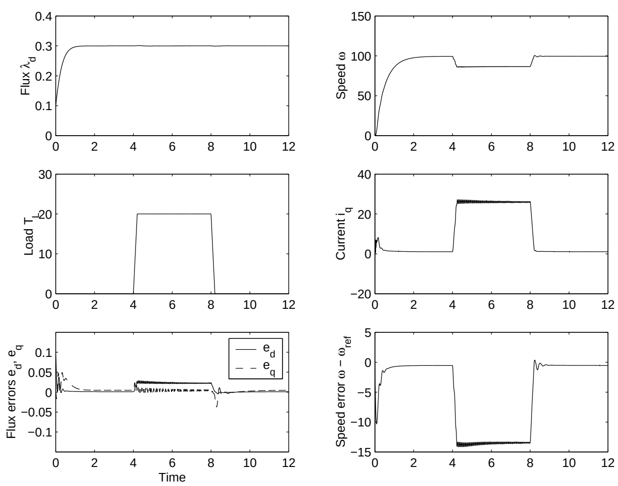

Figure 3: Simulation with 100% increase in R_r and R_s.

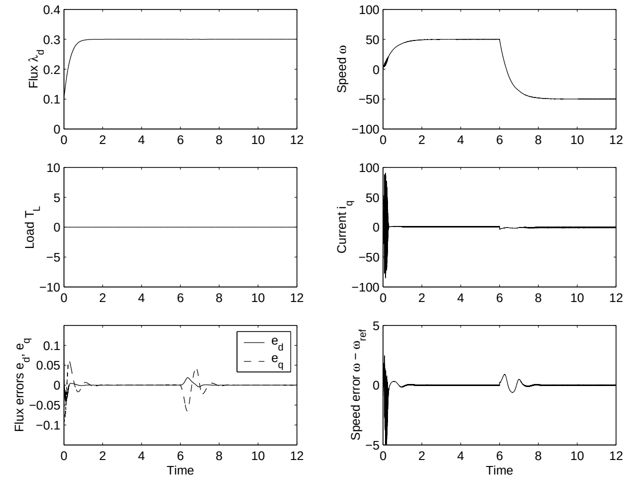

Figure 4: Speed reversal under no load with nominal parameters.

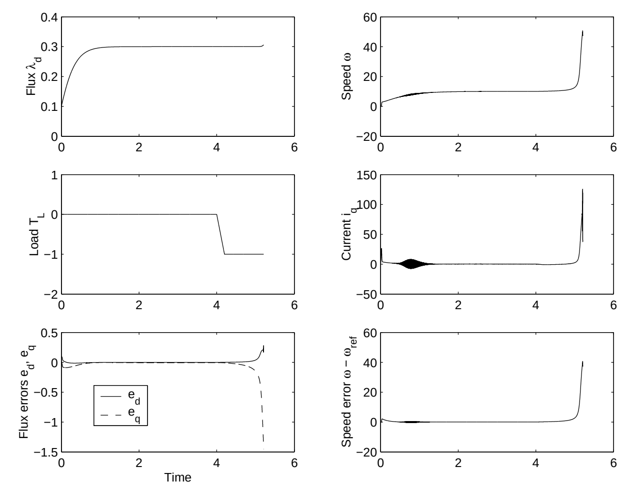

Figure 5: Loss of stability for negative $\omega_c \bar{i}_q$.

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