Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Passivity-Based Attitude Control for an Integrated Power and Attitude Control System using Variable Speed Control Moment Gyroscopes

David A. DeVon
University of Illinois
Urbana-Champaign, IL

Robert J. Fuentes
Boeing - SVS, Inc.
Albuquerque, NM

Jerry L. Fausz
Air Force Research Laboratory
Kirtland AFB, NM

***Abstract*— The attitude control of a rigid spacecraft with a cluster of *N* variable speed control moment gyroscopes is considered from a perspective of passivity. The dynamics are derived using a special form of Euler-Lagrange equations. The spacecraft dynamics and unit quaternion kinematics are shown to be passive. A proportional-integral controller is proposed to make the feedback system passive, which closely resembles previous controller designs using Lyapunov analysis. Consequently, the attitude errors are shown to be globally asymptotically stable. A null space solution is used to provide power tracking of the spacecraft. Numerical simulations are provided for validation.**

I. INTRODUCTION

Most spacecraft use chemical batteries to store energy from solar panels for use during periods of eclipse from the sun. Chemical batteries often require consideration in the spacecraft design given that the batteries have many drawbacks such as weight, efficiency, limited operating conditions, and limited battery life. As an alternative, flywheels offer a promising choice for energy storage given such systems can be lighter, last longer, and provide the capability to not only do power tracking but also attitude control.

Flywheels can be considered as single-gimbal variable speed control moment gyroscopes (VSCMGs) where the varying wheel speeds provide an extra degree of freedom for use in attitude control and power tracking. Energy storage is achieved via converting electrical energy from solar arrays into rotational kinetic energy of the spinning wheels. Power tracking is accomplished by changing the wheel speeds as required, but must be done as not to produce any torques on the spacecraft.

The development of an integrated power and attitude control system (IPACS) using flywheel technology is nontrivial and has been subject of research for many years. The nonlinear equations of motion for a VSCMG cluster in a rigid spacecraft have been derived for different kinematic descriptions of the spacecraft orientation [9][10][14]. The nonlinear equations of motion have been extended to flexible spacecraft [3]. By ignoring the gimbal accelerations, the gimbal rates and wheel spin rates can be taken as control inputs to develop a velocity-based steering control law, which takes advantage of the torque- amplification property of control moment gyroscopes [9][10][14]. A singularity measure is used in defining a weighting matrix to avoid any CMG singularities [10]. Also, an adaptive nonlinear control law for VSCMGs has been previously derived [14].

In addition, passivity properties of rigid spacecraft have been used to derive attitude controllers. Feedback interconnections of passive and strictly passive systems are globally asymptotically stable [5][7]. An adaptive attitude controller using passivity has been derived using a unit quaternion tracking error. [2]. By using passivity theorems, the attitude controllers have been extended to systems without angular velocity measurements [6][11][12].

In this work, a rigid spacecraft with a cluster of VSCMGs is considered. The system energy is used to derive the complete set of nonlinear dynamical equations of motion. Passivity theory, as opposed to Lyapunov analysis, is used to derive the attitude control for the integrated power and attitude control problem. The simultaneous power tracking and attitude control is discussed. Simulations are provided for validation.

