

# Adaptive Fuzzy Attitude Control of Satellite based on Linearization

P. Guan, X.J. Liu, Lara-Rosano F. and J.B.Chen

**Abstract—** Input-output (I/O) linearization is a typical control method in the nonlinear multi-input multi-output (MIMO) system. In this paper, adaptive fuzzy control is combined with input-output linearization control to constitute the hybrid controller. The proposed control method is applied to the attitude maneuver control of the flexible satellite, which is a complex nonlinear system. The basic control structure is presented. The selection of the controller parameter, which guarantee the attitude stabilization of the satellite with parameter uncertainties, has been analyzed. The adaptive fuzzy control compensates for the plant uncertainties to increase the robustness of the controller. Simulation results show that precise attitude control is accomplished in spite of the uncertainty in the system.

## I. INTRODUCTION

ROBUSTNESS against modeling uncertainties and unknown disturbances for the satellite attitude control system is widely treated in the literature. One of the popular techniques is input-output (I/O) linearizing control that is often employed in the nonlinear multi-input multi-output (MIMO) system. Although the technique of I/O linearization results in input-output decoupling, it has limitations. The technique relies on the exact cancellation of nonlinear terms and the resulting controller's robustness cannot be guaranteed for the nonlinear system with uncertainties. In recent years, the enhancing robustness of I/O linearization controllers have been widely discussed. In [1], robustness of linearization control system is obtained by the integral error feedback. For pitch axis maneuver, attitude control is accomplished in spite of the parameter uncertainty in the system. The I/O linearization is combined with variable structure control to form a hybrid

controller in [2]-[4]. It requires that the uncertainty be matched and bounded. The resulting hybrid controllers increase the robustness of the I/O linearization controllers. However, the conventional chattering problem of variable structure control is not completely solved.

In this paper, adaptive fuzzy control is combined with I/O linearization to constitute the hybrid controller. The adaptive fuzzy control can compensate for the system uncertainties so that it can increase the robustness of the controller. For MIMO system, building MIMO fuzzy rules is usually required for the controller, which are time consuming or even difficult in practice. This limits somehow fuzzy control's application to MIMO systems. The proposed technique will exploit the decoupling property of I/O linearization, so as to decompose MIMO fuzzy control rules to double-input and single-output (DISO) rules without any deterioration in the closed-loop system. Simulation results show the effectiveness of the proposed hybrid control by incorporating the merits of both techniques and realizing the accurate attitude control.

The organization of this paper is as follows. Section II describes the attitude control problem. The theory of feedback linearization is applied to the uncertain nonlinear system in section III. The hybrid control law is derived in section IV. Section V presents simulation results.

## II. PROBLEM FORMULATION

The flexible satellite with a solar panel moving in a circular orbit, an inverse square gravitational field, equipped with reaction jets for orientation control is considered. The equations of motion about the center of mass of the satellite is as follows:

$$J\dot{\omega} + \omega^\times J\omega + C\ddot{\eta} = T_d + u \quad (1)$$

$$\ddot{\eta} + 2\xi\Lambda\dot{\eta} + \Lambda^2\eta + C^T\dot{\omega} = 0 \quad (2)$$

$$\dot{q} = \frac{1}{2}(q^\times + q_0I)\omega, \quad \dot{q}_0 = -\frac{1}{2}q^T\omega \quad (3)$$

where  $J \in R^{3 \times 3}$  denotes the inertia matrix of satellite,  $\omega \in R^3$  denotes the angular velocity of the satellite with respect to an inertial frame,  $u \in R^3$  denotes the vector of control torques,  $T_d \in R^3$  denotes the vector of external disturbances,  $C$  is the coupling matrix between rigid body

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and appendage,  $\eta$  is the vector of five-order model displacements,  $\xi$  is the modal damping matrix,  $\Lambda$  is modal frequency matrix,  $q \in R^3$  and  $q_0 \in R$  denote the quaternion of the satellite with respect to an inertial frame and satisfy the constraint  $q^T q + q_0^2 = 1$ , and  $I$  denotes a  $3 \times 3$  identity matrix.

