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The gaptooth scheme, patch dynamics and equation-free controller design for distributed complex/ multiscale processes

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***Abstract*— We present an equation-free multiscale computational framework for the design of "coarse" controllers for spatially distributed processes described by microscopic/mesoscopic evolution rules. In particular, we exploit the smoothness *in space* of the process observables to estimate the unknown coarse system dynamics. This is accomplished through appropriately initialized *and linked* ensembles of microscopic simulations realizing only a small portion of the macroscopic spatial domain (the so-called gaptooth and patch-dynamics schemes, [10]). We illustrate this framework by designing discrete-time, coarse linear controllers for a Lattice-Boltzmann (LB) scheme modelling a reaction-diffusion process (a kinetic-theory based realization of the FitzHugh-Nagumo equation in one spatial dimension).**

I. Introduction

In recent years, there has been an increasing need for the development of controllers for the macroscopic, "coarse" behavior of processes for which physical models are available at the microscopic/stochastic level (e.g. kinetic Monte Carlo, Lattice Boltzmann or Molecular Dynamics codes), but for which no explicit, macroscopic evolutionary partial differential equation (PDE) description is available in closed form. When such explicit evolution equations are available, the "state" whose evolution they describe consists, typically, of the leading moments of stochastically/microscopically evolving distributions (e.g. density and momentum in the case of fluid flow, concentration or surface species coverage in the case of catalytic chemical reactions). The purpose of this contribution is to describe and illustrate a systematic framework that can be used to design controllers for the macroscopic behavior of such spatially distributed processes. Circumventing the derivation of explicit closed equations, we design controllers acting directly on the microscopic simulator.

Our work follows the "equation-free" approach to computer-aided modeling of complex, multiscale systems that we have been developing over the last few years [24], [10]. In this approach, macroscopic modeling tasks yielding information over *long* time and *large* spatial scales, are accomplished through appropriately initialized calls to the microscopic simulator for only *short* times and *small* spatial domains. This constitutes a systems identification based, "closure on demand" computational toolkit, bridging microscopic/stochastic simulation with traditional continuum scientific computation and numerical analysis. Other approaches towards the construction of hybrid multiscale models include linking molecular dynamics with continuum conservation equations [13], [2] and the quasicontinuum method of Phillips, Ortiz and coworkers [15], [16].

The main tool of the equation free approach is the so-called "coarse time-stepper". *Lifting* macroscopic initial conditions to ensembles of consistent microscopic ones, *evolving* based on the microscopic rules, and *restricting* back to macroscopic variables provides an estimate of the time-stepper of the unavailable macroscopic evolution equation. The methodology has been described in detail in a sequence of publications ([24], [5], [10] and references therein).

Performing such steps for appropriately chosen nearby initial conditions allows the estimation of the action of the (slow) linearization of the unavailable coarse model, and can be integrated in computational algorithms for the location of unstable coarse fixed points, as well as filter and controller design. Recently we have been able to design such linear, discrete-time, coarse controllers for microscopic processes (kinetic Monte Carlo simulations of surface reactions [22] as well as Brownian Dynamics simulators of nematic liquid crystal dynamics [21]). We also demonstrated that, if one has the computational power to evolve the microscopic model over *macroscopic* spatial domains, the coarse time-stepper can be used for coarse controller design of spatially distributed systems [1]. In this case, estimation of the action of the slow components of the system linearization requires interfacing with matrix free numerical techniques (such as the Recursive Projection Method [20], or GMRES [8]).

Other recent approaches to the control of systems described by microscopic evolution rules include model reduction of a bilinear Master Equation [4] and the design of observers based on Monte-Carlo simulations and process measurements followed by controller design [12].

Macroscopic size domains require a formidable, often prohibitive computational effort to directly evaluate the coarse time-stepper. The alternative is to exploit the smoothness *in macroscopic space* of the coarse system state,

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and estimate the coarse time-stepper through the so-called *gaptooth* scheme [10], [6], [19], [18], [9].