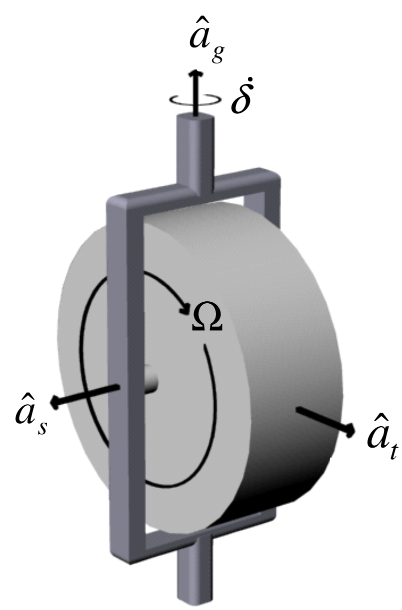

Fig. 1. Variable Speed Control Moment Gyroscope

II. SYSTEM MODEL

Consider a rigid body spacecraft with a cluster of N variable speed control moment gyroscopes (VSCMGs). Let a body-fixed reference frame associated with the spacecraft be denoted by $\mathscr{B}$, and let a inertial frame be denoted by $\mathscr{I}$. The cluster is fixed in $\mathscr{B}$ with the gimbal axes of each VSCMG fixed within the cluster. Let $\hat{a}_{g_i}$, $\hat{a}_{s_i}$, and $\hat{a}_{t_i}$ represent the i-th VSCMG's directional unit vectors of the gimbal, spin, and transverse axes expressed in $\mathscr{B}$ as shown in Fig. 1. Define the matrix $A_g \in \mathbb{R}^{3\times N}$ such that i-th column of A_g is the directional unit vector of the i-th VSCMG's gimbal axes. The matrices, $A_s \in \mathbb{R}^{3\times N}$ and $A_t \in \mathbb{R}^{3\times N}$ are defined similarly for the spin and transverse directions [3][9][10][14]. Given the initial unit vectors, the matrices are functions of the gimbal angles, denoted by $\delta \in \mathbb{R}^N$, and can expressed as $A_g = A_{g_0}$, $A_s = A_{s_0}[\cos\delta]^d - A_{t_0}[\sin\delta]^d$, and $A_t = A_{t_0}[\cos\delta]^d + A_{s_0}[\sin\delta]^d$, where $[x]^d \in \mathbb{R}^{N\times N}$ denotes a diagonal matrix with elements $x \in \mathbb{R}^N$. Given $x \in \mathbb{R}^3$, define the skew symmetric operator $x^\times \in \mathbb{R}^{3\times 3}$, which represents the cross-product operation, as

$$x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

Next, the constant inertia matrices of the system are defined. $I_{s/c} \in \mathbb{R}^{3\times 3}$ is the total inertia matrix of the spacecraft including the contributions from the center of mass of each VSCMG. The matrices $I_{gw\diamondsuit} \in \mathbb{R}^{N\times N}$ and $I_{w\diamondsuit} \in \mathbb{R}^{N\times N}$ are diagonal matrices with elements being the inertias of the gimbal including the wheel and inertias of just the wheel of each VSCMG corresponding to the triad of unit directions vectors, e.g. gimbal axes ($\diamondsuit$ = g), spin axes ($\diamondsuit$ = s), or transverse axes ($\diamondsuit$ = t). Consequently, the matrix $I_{ws} \in \mathbb{R}^{N\times N}$ is a diagonal matrix with elements of each VSCMG wheel inertia along the spin axis. The inertia matrix $I_\diamondsuit = I_{gw\diamondsuit} + I_{w\diamondsuit} \in \mathbb{R}^{N\times N}$ is the sum of diagonal matrices $I_{gw\diamondsuit}$ and $I_{w\diamondsuit}$.

A. Dynamics

In general, the system's equations of motion can be derived using either classical mechanics or Lagrangian mechanics. The former uses Newton's and Euler's laws to provide some physical intuition into the system dynamics. The latter approach uses the system's energy and Euler-Lagrange equations to derive the equations of motion. The Lagrangian approach is considered using the system's kinetic energy and a special form of the Euler-Lagrange equations.

1) Kinetic Energy: The total kinetic energy of the spacecraft including N VSCMGs can be expressed as the sum of the kinetic energy of the spacecraft plus the kinetic energy of the gimbals and wheels, $T = T_{s/c} + T_g + T_w$. Let $N \in \mathbb{R}$ be arbitrary. By considering the inertia and angular velocities w.r.t. $\mathscr{I}$ expressed in $\mathscr{B}$, the total kinetic energy of the system is given by

$$T = \frac{1}{2}\omega^T J\omega + \frac{1}{2}\dot{\delta}^T I_g \dot{\delta} + \omega^T A_g I_g \dot{\delta} + \frac{1}{2}\Omega^T I_{ws}\Omega + \omega^T A_s I_{ws}\Omega$$

where $\omega \in \mathbb{R}^3$ is the spacecraft's angular velocity, $\dot{\delta} \in \mathbb{R}^N$ is a column vector of the N VSCMGs' gimbal rates, $\Omega \in \mathbb{R}^N$ is a column vector of the N VSCMGs' wheel spin rates, and $J \in \mathbb{R}^{3\times 3}$ is the inertia matrix of the spacecraft including the variable speed control moment gyroscopes,

$$J = I_{s/c} + A_s I_s A_s^T + A_t I_t A_t^T + A_g I_g A_g^T$$

The time derivative of J is given by

$$\dot{J} = A_t\left[\dot{\delta}\right]^d (I_t - I_s) A_s^T + A_s\left[\dot{\delta}\right]^d (I_t - I_s) A_t^T$$

where $\dot{A}_s = -A_t\left[\dot{\delta}\right]^d$ and $\dot{A}_t = A_s\left[\dot{\delta}\right]^d$. The total kinetic energy can be written in matrix form as

$$T = \frac{1}{2}\dot{r}^T M(\delta)\dot{r} \tag{1}$$

where

$$M(\delta) = \begin{bmatrix} I_g & 0 & I_g A_g^T \\ 0 & I_{ws} & I_{ws} A_s^T \\ A_g I_g & A_s I_{ws} & J \end{bmatrix} > 0$$

and $\dot{r} = \begin{bmatrix} \dot{\delta} & \Omega & \omega \end{bmatrix}^T \in \mathbb{R}^{3+2N}$.