For any vector  $r = [r_1 \ r_2 \ r_3]^T$ , the notation  $r^\times$  stands for cross product matrix  $r^\times = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}$

The inertia matrix of satellite is usually not known exactly. In this paper, the inertia matrix is assumed that  $J = J^* + \Delta J$ , where  $J^*$  is a known matrix,  $\Delta J$  denotes the uncertain parts of the matrix  $J$ . In order to apply the proposed design method, we define state variable  $x = (q^T, \omega^T)^T$  and the controlled output  $y = q$ . The equations of motion of the satellite (1)~(3) can be put in the form of MIMO uncertain nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + \Delta f(x) + [g(x) + \Delta g(x)]u \\ y &= h(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} x \end{aligned} \quad (4)$$

$$\text{where } f(x) = \begin{bmatrix} \frac{1}{2}(q^\times + q_0 I)\omega \\ (J^*)^{-1}(-\omega^\times J^* \omega) \end{bmatrix}, \quad \Delta f(x) = \begin{bmatrix} 0 \\ \Delta f_2 \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 0 \\ (J^*)^{-1} \end{bmatrix}, \quad \Delta g(x) = \begin{bmatrix} 0 \\ \Delta g_2 \end{bmatrix},$$

$u = [u_1, u_2, \dots, u_m]^T$ ,  $y = [y_1, y_2, \dots, y_m]^T$ ,  $m=3$ .  $\Delta f(x)$  and  $\Delta g(x)$  denote the terms in (4) which arise due to satellite uncertainties.  $\Delta f_2$  is a function of  $\Delta J, T_d$  and  $\dot{\eta}$ .  $\Delta g_2$  is a function of  $\Delta J$ . The control objective is to design a control law  $u(t)$  which make the output  $y$  tracks a desired trajectory  $y_d = q_d(t) = (q_{d1}, q_{d2}, q_{d3})^T$  in the presence of bounded disturbance  $\Delta f$  and  $\Delta g$ .

In the following section, we will first apply I/O linearization control to (4). The error dynamics for the uncertain system are derived. Then the adaptive fuzzy controller is designed to achieve zero tracking error and to increase the robustness of I/O linearization controller.

### III. I/O LINEARIZATION OF SATELLITE ATTITUDE SYSTEM WITH UNCERTAINTIES

If  $\Delta f = 0$  and  $\Delta g = 0$ , the nonlinear system (4) corresponds to the nominal system. The I/O linearization process is performed on the nominal system by assuming that the nonlinear system (4) has a vector relative degree

$$[r_1, \dots, r_m] \quad \begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = B(x(t)) + A(x(t)) \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \quad (5)$$

where

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{(r_1-1)} h_1(x) & \dots & L_{g_m} L_f^{(r_1-1)} h_1(x) \\ \dots & \dots & \dots \\ L_{g_1} L_f^{(r_m-1)} h_m(x) & \dots & L_{g_m} L_f^{(r_m-1)} h_m(x) \end{bmatrix},$$

$$B(x) = \begin{bmatrix} L_f^{(r_1)} h_1(x) \\ \vdots \\ L_f^{(r_m)} h_m(x) \end{bmatrix} \quad (6)$$

In (6),  $L^k$  ( $k=r_1-1, \dots, r_m-1$ ) denotes the  $k$ th successive Lie derivative. A major drawback of I/O linearization is that it relies on the exact cancellation of nonlinear terms in order to achieve linear input-output relation. Thus the presence of uncertainties causes loss of I/O decoupling, steady-state tracking errors and deteriorated transient responses. However, for a certain class of uncertainties which obey the so called ‘matching condition’, I/O linearization is guaranteed[3]-[4].