In the gaptooth scheme, macroscopic space is tiled with *teeth* and intervening *gaps* (in one dimension); the teeth correspond, qualitatively, to the mesh points in a discretization of the unavailable macroscopic evolution equation. The microscopic evolution is performed in the interior of each tooth only. Clearly, appropriate boundary conditions have to be provided for the edges of each tooth; these boundary conditions implement *effective smoothness* of the macroscopic state profile. These macroscopically inspired, coarse boundary conditions for the microscopic simulation are crucial for the success of the scheme, as discussed extensively in [10]; ways to impose them (such as the Optimal Particle Controller [11]) have been the subject of extensive research in computational materials science. Lifting from macro- to micro-initial conditions, microscopic evolution, and restriction back to macro-variables occur only in each tooth. The important new element is that periodically the computation is stopped, the macroscopic profile interpolated smoothly from the microscopic runs, and new boundary conditions computed, allowing thus for communication between "teeth" (in effect, informing the "teeth" of the evolution of the macroscopic field in which they participate). Each "tooth" thus runs coupled with a macroscopic coarse field arising from the interpolation between all the (neighboring) teeth, and the gaptooth scheme is thus related to the so-called *hybrid* multiscale schemes and domain decomposition algorithms. Evolving appropriately chosen initial conditions over a lattice of "teeth" (1D) or "patches" (higher dimensions) for relatively short times, coupled with variance reduction and system identification techniques, provides an estimate of the full-space coarse time-stepper. A number of computational tasks, such as coarse projective integration ("patch dynamics"), as well as coarse stability and bifurcation analysis are thus enabled.

In this work we address the issue of coarse controller design for spatially distributed processes, where the computational cost associated with full space microscopic simulations during the identification/controller design step may be prohibitive (due to the large spatial extent of the system). Obtaining a coarse time-stepper through the gaptooth scheme is used to identify the dominant spatial behavior of macroscopic distributed processes and design coarse controllers using well-established methodologies for linear discrete-time controller design, such as pole placement and the Riccati Equation. Discrete-time controller design is validated on a mesoscopic, kinetic theory based *full space* realization of the FitzHugh-Nagumo PDE, widely used to describe the formation of patterns in reacting and biological systems. In the presence of open-loop oscillatory behavior, the approach stabilizes (both for the continuum and for the coarse microscopic realizations) the unstable, nonuniform in space, steady (microscopically stationary) state. It is interesting to note that this approach, used here as a "wrapper" around a mesoscopic simulator, can be also used as a wrapper around black-box proprietary or legacy large scale simulation codes [23].

II. Gaptooth time-steppers for controller design

In this section we discuss the construction of linear discrete-time controllers for spatially distributed processes using a combination of coarse time-steppers and the gaptooth scheme. In previous work, we combined existing *full space* coarse time-steppers with the Recursive Projection Method to identify the slow coarse process behavior and design coarse linear controllers. The *gaptooth* time-stepper is repeatedly called as a black box subroutine for short times, using nearby coarse initial conditions and parameter values. These calls are initiated by a computational superstructure which processes the results, and iteratively computes a fixed point of the unavailable coarse system behavior. The computational superstructure thus enables the time-stepper to perform coarse fixed point computations (using RPM). An estimate of the coarse slow Jacobian is a byproduct of RPM upon convergence; if necessary, an Arnoldi procedure can be used for a refined estimate of this linearization [3].

In the controller design stage, linear discrete-time controllers are designed based on the identified coarse slow linear system using pole placement or optimal control techniques.

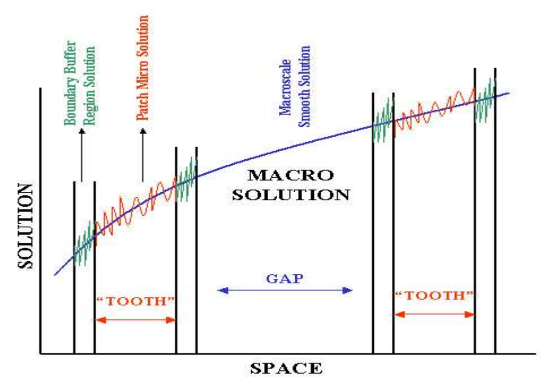

Fig. 1. Gaptooth scheme to discretize the spatial domain.