2) Euler-Lagrange Equations: Consider a system using generalized coordinates, q, possibly subject to either holonomic or non-holonomic constraints. Assuming no potential field, the Euler-Lagrange equations can be written as

$$\frac{dt}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = Q + F^T\lambda$$

where Q is the generalized force moment, F is Jacobian of the constaint vector, and λ are Lagrange multipliers [4].

The standard form of Euler-Lagrange equations use generalized coordinates where the generalized velocities are integrable with respect to time, e.g. $\int \dot{q}_i = q_i$. When the velocities are not integrable, the standard form of Euler-Lagrange equations do not apply. These velocities, known as quasi-velocities, are considered as time derivatives of quasi-coordinates. Quasi-coordinates, r, are coordinates that only the time derivatives have any physical meaning [1]. The spacecraft's angular velocity is an example of a quasi-velocity since it is not integrable and can be considered the time derivatives of some quasi-coordinates [1][8]. Given the kinetic energy is expressed in terms of true and quasi-coordinates, this leads to using a special form of Euler-Lagrange equations.

3) Boltzmann-Hamel Equations: The Lagrange equations for quasi-coordinates are called Boltzmann-Hamel equations [1][13]. Let the relationship between the quasi-velocities and true velocities be given by $\dot{r} = S(q)\dot{q}$. Since the kinetic energy does not depend on the true coordinates and assuming no potential field, the usable form of the Boltzmann-Hamel equations become

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right) + \begin{bmatrix} 0 & 0 \\ 0 & \omega^\times \end{bmatrix}\left(\frac{\partial T}{\partial \dot{r}}\right) - S(q)^{-1}\left(\frac{\partial T}{\partial q}\right) = \tau \tag{2}$$

where $\tau = \begin{bmatrix} \tau_{\dot{\delta}} & \tau_\Omega & \tau_{ext} \end{bmatrix}^T \in \mathbb{R}^{3+2N}$ is column vector of the internal moments associated with the gimbals/wheels of the VSCMGs and the external moment on the spacecraft.

By using Euler angles to represent the spacecraft orientation, the true coordinates become $q = \begin{bmatrix} \delta & \alpha & n \end{bmatrix} \in \mathbb{R}^{3+2N}$ where $n \in \mathbb{R}^3$ is a vector of the Euler angles and $\alpha \in \mathbb{R}^N$ is a vector of position angles of an arbitrary reference point on each VSCMG wheel. Considering Eq. (1) and Eq. (2), the dynamical equations of motion for the system are given by

$$I_g\ddot{\delta} + I_g A_g^T \dot{\omega} - \beta\omega + [\Omega]^d I_{ws} A_t^T \omega = \tau_\delta \tag{3a}$$

$$I_{ws}\dot{\Omega} + I_{ws}\dot{A}_s^T \omega + I_{ws} A_s^T \dot{\omega} = \tau_\Omega \tag{3b}$$

$$\begin{aligned} \dot{J}\omega + J\dot{\omega} + A_g I_g \ddot{\delta} + A_s I_{ws}\dot{\Omega} + \dot{A}_s I_{ws}\Omega + \\ [\omega]^\times \left(J\omega + A_g I_g \dot{\delta} + A_s I_{ws}\Omega\right) = \tau_{ext} \end{aligned} \tag{3c}$$

where $\beta = \frac{1}{2}\left[A_s^T\omega\right]^d (I_t - I_s) A_t^T + \frac{1}{2}\left[A_t^T\omega\right]^d (I_t - I_s) A_s^T$.

B. Kinematics

The spacecraft orientation is represented by using an unit quaternion, or Euler parameters, corresponding to the transformation from $\mathscr{B}$ frame to the inertial reference frame, $\mathscr{I}$. Define the unit quaternion as $q = \begin{bmatrix} q_o & q_v \end{bmatrix}^T \in \mathbb{R}^4$ where $q_o \in \mathbb{R}$ and $q_v \in \mathbb{R}^3$ are the scalar and vector components. The unit quaternion satisfies the holonomic constraint

$$q^T q = q_o^2 + q_v^T q_v = \sum_{i=0}^{3} q_i = 1$$

The kinematic differential equation can be given as

$$\dot{q} = \frac{1}{2} Q(q)\omega \tag{4}$$

where the matrix $Q(q) \in \mathbb{R}^{4\times 3}$ is defined as

$$Q(q) \triangleq \begin{bmatrix} -q_v^T \\ q_o I_{3\times 3} + q_v^\times \end{bmatrix} = \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_o & -q_3 & q_2 \\ q_3 & q_o & -q_1 \\ -q_2 & q_1 & q_o \end{bmatrix} \tag{5}$$

Equations (3c) and (4) form the spacecraft's equations of motion.