Matching condition: If the system has a vector relative degree  $[r_1, \dots, r_m]$ , then the addition of perturbations  $\Delta f$  and  $\Delta g$  do not change the relative degree of the system if,

$$\Delta f(x, t) \quad \text{and} \quad \Delta g_j(x, t) \in \ker \left[ dh_i, dL_f h_i, dL_f^2 h_i, \dots, dL_f^{r_i-2} h_i \right] \quad \text{for } i = j=1, 2, \dots, m.$$

where  $\ker(\dots)$  denotes the kernel of a matrix. This ‘matching condition’ guarantees that the perturbations  $\Delta f$  and  $\Delta g$  do not appear in derivatives of  $y_i$  of order less than  $r_i$ . Therefore if the uncertainties  $\Delta f$  and  $\Delta g$  satisfy ‘matching condition’, there exists diffeomorphic coordinate transformation  $T(x) = (\xi, \eta)$  which transforms system (4) into the nominal form[3]-[4]:

$$\begin{cases} \dot{\xi}^{(r)} = B + \Delta B + (A + \Delta A)u \\ \dot{\eta} = W \end{cases}$$

where  $\xi^{(r)} = [(\xi_1^1)^{(r_1)}, (\xi_1^2)^{(r_2)}, \dots, (\xi_1^m)^{(r_m)}]^T$

$$y^{(r)} = \xi^{(r)} = B + \Delta B + (A + \Delta A)u \quad (7)$$

where  $y^{(r)} = [y_1^{(r_1)}, y_2^{(r_2)}, \dots, y_m^{(r_m)}]^T$ .  $\Delta A$  and  $\Delta B$  arise due to uncertainty  $\Delta f$  and  $\Delta g$  in linearization process. The matrix  $A$  and  $B$  are defined similarly as (6) with the argument  $x$  replaced with  $T^{-1}(\xi, \eta)$ . If the matrix  $A$

is nonsingular, the control law is

$$u = A^{-1}(v - B) \quad (8)$$

where  $v$  is a new synthetic input. Substituting (8) into (7) gives:

$$y^{(r)} = v + \Delta B + \Delta A A^{-1}(v - B) \quad (9)$$

Equation (9) clearly demonstrates that the linear feedback control law (8) alone is not adequate to achieve zero tracking error. Consequently, the adaptive fuzzy controller will be added to the I/O control law(8).

#### IV. HYBRID CONTROLLER DESIGN

##### A. Hybrid controller structure

Under the control law (8), the input-output relation can be rewritten as follows

$$y^{(r)} = v + E(x, v) \quad (10)$$

where  $E(x, v) = \Delta B + \Delta A A^{-1}(v - B)$ ,  $E = [E_1, E_2, \dots, E_m]^T$  denotes the error which arise due to the uncertainties in the system. The  $i$ th output is expressed as  $y_i, i=1,2,\dots,m$ ,

$$y_i^{(r_i)} = v_i + E_i(x, v) \quad (11)$$

To compensate for the uncertainties in the system, the adaptive fuzzy control  $v_{fi}$  is added to the linear feedback control law  $v_i$ . The modified  $v_i$  is defined as

$$v_i = y_{di}^{(r_i)} - (k_{i(r_i-1)}e_i^{(r_i-1)}(t) + \dots + k_{i1}e_i^{(1)}(t) + k_{i0}e_i(t)) + v_{fi}(t) \quad i=1,2,\dots,m \quad (12)$$

where  $y_{di}$  is the desired trajectory,  $e_i = y_i - y_{di}$ ,  $v_{fi}$  is the output of the  $i$ th fuzzy neural network which compensate for the uncertainties. The hybrid controller structure is shown in Fig.1. The control system comprises two parts. One part is the conventional linear feedback law and the other part is the adaptive fuzzy control  $v_{fi}$  that compensate for the uncertainties  $\Delta f$  and  $\Delta g$ .