A. Obtaining the coarse slow linearization

For the LB-BGK realization of the FHN equation at each lattice point the "fine" description is a distribution of particle density over a discrete velocity space: right-moving, left-moving and non-moving particles. The sum of these (the zeroth moment of the discrete distribution) is the coarse description (details about the LB-BGK scheme can be found in [17]).

To construct the coarse time-stepper based on the gaptooth scheme we select 101 equidistant points, where we place the center of the "teeth" of the gaptooth time-stepper. In Figure II the gaptooth discretization scheme is depicted. The spatial domain is decomposed in a series of teeth (which represent 10% of the extent of the spatial domain) and the gaps between these teeth. The microscopic evolution is simulated *in the interior of each tooth only* with appropriate initial conditions for each tooth and boundary conditions at each tooth edge arising from the macroscopic spatial profile of the coarse variables.

Lifting is performed by initializing the three components of the distribution at local equilibrium, but different liftings (e.g. the entire local particle density assigned to right-moving particles) "heal" quickly, over a small fraction of the reporting horizon of the coarse time-stepper. Restriction simply involves computation of the zeroth moments of the discrete distribution over velocities and averaging over each tooth. The remaining moment fields ("momentum" field and "energy" field) evolve very fast to functionals of the smooth zeroth moment (concentration) field.

Periodically the microscopic computations are stopped, the macroscopic profile is interpolated from the microscopic runs, and new macroscopic boundary conditions for the teeth are computed, allowing thus for communication between "teeth" (in effect, informing the "teeth" of the evolution of the macroscopic field in which they participate). It is important that the microscopic simulations be long enough in time for "healing" to take place before the communication instance between the teeth, leading to accurately inferred macroscopic profiles and, by extension, to effective communication between the teeth. The reporting horizon for the tooth computation is set to be $T = 0.5$ for our application, a time interval which lies in the gap between the fast and the slow dynamics of the FHN equation (for the process parameters of Table I). In each tooth, the LB-BGK simulations we employed spanned 12 lattice points, while the time step was set to $\delta t = 5 \times 10^{-6}$. The communication time-interval between the teeth was set to $T_{comm} = 0.00075$.

During the communication operation, the coarse variables are evaluated at all teeth and the profile of the coarse variable profiles at the gaps is interpolated in order to provide the new boundary conditions to the time-steppers [18], [19].

Open-loop stationary states are located by RPM wrapped around the coarse gaptooth time-stepper. RPM is based on a Newton-type iteration on the recursively identified coarse slow subspace, and can thus be used to locate both stable and unstable stationary states, perform continuation and stability analysis tasks and, in general, compute coarse bifurcation diagrams [24], [5], [10]. The method is particularly efficient when a large time-scale separation exists between the system coarse eigenmodes: a relatively small number of eigenvalues (stable or unstable) lie close to the imaginary axis, complemented by many strongly stable eigenmodes in the far left hand plane. Upon convergence of the RPM to the target stationary state, the effect of control actuators can be obtained by perturbing the system and estimating the corresponding coarse derivatives. It is important to confirm that the "slow manifold" of the open-loop system remains a good description of the slow response to these perturbations; else the dimension of the slow subspace should be augmented. RPM provides an estimate of the coarse linearized discrete time model (which can be refined, if necessary, through an Arnoldi procedure). The slow eigenvalues of the continuous system λ can be computed from the discrete linearization "multipliers" μ through the formula $\mu_i = exp(\lambda_i T)$, where T is the time-stepper reporting horizon.

B. Controller design

The dynamics of the system in the neighborhood of the steady state x_{ss} can be described by the unknown discrete-time coarse linear model

$$x_{k+1} - x_{ss} \quad = \quad G(x_k - x_{ss}) + Hu_k \tag{1}$$

with sampling interval T. Here $x \in \mathbb{R}^N$ denotes the state of the coarse linearization, G is the Jacobian matrix at the target stationary state, and H is an $N \times k$ matrix describing the effect of the k control actuators on the system dynamics.