III. PASSIVITY-BASED ATTITUDE CONTROL

Let the desired spacecraft attitude be described by the body-fixed frame, $\mathscr{D}$, which gives a desired attitude quaternion, $q_d = \begin{bmatrix} q_{o_d} & q_{v_d} \end{bmatrix}^T$, and an angular velocity of $\mathscr{D}$ with respect to $\mathscr{I}$ expressed in $\mathscr{D}$, ω_d. The desired orientation of the spacecraft is represented by the rotation matrix, $R_d \in SO_3$, where the current attitude is given by the rotation matrix, $R \in SO_3$. SO_3 defines the special orthogonal group, which is the subset of 3×3 matrices that satisfy $R^T R = 1$ and $det(R) = 1$. Define the attitude error rotation matrix, $R_e = R_d R \in SO_3$, and the error quaternion, $e^T = \begin{bmatrix} e_o & e_v \end{bmatrix}^T$ from

$$e = \begin{bmatrix} q_{0d} \\ q_{vd} \end{bmatrix} \otimes \begin{bmatrix} q_0 \\ q_v \end{bmatrix} = \begin{bmatrix} e_o \\ e_v \end{bmatrix}$$

The error rotation matrix between $\mathscr{D}$-frame and $\mathscr{B}$-frame can be written as $R_e = \left(e_o{}^2 + e_v{}^T e_v\right) I_3 + 2 e_v e_v{}^T - 2 e_o e_v^\times$. Define the angular velocity error as $\omega_e = \omega - \omega_d$.

Using the error quaternion and angular velocity error, the spacecraft tracking error dynamics and kinematics can be written as

$$\frac{1}{2}\dot{J}\omega_e + J\dot{\omega}_e = -\omega_e^\times J\omega_e + z \tag{6}$$

$$\dot{e} = \frac{1}{2} E(e)\omega_e \tag{7}$$

where

$$E(e) = \begin{bmatrix} -e_v{}^T \\ e_o I_{3\times 3} + e_v^\times \end{bmatrix} = \begin{bmatrix} -e_1 & -e_2 & -e_3 \\ e_o & -e_3 & e_2 \\ e_3 & e_o & -e_1 \\ -e_2 & e_1 & e_o \end{bmatrix}$$

and $z = -\frac{1}{2}\dot{J}(\omega + \omega_d) - J\dot{\omega}_d - \omega_d^\times J\omega - \omega^\times J\omega_d + \omega_d^\times J\omega_d - \omega^\times \left(A_g I_g \dot{\delta} + A_s I_{ws}\Omega\right) - A_g I_g \ddot{\delta} + A_t \left[\dot{\delta}\right]^d I_{ws}\Omega - A_s I_{ws}\dot{\Omega}$. Using the error dynamic and kinematic equations, passivity theory yields the following proposition

Proposition 1: (i.) The mapping $z \to \omega_e$ is passive.
(ii.) The mapping $\omega_e \to e_v$ is passive.
Proof: (i) Define the function $V = \frac{1}{2}\omega_e^T J\omega_e \geq 0$ where J is the positive definite function defined previously. The derivative along the trajectories of Eq. (6) yields $\dot{V} = \frac{1}{2}\omega_e^T \dot{J}\omega_e + \omega_e^T J\dot{\omega}_e = \omega_e^T z$. Hence, $\int_0^T \omega_e^T z = V(T) - V(0) \geq -V(0)$ where $T \geq 0$. Thus, the system with input z and output ω_e is passive. (ii) [2][6] Given the following from Eq. (7), $\dot{e}_0 = -\frac{1}{2}\omega_e^T e_v$. Hence, $\int_0^T \omega_e^T e_v = 2\left[e_o(0) - e_o(T)\right] \geq -4$, where $|e_o(0) - e_o(T)| \leq 2$. Thus, the system with input ω_e and output e_v is passive. □

A. Feedback Passive System

From Proposition (1), Eq. (6) and Eq. (7) represent a cascade interconnection of two passive systems. By using feedback, the first subsystem can be made output strictly passive. Then, closing the outer loop with the second subsystems output can yield a feedback interconnection of a output strictly passive and passive system. As an alternative, the system could be designed such that the first subsystem is passive, and the system in feedback is any strictly passive system [6][11]. Passivity theorems state that a feedback interconnection of a passive and strictly passive system is globally asymptotical stable [5][7][11].