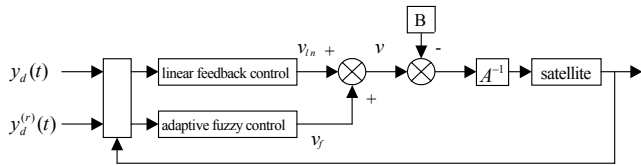


Fig. 1 Diagram of adaptive fuzzy linearization control system

According to (11) and (12), the error dynamic equation is yielded

$$e_i^{(r_i)}(t) + k_{i(r_i-1)}e_i^{(r_i-1)}(t) + \dots + k_{i1}e_i^{(1)}(t) + k_{i0}e_i(t) = v_{fi} + \hat{E}_i \quad (13)$$

$$\text{where } \hat{E}_i = E_i(x, v) \Big|_{v_i = y_{di}^{(r_i)} - \sum_{j=0}^{r_i-1} k_{ij}e_i^{(j)} + v_{fi}}$$

The error dynamics can be defined as a linear dynamic system containing uncertain elements, i.e.

$$\dot{z}_i = A_i z_i + B_i v_{fi} + B_i \hat{E}_i(x, t) \quad (14)$$

$$\text{where } \|\hat{E}_i\| \leq D_i(x, t), A_i = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -k_{i0} & -k_{i1} & \dots & -k_{i(r_i-1)} \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, z_i = (e_i, e_i^{(1)}, \dots, e_i^{(r_i-1)}). k_i \text{ elements are selected}$$

such that  $A_i$  is hurwitz where  $k_i = (k_{i0}, k_{i1}, \dots, k_{i(r_i-1)})$ .

Thus there exists a positive definite matrix  $P_i$  that satisfies the Lyapunov equation  $A_i^T P_i + P_i A_i = -Q_i$ .

##### B. Adaptive fuzzy control

The adaptive fuzzy controller is implemented by the Takagi-Sugeno(T-S) model with constant consequents. The output of the adaptive fuzzy controller is  $y_{TS} = C_f^T \Psi(x)$ .

Where

$$C_f = [C_1, C_2, \dots, C_L]^T, \Psi(x) = [\psi_1(x), \psi_2(x), \dots, \psi_L(x)]^T,$$

$$\psi_l(x) = \frac{\prod_{i=1}^L \mu_{A_i^l}(x_i)}{\sum_{l=1}^L (\prod_{i=1}^L \mu_{A_i^l}(x_i))}, l=1,2,\dots,L \text{ (L is number of}$$

rules).  $A_i^l$  represent the fuzzy set of input variables  $x_i (i=1,2)$ .  $\mu_{A_i^l}(x_i)$  is Gaussian membership function. On the premise of precise attitude control, the only rule consequent parameters  $C_l (l=1,2,\dots,L)$  are on-line adjusted.

There are  $m$  T-S models that are employed to compensate for those  $m$  output tracking errors in attitude control system. Let

$$v_{fi} = C_{fi}^T \Psi_i(z_i) \quad i=1,2,\dots,m \quad (15)$$

Substituting (15) into (14) gives

$$\dot{z}_i = A_i z_i + B_i C_{fi}^T \Psi_i(z_i) + B_i \hat{E}_i(x, t) \quad (16)$$

Let  $\hat{z}_i = A_i \hat{z}_i + B_i C_{fi}^{*T} \Psi_i(\hat{z}_i)$  be an identification model and  $\varepsilon_i = z_i - \hat{z}_i$ , where  $C_{fi}^*$  denotes the optimal  $C_{fi}$ , defined as

$$C_{fi}^* = \arg \min_{|C_{fi}| \leq M} \left[ \sup |v_{fi}(z_i | C_{fi}^*) - v_{fi}(z_i | C_{fi})| \right] \quad (17)$$

$$\text{Therefore } \dot{\varepsilon}_i = A_i \varepsilon_i + B_i \phi_i^T \Psi_i(\varepsilon_i) + B_i \hat{E}_i \quad (18)$$