The discrete-time coarse linear system of Eq.1 can be equivalently written in the control form

$$\begin{aligned} \xi_{k+1} &= G\xi_n + Hu_k \\ y_c &= C\xi_k \end{aligned} \tag{2}$$

where $C \in \mathbb{R}^{M \times N}$, $\xi = x - x_{ss} \in \mathbb{R}^N$ is the deviation from the coarse stationary state, and $y_c \in \mathbb{R}^M$ is the controlled output vector of dimension M.

We initially design state-feedback discrete-time Linear Quadratic Regulators (LQRs) by solving an optimal control problem with cost function:

$$J = \sum_{k=0}^{\infty} {y_c}^* Q y_c + u_k R u_k = \sum_{k=0}^{\infty} \xi_k^* C^* Q C \xi_k + u_k R u_k \tag{3}$$

where Q, R are positive semidefinite matrices; A^* denotes the conjugate transpose of A. The optimal feedback controller gain P_c is computed solving the algebraic Riccati equation [14]:

$$\begin{aligned} &P = Q + G^*[P - PH(R + H^*PH)^{-1}H^*P]G \\ &P_c = -(R + H^*PH)^{-1}H^*PG \end{aligned} \tag{4}$$

and the control action is given from $u_k = P_c \xi_k$.

For an output feedback LQR formulation, the discrete-time system is equivalently written in the form:

$$y_{c k+1} = CGC^{\perp} y_{c k} + CHu_k \tag{5}$$

where $C^{\perp} = C^*(CC^*)^{-1}$. A special case is $C = V_F (V^* V)^{-1} V^*$, $V \in \mathbb{R}^{N \times M}$ is the matrix with columns the RPM estimated eigenvectors (in descending order of eigenvalue values) of the first M coarse slow eigenmodes, and V_F is defined below. In this case $y_c \in \mathbb{R}^M$ denotes the projection of the state on the first M, RPM identified, slow eigenmodes of the coarse linearization. Furthermore, $C^{\perp} = V\, V_F^{-1}$ and $CGC^{\perp} \in \mathbb{R}^{M \times M}$ is the coarse slow Jacobian matrix at the target stationary state, which is estimated by RPM. V_F is a matrix with columns the eigenvectors of $CGC^{\perp}$ in descending order of associated eigenvalue magnitudes. $CH \in \mathbb{R}^{M \times k}$ expresses the influence of the control actuators on the coarse slow dynamics of these modes.

III. An illustrative model: the FitzHugh-Nagumo equation

The controller design methodology is validated using a *full space* Lattice Boltzmann based realization of the FitzHugh-Nagumo (FHN) equation, a widely used model of wave behavior in excitable media in biology [7] and chemistry [24]:

$$\begin{aligned} \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial z^2} + v - w - v^3 + b(z)u(t) \\ \frac{\partial w}{\partial t} &= \delta \frac{\partial^2 w}{\partial z^2} + \epsilon(v - p_1 w - p_0) \end{aligned} \quad (6)$$

with boundary conditions:

$$\frac{\partial v}{\partial z}|_0 = \frac{\partial v}{\partial z}|_L = 0, \quad \frac{\partial w}{\partial z}|_0 = \frac{\partial w}{\partial z}|_L = 0 \quad (7)$$

and initial condition

$$v(0,z) = v_0(z), \quad w(0,z) = x_0(z) \quad (8)$$

where $v(t,z),\ w(t,z) \in \mathbb{R}$ are the system variables, $u(t) \in \mathbb{R}^m$ is the array of manipulated variables, t is the time, z is the spatial coordinate, $b(z)$ is a row vector describing the control actuators, ϵ, δ, p_1, p_0 are process parameters and L is the length of the spatial domain. We assume that three control actuators are available:

$$b(z) = [g(z, 0.25L) \;\; g(z, 0.50L) \;\; g(z, 0.75L)]$$

where $g(z,\zeta) = exp(-0.3(z-\zeta)^2)$; note that this choice of actuator influence functions extends over the entire spatial domain of the process. In the simulations that are presented, unless otherwise noted, the initial condition was $v_0 = 0.5cos(\pi z/L)$ and $w_0 = 0.5cos(\pi z/L)$.