Proposition 2: Consider the system in Eq. (6) with the feedback control law $z = -K_v\omega_e + v$, where K_v is a positive definite constant gain matrix. The system with input v and output ω_e is output strictly passive.
Proof: Define the positive definite function $V = \frac{1}{2}\omega_e^T J\omega_e$. The derivative along the trajectories is given by $\dot{V} = -\omega_e^T K_v \omega_e + \omega_e^T v$. Let $K_v = k_v I_{3\times 3}$, where $k_v > 0$ is a scalar. The integral of the supply rate[8], $\omega_e^T v$, is $\int_0^T \omega_e^T v \geq -V(0) + k_v \int_0^T \|\omega_e\|^2, T \geq 0$. Thus, the system with input v and output ω_e is output strictly passive (OSP) [7]. □

Considering the output strictly passive system as in Fig. 2, the quaternion error is used to form a feedback

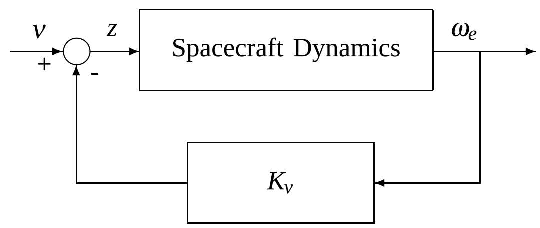

Fig. 2. Output Strictly Passive System.

connection of an output strictly passive and passive system. This will globally asymptotically stabilize the system to the origin, e.g. $\omega_e = 0$. See Fig. 3.

Theorem 1: The linear control law $z = -k_v\omega_e - k_p e_v$, where $k_p > 0$ and $k_v > 0$, globally asymptotically stabilizes the system about the origin given by the spacecraft tracking error dynamics and kinematics.

Proof: The proof is similar to that used by Tsiotras [12] with Rodrigues parameters. Consider a Lyapunov function that is the sum of the individual storage functions for the spacecraft tracking error dynamics and kinematics. This yields the positive definite function

$$V(\omega_e, e) = \frac{1}{2}\omega_e^T J\omega_e + k_p\left[(e_o-1)^2 + e_v{}^T e_v\right] > 0$$

where $k_p > 0$. The derivative along the trajectories is

$$\dot{V}(\omega_e, e) = \omega_e^T z + k_p\omega_e^T e_v = -k_v\|\omega_e\|^2 \leq 0$$

This gives Lyapunov stability. Let Λ be the largest invariant set in $\Psi = \{(e,\omega_e)\,|\,\dot{V} = 0\} = \{(e,\omega_e)\,|\,\omega_e = 0\}$. Since Λ is invariant, this implies that $\dot{\omega}_e = 0$ on Λ. From the linear control law and Eq. (6), $\dot{\omega}_e = 0$ implies $k_p e_v = 0$ or $e_v = 0$. Hence, $\Lambda = \{(e,\omega_e)\,|\,\omega_e = 0, e_v = 0\}$. By LaSalle's Invariance principle, all trajectories converge to the invariant set, Λ; thus, the linear control law asymptotically stabilizes the system in the (e,ω_e) space. In addition, since $V(\omega_e, e)$ is radially unbounded, the closed-loop system trajectories are globally asymptotically stable [5].

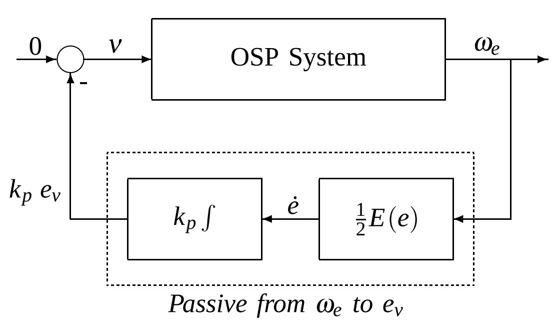

Fig. 3. Feedback Interconnection of an Output Strictly Passive System and Passive System.

B. Required Control Torque

The control law $z = -k_v\omega_e - k_p e_v$ and Eq. (6) give the equation

$$B\ddot{\delta} + C\dot{\delta} + E\dot{\Omega} = L_r$$

where

$$\begin{aligned} B &= A_g I_g \\ C &= \omega^{\times} A_g I_g + \frac{1}{2}A_t\left(I_t - I_s\right)\left[A_s^T\left(\omega + \omega_d\right)\right]^d \\ &\quad + \frac{1}{2}A_s\left(I_t - I_s\right)\left[A_t^T\left(\omega + \omega_d\right)\right]^d - A_t I_{ws}\left[\Omega\right]^d \\ E &= A_s I_{ws} \end{aligned}$$

The required control torque for attitude tracking, L_r, is given by

$$L_r = k_v\omega_e + k_p e_v - J\dot{\omega}_d - \omega_d^{\times} J\omega_e - \omega^{\times}\left(J\omega_d + A_s I_{ws}\Omega\right) \quad (8)$$

and closely follows the nonlinear control law from Lyapunov analysis [3][9][10][14]. In general, the feedback gain, k_v, need not be scalar but a positive definite matrix.