Where  $\phi_i = C_{fi} - C_{fi}^*$

In the following, the adaptive control law for updating

rule parameter  $C_{\hat{f}_i}$  is derived in order to ensure that  $\varepsilon_i \rightarrow 0$  as  $t \rightarrow \infty$ . A candidate Lyapunov function is chosen as:

$$V_i = \frac{1}{2} (\varepsilon_i^T p_i \varepsilon_i + \frac{\phi_i^T \phi_i}{r_i \|\hat{E}_i\|}) \quad (19)$$

where  $r_i$  is a design parameter. The time derivative of  $V_i$  is

$$\dot{V}_i = -\varepsilon_i^T Q_i \varepsilon_i + \varepsilon_i^T p_i B_i \hat{E}_i + \phi_i^T (r_i \|\hat{E}_i\| \|\varepsilon_i^T p_i B_i\| \Psi_i(\varepsilon_i) + \dot{\phi}_i) \quad (20)$$

Choosing the adaptive law (recalling that  $\dot{\phi}_i = \dot{C}_{\hat{f}_i}$ )

$$\dot{C}_{\hat{f}_i} = -r_i \|\hat{E}_i\| \|\varepsilon_i^T p_i B_i\| \Psi_i(\varepsilon_i) \quad (21)$$

Equation (20) is reiterated using vector norms

$$\dot{V}_i = -\lambda_{\min}(Q_i) \|\varepsilon_i\|^2 + \|\varepsilon_i^T p_i B_i\| \|\hat{E}_i\| \quad (22)$$

When the adaptive fuzzy control  $v_{f_i}$  is added. Let

$$\|\hat{E}_i\| \leq \frac{\lambda_{\min}(Q_i) \|\varepsilon_i\|^2 - \alpha_i \|\varepsilon_i\|}{\|\varepsilon_i^T p_i B_i\|} \quad (23)$$

where  $\alpha_i > 0$

Substituting (23) into (22) gives  $\dot{V}_i \leq -\alpha_i \|\varepsilon_i\|$

To avoid that  $C_{\hat{f}_i}$  take arbitrarily large values, the estimate of  $C_{\hat{f}_i}$  is restricted to a compact set B(M) (where

$\|C_{\hat{f}_i}\| \leq M$  denotes a ball of radius M). Using a Lyapunov function, an adaptive law with projection can be defined as

$$\dot{C}_{\hat{f}_i} = -r_i \|\hat{E}_i\| \|\varepsilon_i^T p_i B_i\| \Psi_i(\varepsilon_i) + \alpha_0 r_i \|\hat{E}_i\| \|\varepsilon_i^T p_i B_i\| \frac{C_{\hat{f}_i} C_{\hat{f}_i}^T \Psi_i(\varepsilon_i)}{|C_{\hat{f}_i}|^2} \quad (24)$$

where

$$\alpha_0 = \begin{cases} 1 & \text{if } |C_{\hat{f}_i}| = M \text{ and } \|\hat{E}_i\| \|\varepsilon_i^T p_i B_i\| C_{\hat{f}_i}^T \Psi_i(\varepsilon_i) > 0 \\ 0 & \text{if } |C_{\hat{f}_i}| \leq M \text{ and } \|\hat{E}_i\| \|\varepsilon_i^T p_i B_i\| C_{\hat{f}_i}^T \Psi_i(\varepsilon_i) < 0 \end{cases} \quad (25)$$

Based on the above discussion, the proposed hybrid control design is outlined as follows. In satellite attitude control system, the overall control law is

$$u = A^{-1}(x) [v_{\ln} + v_f - B(x)] \quad (26)$$

where  $v_{\ln} = [v_{\ln 1}, v_{\ln 2}, v_{\ln 3}]^T$ ,  $v_f = [v_{f1}, v_{f2}, v_{f3}]^T$ ,

$$v_{\ln i} = y_{di}^{(r_i)} - (k_{i(r_i-1)} e_i^{(r_i-1)}(t) + \dots + k_{i1} e_i^{(1)}(t) + k_{i0} e_i(t)),$$

$$v_{f_i} = C_{\hat{f}_i}^T \Psi_i(\varepsilon_i), \quad i=1,2,3.$$

In order to incorporate the adaptive fuzzy control  $v_{f_i}$  into (26), the premise condition is to make the error  $\|\hat{E}_i\|$  satisfy (23). This involves choosing a value for  $\alpha_i$  which clearly depends on bounds of the satellite

uncertainty,  $i=1,2,3$ .