TABLE I

Process parameters

L	20	δ	4.0	p_1	2.0
T	0.5	ϵ	0.017	p_0	-0.03

The FHN exhibits multiple steady solutions (spatially uniform as well as nonuniform ones) and spatially nonuniform periodic solutions, depending on the values of the process parameters. For the parameter values of Table I, the system has four spatially nonuniform and three spatially uniform steady-states, presented in Figures 2a and 2b for v and w respectively. Linearizing the discretized FHN in the neighborhood of the steady-states and computing the eigenvalues we conclude that the system is locally unstable in the neighborhood of steady states one, two and three, and locally stable in the neighborhood of steady states four, five, six and seven. Furthermore, simulating Eq.6 with $u(t) \equiv 0$ and initial conditions far from the stable steady-states, we observe the locally stable, spatially nonuniform

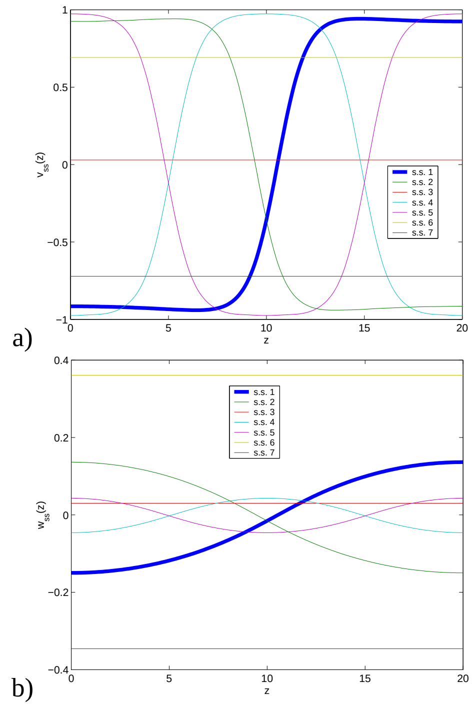

Fig. 2. Open-loop unstable steady states of (a)v, (b) w.

periodic orbit shown in Figures 3a and 3b for $v(t)$ and $w(t)$ respectively.

Representing the system in discrete time, with sampling time $T = 0.5$, in the neighborhood of the first steady-state, depicted as thick lines in Figures 2a and 2b for v and w respectively (denoted as $x_{ss,1}$ for the rest of the section), we observe in Table II, that there is a finite number of eigenvalues close to the unit circle, while an infinite number of them (in the limit of infinite discretization points) lie close to zero. Moreover we observe that a large spectral gap exists between two consecutive eigenvalues. This time-scale separation implies that a few dominant modes may be able to accurately parameterize the long term dynamics of the open-loop process.

In [1], using RPM methodology with time-steppers based on either the discretized PDE or *full space* LB-based simulations we are able to locate target unstable spatially nonuniform fixed points and identify low-dimensional slow linearizations of the system with the intention to design linear discrete time controllers that enforce closed-loop stability. In the current work, we show that using the proposed gaptooth discretization with kinetic theory based LB-BGK scheme in the teeth, we can circumvent the computational costs associated with full spatial domain simulation. We constructed a coarse time-stepper with a time-reporting horizon of $T = 0.5$ and 101 teeth, that computes the zeroth moment fields that approximately satisfy the FHN equation (this time-stepper is denoted as GT for the rest of the section).

Specifically, in each tooth, the time-stepper combined lifting, from zeroth moment fields to full LB state fields (employing a local equilibrium assumption), LB-BGK "meso-

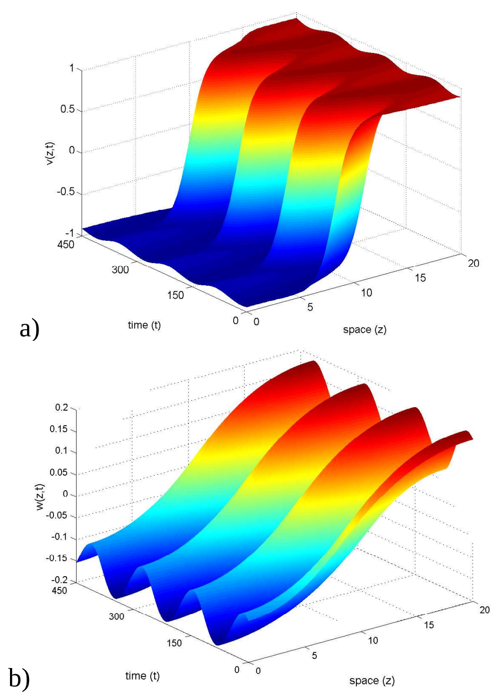

Fig. 3. Open-loop stable periodic orbit of (a) v, (b) w.