The gimbal acceleration term is considered small and is ignored with control inputs given by $\dot{\delta}$ and $\dot{\Omega}$. The control inputs are chosen to satisfy the following velocity-based steering law

$$C\dot{\delta} + E\dot{\Omega} = L_r \quad (9)$$

or in matrix form as

$$\begin{bmatrix} E & C \end{bmatrix}\begin{bmatrix} \dot{\Omega} \\ \dot{\delta} \end{bmatrix} = L_r \quad (10)$$

Given any required control torque, L_r, the spacecraft will track the desired attitude given that Eq. (10) gives a solution. Depending on N, the number of VSCMGs, and the orientation of each VSCMG, the VSCMG cluster may not be able to produce the required torque vector. This constitutes a singularity. As long as the matrix $D \triangleq \begin{bmatrix} E & C \end{bmatrix}$ has full rank and $N \geq 2$, the range space of D will span 3 dimensions, and attitude tracking can be achieved.

The general solution to Eq. (10) can be expressed in terms of the particular (Range) and homogeneous (Null) solutions. This is given by[10,12,16]

$$\begin{bmatrix} \dot{\Omega} \\ \dot{\delta} \end{bmatrix} = WD^T\left(DWD^T\right)^{-1} L_r + \begin{bmatrix} \dot{\Omega}_h \\ \dot{\delta}_h \end{bmatrix} \quad (11)$$

The weighting matrix, $W \in \mathbb{R}^{2N\times 2N}$, is a diagonal matrix defined as $W = \mathrm{diag}\left[W_s, W_g\right]$, where $W_g = \mathrm{diag}\left[W_{g_i}\right]$ has elements that are constant weights for maximizing the torque-amplification property of the CMGs, and $W_s = \mathrm{diag}\left[W_{s_i}\right]$ has weighting elements that are important for use near a CMG singularity and are given by

$$W_{s_i} = W_{sc_i} e^{-\mu\zeta}, \quad i = 1, \ldots, N \quad (12)$$

W_{sc_i} and μ are constant design parameters [9]. The variable ζ is the minimum singular value of D and is used to describe proximity to a CMG singularity [3][10].

IV. Power Tracking

The kinetic energy of the VSCMGs' wheels is given by $T_w = \frac{1}{2}\Omega^T I_{ws}\Omega = \frac{1}{2}\sum_{i=1}^{N} I_{ws_i}\Omega_i^2$. By taking the derivative of

T_w and using the null space solution of Eq. (11), the ideal power generated by the wheels is given by

$$P = \Omega^T I_{ws} \dot{\Omega}_h = P_{out} = \begin{bmatrix} \Omega^T I_{ws} & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\Omega}_h \\ \dot{\delta}_h \end{bmatrix} \quad (13)$$

The homogeneous solution of Eqn. 11 is

$$\begin{bmatrix} \dot{\Omega}_h \\ \dot{\delta}_h \end{bmatrix} = \underbrace{\left(I - WD^T \left(DWD^T\right)^{-1} D\right)}_{\Pi_\perp} \begin{bmatrix} \alpha_{arb} \\ \beta_{arb} \end{bmatrix} \quad (14)$$

where $\alpha_{arb} \in \mathbb{R}^N$ and $\beta_{arb} \in \mathbb{R}^N$ are arbitrary vectors. By substituting Eq. (14) into Eq. (13), the power output can be written as

$$P = \Omega^T I_{ws} \Phi \alpha_{arb} - \Omega^T I_{ws} \Psi \beta_{arb} \quad (15)$$

Since α_{arb} and β_{arb} can be chosen arbitrarily, this motivates the following definition.

Definition: Given Eq. (15), a *power singularity* is defined when the axes configuration and wheel speeds satisfy both of the following conditions:

i. $\Omega^T I_{ws} \underbrace{\left(I_{N\times N} - W_s E^T \left(E W_s E^T + C W_g C^T\right)^{-1} E\right)}_{\Phi} = 0$

ii. $\Omega^T I_{ws} \underbrace{W_s E^T \left(E W_s E^T + C W_g C^T\right)^{-1} C}_{\Psi} = 0$

When a power singularity is encountered, the VSCMGs can not achieve the required power output. It is sufficient that C have rank 3, or equivalently, not being at a CMG singularity. In this case, no power singularity is encountered via the following argument: *Assume not:* C has rank 3 and the system is at a power singularity. Then $\Omega^T I_{ws} = \Omega^T I_{ws} W_s E^T \left(E W_s E^T + C W_g C^T\right)^{-1} E \neq 0$ for all non-zero wheel speeds. This implies that $\Omega^T I_{ws} W_s E^T \left(E W_s E^T + C W_g C^T\right)^{-1} \neq 0$ is a non-zero 3x1 column vector, so (ii.) is never satisfied since C is rank 3. Therefore, C having rank 3 is a sufficient condition for power singularity avoidance.