In (26),  $y_{di}^{(r_i)}$  denotes the derivatives of the desired quaternion  $q_{di}$  of order  $r_i$ ,  $e_i = q_i - q_{di}$ ,  $q_i$  denotes the actual quaternion,  $i=1, 2, 3$ .  $\varepsilon_i$  denotes the terms of the difference between the nominal satellite model error in tracking the desired  $q_{di}$  and the error of the uncertain satellite model in tracking  $q_{di}$ , i.e.  $\varepsilon_i = e_i - \hat{e}_i$ . The input of the  $i$ th T-S model are  $\varepsilon_i$  and its rate  $\dot{\varepsilon}_i$ . The output is  $v_{f_i}$ ,  $i=1, 2, 3$ . In simulation, the fuzzy sets of input variables ( $\varepsilon_i, \dot{\varepsilon}_i$ ) are defined as negative big(NB), zero(E), and positive big(PB). The initial mean of NB, E, and PB in  $\varepsilon_i$  is respectively -0.01, 0 and 0.01. The corresponding initial variance is 0.0047. The initial mean of NB, E, and PB in  $\dot{\varepsilon}_i$  is respectively -0.008, 0 and 0.008. The corresponding initial variance is 0.0038. In order to improve the self-learning speed, the parameter vector  $C_{\hat{f}_i}$  is initialized as a linear PD controller with an acceptable performance. The initial fuzzy rules are given in Table I,  $i=1, 2, 3$ .

The satellite dynamic (4) can easily be verified that it has a relative degree of [2,2,2]. Integral error feedback is added to the linear feedback control law  $v_{\ln}$ [1]. Thus  $v_{\ln}$  is written as

$$v_{\ln i} = \ddot{q}_{di} - k_{i1} \dot{e}_i - k_{i0} e_i - p_i d_i \quad i=1, 2, 3 \quad (27)$$

where  $\dot{d}_i = \dot{e}_i$ . The parameters  $k_{i1}$ ,  $k_{i0}$  and  $p_i$  are selected such that  $\ddot{e}_i + k_{i1} \dot{e}_i + k_{i0} e_i + p_i e_i = 0$  is Hurwitz. The desired trajectory  $q_{di}(t)$  is selected as

$$\ddot{q}_{di} + 2\xi w_n \dot{q}_{di} + w_n^2 q_{di} = w_n^2 R_i \quad (28)$$

where  $\xi = 0.707$ ,  $w_n = 0.08$ , external input  $R_i = q_i^* = 0$ ,  $i=1, 2, 3$ .

TABLE I  
THE INITIAL FUZZY RULES

$\dot{\varepsilon}_i$ \ $\varepsilon_i$	NB	E	PB
NB	-0.001	-0.0005	0
E	-0.0005	0	0.0005
PB	0	0.0005	0.001

## V. SIMULATION

The first elastic mode is considered for the simplification. The parameter values used for the flexible satellite are as follows: inertia matrix

$$I = \begin{bmatrix} 6100 & -90 & 20 \\ -90 & 5070 & -1100 \\ 20 & -1100 & 8400 \end{bmatrix} \text{kg} \cdot \text{m}^2, \quad \text{coupling} \quad \text{matrix}$$

$C = [0.3 \ 18 \ -21]^T$  ( $\text{kg}^{1/2}\text{m}$ ), modal frequency  $\Lambda = 1.02 \text{ rad/s}$ , modal damping  $\xi = 0.001$ , control torque range  $[-10, 10]$ , orbital rate  $\omega_0 = 1.078 \times 10^{-3} \text{ rad/s}$ .