scopic" evolution, and restriction back to zeroth moments corresponding to the local values of v and w. The LB-BGK based simulators employed by the 101 teeth represented a total 10% of the spatial domain of the process; the lattice size of 12 points was chosen to accurately capture the process behavior in the teeth. With set time integration step to $\delta t = 5 \times 10^{-6}$ and communication time-interval of $T_{comm} = 0.00075$ between the teeth, the coarse time-stepper was able to accurately capture the spatiotemporal profiles of the coarse variables.

The constructed controllers were then applied to a kinetic theory based full-space LB-BGK realization of the FHN equation [17], [24]. The combination of proposed gaptooth time-stepper with RPM located the target coarse stationary state, inferred its coarse stability properties through estimates of the leading coarse eigenvalues/vectors, and led to a third order approximate linear discrete-time coarse slow subsystem in its neighborhood. Using as an initial guess the stable coarse stationary profile at $\epsilon = 0.1$ and $\epsilon = 0.11$ we converged on the unstable nonuniform coarse stationary profile at the target value of $\epsilon = 0.017$ beyond the Hopf bifurcation at $\epsilon = 0.019$. We also approximated the coarse slow eigenvalues, their respective eigenvectors and the Jacobian $CGC^{\perp}$ in Eq.5 of the coarse slow subsystem. In Table II we present the open-loop eigenvalues of the coarse gaptooth LB system (denoted as GT), and compare with the ones calculated based on the coarse full space LB realization (denoted as LB). This comparison partially corroborates the correspondence between the coarse LB-RPM model and the Gaptooth LB-RPM model.

Following the coarse open-loop analysis we computed the coarse gaptooth process model response to actuators' perturbations, and subsequently obtained a linearized expression of their effect on the slow discrete-time subsystem (matrix CH of Eq.5). Two eigenvalues of the identified local coarse slow linear system lie outside the unit circle in the complex plane (Table II). As a result, for the LQR to stabilize the system at $x_{ss,1}$, our control objective becomes the placement of the closed-loop eigenvalues corresponding to the unstable slow eigenmodes within the unit circle. To retain the time-scale separation between the slow and the fast subsystems, the resulting closed-loop eigenvalues are set inside but close to the unit circle.

We designed an LQR discrete-time controller for the identified discrete-time coarse linear model solving the Riccati equation with cost function weights in Eq.3 $Q = 0.5I_{3\times3}$, and $R = 10I_{3\times3}$.

In Table II we present the eigenvalues of the closed-loop full-space LB model in the neighborhood of $x_{ss,1}$ and compare them with the eigenvalues when no control is used. We observe that that LB model is stabilized in closed loop, and the time-scale separation between the slow eigenmodes and the fast ones persists (only four fast eigenvalues were identified using an Arnoldi scheme): spillover did not change the dimension of the slow closed-loop subsystem. We also observe that the controller fails to assign all the eigenvalues at the desired locations, in part due to spillover, and in part due to the inaccuracy of coarse slow eigenvalue/eigenvector estimates (which, however, can be refined).

The effect of the coarse three-dimensional "3:GT-RPM LQR" on the full space LB dynamics is shown in the simulation of figure 5a where the time-profile of the L_2 norm of the coarse state of the LB-BGK model converges to the stationary value rapidly and smoothly. Figures 6a and 6b present the spatiotemporal profiles of the zeroth moments of the full space LB that correspond to $v(z,t)$ and $w(z,t)$ respectively. In Figure 5b we also present the time-profile of the coarse control action. We observe that the control action tends to zero as time progresses, and it achieves coarse stabilization of the LB-BGK model at $x_{ss,1}$ without chattering.