The open loop solution to the power tracking problem in the null space of Eq. (10) is given by

$$\begin{bmatrix} \dot{\Omega}_h \\ \dot{\delta}_h \end{bmatrix} = \Pi_\perp \begin{bmatrix} I_{ws}\Omega \\ 0 \end{bmatrix} \left(\begin{bmatrix} \Omega^T I_{ws} & 0 \end{bmatrix} \Pi_\perp \begin{bmatrix} I_{ws}\Omega \\ 0 \end{bmatrix} \right)^{-1} P_n \quad (16)$$

where $P_n = P - \begin{bmatrix} \Omega^T I_{ws} & 0 \end{bmatrix} \cdot WD^T \left(DWD^T\right)^{-1} L_r \in \mathbb{R}$.

Equations (11) and (16) complete the simultaneous attitude and power tracking solution of the velocity-based steering equation, Eq. (10).

V. NUMERICAL EXAMPLES

Numerical simulations were performed using the described control algorithms for simultaneous attitude control and power tracking. The gimbal axes of four VSCMGs are aligned to create a pyramid configuration with respect to the body. The reference trajectory used in the numerical simulation is a near-polar LEO that provides both required sun and ground tracking, similar to that of an Iridium 25778 satellite. Specifically, the symmetric axis of the spacecraft must track Albuquerque, New Mexico, while rotating about the symmetric axis to keep the solar panels perpendicular to the sun. In addition, the spacecraft must follow a given power profile. Table 1. contains the configuration data. The results show the spacecraft is tracking the desired trajectory while maintaining the required power profile.

TABLE I

SIMULATION PARAMETERS

Parameter	Value
N	4
θ	54.75 deg
$\omega(0)$	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ rad/sec
$q(0)$	$\begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$
$\delta(0)$	$\begin{bmatrix} \frac{\pi}{4} & -\frac{\pi}{4} & -\frac{\pi}{4} & \frac{\pi}{4} \end{bmatrix}$ rad
$\dot{\delta}(0)$	$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$ rad/sec
$\Omega(0)$	$\begin{bmatrix} 50000 & 45000 & 40000 & 37000 \end{bmatrix}$ rpm
I_{w_i}	diag$\begin{bmatrix} 0.7 & 0.2 & 0.2 \end{bmatrix}$ $Kg\ m^2$
I_{w_i}	diag$\begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}$ $Kg\ m^2$
k_v	700
k_p	35
μ	1e^{-4}
W_{s_i}	40
W_{g_i}	1

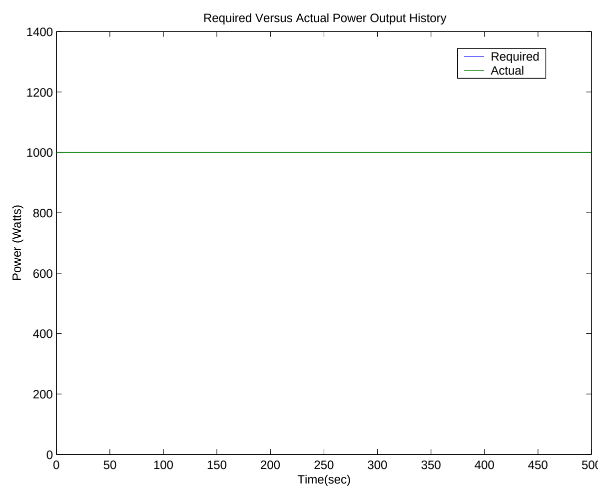

Fig. 4. Spacecraft Output Power

VI. CONCLUSIONS

In this paper, the attitude control problem using variable speed control moment gyroscopes was developed using the system energy and passivity. The resulting control algorithm closely follows previous results using Lyapunov analysis. By using the null space and avoiding singularities, power tracking can be achieved without exerting any torques on the spacecraft. A numerical simulation is given to illustrate the control method validity. Future work will extend the ideal power tracking approach to a closed form methodology that compensates for power losses and system disturbances.

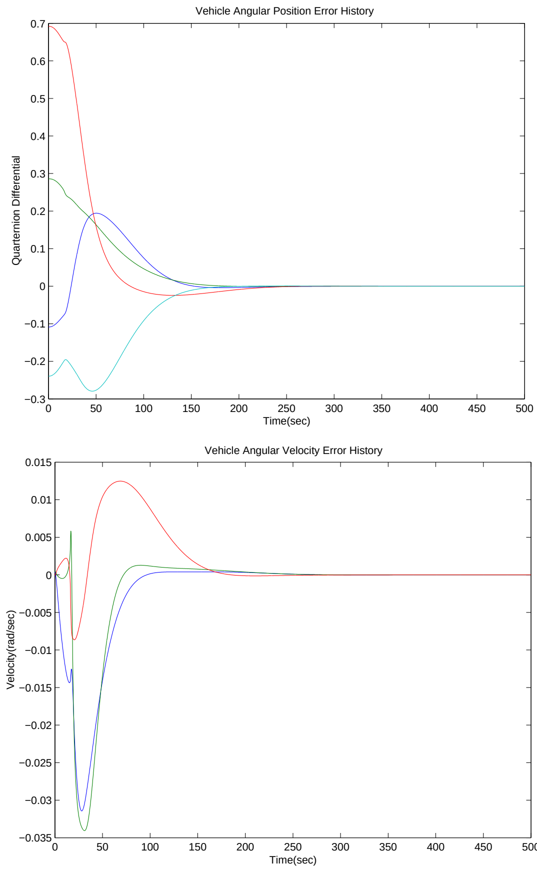

Fig. 5. Spacecraft Attitude Error.