Further, the follow initial conditions are chosen as: the actual attitude  $q(0) = [-0.1070 \ 0.6461 \ 0.5327]$ ,  $q_0(0) = 0.5361$ , (namely roll angle  $\theta_1 = 35^\circ$ , pitch angle  $\theta_2 = 80^\circ$ , yaw angle  $\theta_3 = 60^\circ$ ),  $\omega(0) = [0.04 \ 0.04 \ 0.04]^\circ/\text{s}$ ,  $\eta(0) = 0, \dot{\eta}(0) = 0$ , the desired attitude  $q_0 = 1, q = [0 \ 0 \ 0]$ . The linear feedback parameters are selected as follows by observing the simulated response of the satellite,  $k_0 = [0.05 \ 0.06 \ 0.056]$ ,  $k_1 = [0.4 \ 0.5 \ 0.46]$ ,  $p = [1.1 \ 1.6 \ 1.4] \times 10^{-4}$ . For the adaptive fuzzy control, the parameters are set as follows  $Q_1 = Q_2 = Q_3 = \text{diag}(1,1)$ ,  $\alpha_1 = \alpha_2 = \alpha_3 = 0.06$ ,  $r_1 = 0.1$ ,  $r_2 = 0.15$ ,  $r_3 = 0.12$ .

The proposed hybrid control is compared with the conventional PID control. Fig.2 shows the three attitude angles under PID control and the hybrid control in the case of nominal parameters. Fig.3 shows the result in the case of inertia matrix increment of 20%. Under the hybrid control, attitude angles converge to zero in around 100s and the overshoot is  $-0.17^\circ$ . Under PID control, the response time is around 160s and the overshoot is  $-6.23^\circ$ .

To further demonstrate the effectiveness of the hybrid control under larger inertia matrix variations, the inertia matrix is increased to 150%J. Fig.4 shows the response of the satellite using the hybrid control and PID control. Because of space limitation, we only show modal displacement in the last case. Under PID control, the response time is extended to around 220s and the overshoot arrived to  $-9.45^\circ$ . The modal displacement of panel is between  $-0.003\text{m}$  and  $0.003\text{m}$ . Under the hybrid control, the response time is around 110s and the overshoot is  $-2.6^\circ$ , the oscillation of panel attenuates to zero in around 200s.

Comparing with PID control, the hybrid control offers quicker response and is insensitive to the parameter change of the flexible satellite. It could effectively damp out the oscillation of the solar panel that is yielded in the attitude maneuver process, so as to obtain the accurate attitude control of flexible satellite. Furthermore, since the parameter vector  $C_{fi}$  is updated from the set initial values incorporating the expert's knowledge, the attitude tracking error could converge rapidly to zero so as to offer a faster process response.