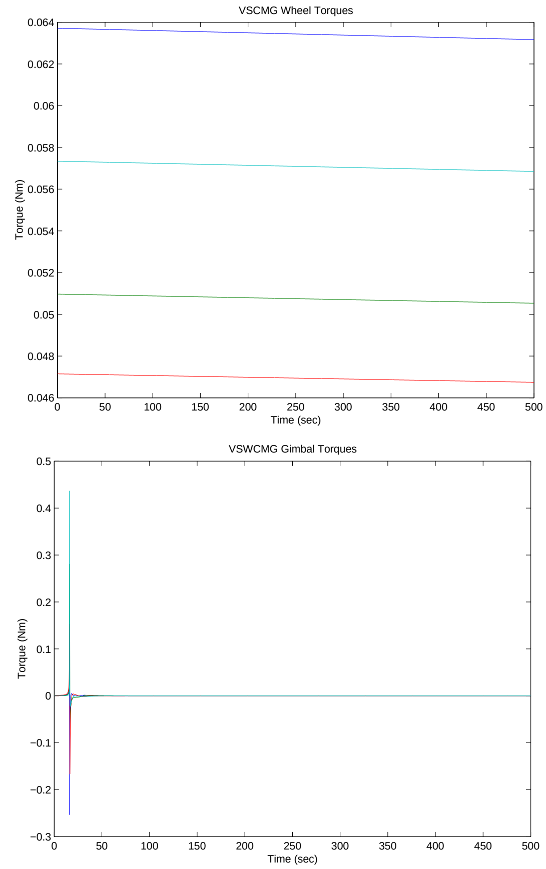

Fig. 6. Spacecraft Control Torques.

VII. Acknowledgments

This work was carried out during a visit of the first author to the AFRL/VS Directorate through the Space Scholar program.

References

[1] Ebert-Uphoff, I. and Kozak, K. "Review of the Role of Quasi-Coordinates for the Kinematic and Dynamic Modeling of Parallel Manipulators." Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators, Canada; 2002.

[2] Egeland, O. and Godhavn, J. M. "Passivity-Based Adaptive Attitude Control of a Rigid Spacecraft." *IEEE Transactions on Automatic Control*, Vol. 41(3): pp. 468 - 472; 1996.

[3] Ford, K.A. "Reorientations of Flexible Spacecraft Using Momentum Exchange Devices." Ph.D. Dissertation, Air Force Institute Technology, WPAFB, OH; 1997.

[4] Greenwood, D.T. *Principles of Dynamics*. Prentice Hall, Englewood Cliffs, NJ; 2nd ed. 1988.

[5] Khalil, H.K. *Nonlinear Systems*. Prentice Hall, Upper Saddle River, NJ; 3rd ed. 2002.

[6] Lizarralde, F. and Wen, J. T. "Attitude Control without Angular Velocity Measurement: A Passivity Approach." *IEEE Transactions on Automatic Control*, Vol. 41(3); 1996.

[7] Lozano, R., Brogliato, B., Egeland, O., and Maschke B. *Dissipative Systems Analysis and Control*. Springer-Verlag, London; 2000.

[8] Meirovitch, L. *Methods of Analytical Dynamics*. McGraw Hill; 1970.

[9] Richie, D.J., Tsiotras, P. and Fausz, J.L. "Simultaneous Attitude Control and Energy Storage using VSCMGs: Theory and Simulation." American Control Conference, Arlington, VA; 2001.

[10] Schaub, H. and Junkins, J. L. *Analytical Mechanics of Space Systems*. AIAA, Reston, VA; 2003.

[11] Tsiotras, P. "A Passivity Approach to Attitude Stabilization using Nonredundant Sets of Kinematic Parameters." *Proceedings of the 34th Conference on Decision and Control*, New Orleans; 1995.

[12] Tsiotras, P. "Futher Passivity Results for the Attitude Control Problem." *IEEE Transactions on Automatic Control*, Vol. 43(11):1597-1600; 1998.

[13] Whittaker, E.T. *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*. Cambridge University Press, NY; 1965.

[14] Yoon, H. and Tsiotras, P. "Spacecraft Adaptive Attitude Control and Power Tracking with Single-Gimballed VSCMGs and Wheel Speed Equalization." AAS/AIAA Astrodynamics Conference, Quebec City; 2001.