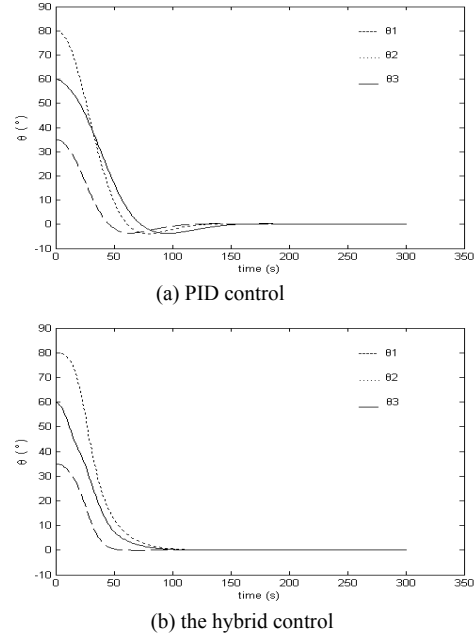


Fig.2 Attitude angle in the case of nominal parameter

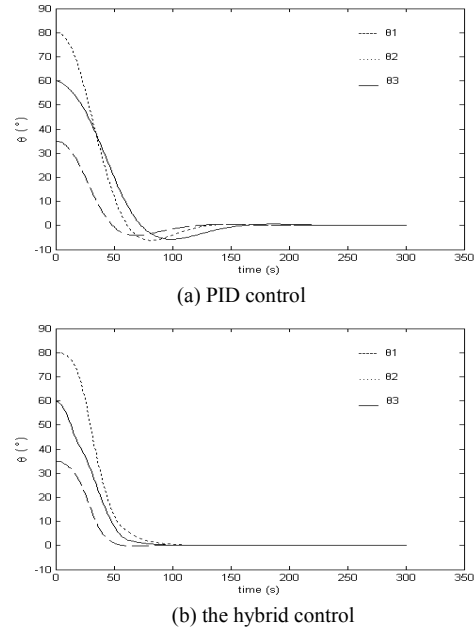
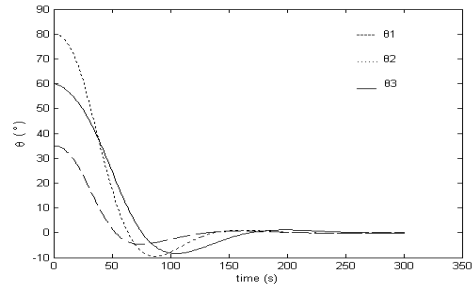
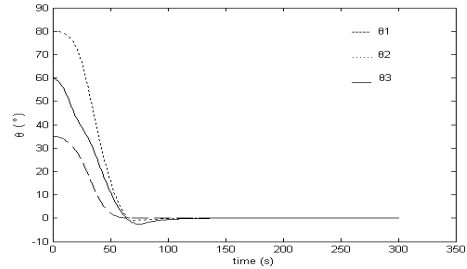


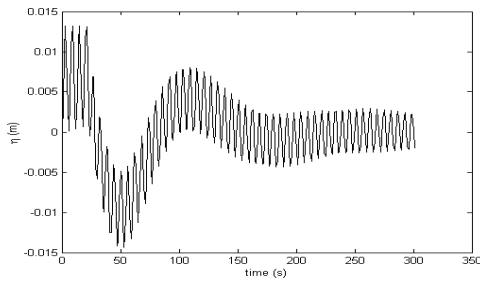
Fig.3 Attitude angle in the case of inertia matrix increment of 20%



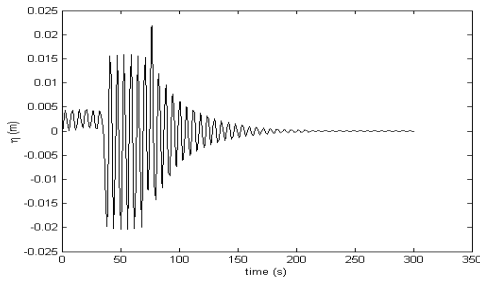
(a) attitude angle under PID control



(b) attitude angle under the hybrid control



(c) modal displacement under PID control



(d) modal displacement under the hybrid control

Fig. 4 Satellite attitude in the case of inertia matrix increment of 50%

## VI. CONCLUSION

The proposed hybrid control structure that combines I/O linearization with adaptive fuzzy control has been outlined. The adaptive fuzzy control compensates for the uncertainties of satellite attitude system so as to enhance the closed-loop system performance by reducing the output tracking error. Based on I/O linearization, the fuzzy control rules employed in MIMO system are decomposed to DISO fuzzy rules, which facilitates the incorporation of experts' experience into the control system. In the practice realization, the hybrid control method needs good quality

quaternion measurements, computations and high-speed control loop closure. The proposed control law has the analytical form and can be implemented using the microprocessor. Simulation results show the derived controller can achieve robust tracking performance for large parameter uncertainty.

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