To test the basin of attraction of the closed-loop LB-BGK coarse stationary state under the designed controller, we simulated the system for an initial condition on the stable limit cycle, activating the controller at different time-instants. In Figure In Figures 7a and 7b we present the spatiotemporal profiles of the moments corresponding to v and w, respectively, when the 3:GT-RPM controller is activated at a particular point on the limit cycle. We see that both coarse fields asymptotically approach the open-loop unstable stationary-state profiles of $x_{ss,1}$ smoothly. The control action action for the specific simulation tends to zero as time progresses, with a smooth time-profile (no chattering).

In Figure 4 we present a phase portrait in terms of the zeroth Fourier modes of the coarse fields corresponding to v and w multiplied by the square root of the domain size; the

TABLE II

EIGENVALUES OF LINEARIZED DISCRETE-TIME FHN IN THE NEIGHBORHOOD OF $x_{ss,1}$.

Open-loop				Closed-loop [LQR]	Closed-loop [PPC]		
LB-Arnoldi	LB-RPM	GT-Arnoldi	GT-RPM	[3:GT-RPM]	Objective	[3:GT-RPM]	[3:LB-RPM]
$1.000 + 0.0237i$	$0.9989 + 0.0249i$	$1.004 + 0.0207i$	$1.00 + 0.0237i$	0.9457	0.980	0.9836	0.9836
$1.000 - 0.0237i$	$0.9989 - 0.0249i$	$1.004 - 0.0207i$	$1.00 - 0.0237i$	0.9455	0.970	0.9673	0.9673
0.9297	0.9297	0.9304	0.9457	$0.876 + 0.033i$	0.930	0.9439	0.9295
0.8054	–	0.8086	–	$0.876 - 0.033i$	–	0.8776	0.8076

zeroth Fourier modes represent the average of these fields in the spatial domain. The dotted black lines denote the system evolution when the 3:GT-RPM LQR is activated with the system on the open-loop stable limit cycle. We observe that the designed 3:GT-RPM LQR achieves driving the closed-loop system to $x_{ss,1}$ for all initial positions on the coarse limit cycle that we have tested.

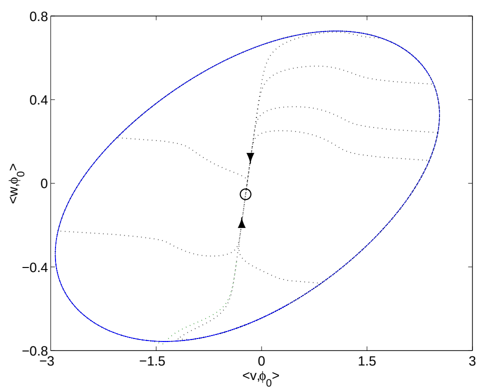

Fig. 4. Phase portrait projection of open loop (blue solid line) and closed-loop (dotted lines) system evolution on the zeroth Fourier modes of $v(z,t)$ and $w(z,t)$ multiplied by $\sqrt{20}$.

We also designed a third order controller, denoted PPC, using pole placement techniques [14] on the same coarse linear model. The objective eigenvalues are presented in Table II; they are placed to values less than unity (for stability) but large enough to retain the time-scale separation. We observe that the coarse eigenvalues of the closed-loop system were close to the objective ones, and that the time scale separation was not perturbed. For comparison purposes we present a controller derived using the full space LB realization of the FHN equation. We observe that the two controllers have similar success in assigning the closed-loop eigenvalues at the desired location, while the spillover effect is minimal in both cases.

IV. CONCLUSIONS

We have extended our equation-free coarse controller design framework to allow for gaptooth coarse time-steppers (microscopic simulation over only part of the spatial domain). The method operates at the coarse level as if closed macroscopic equations were available; but the quantities necessary during computation, instead of being evaluated using a macroscopic equation, are *estimated* using short bursts of appropriately initialized calls to the alternative, microscopic evolution description. These computations are performed over "teeth" (parts of the spatial domain) separated by "gaps", where no simulation is performed; the teeth are connected via effective smoothness boundary conditions. The approach was illustrated by stabilizing through linear, discrete-time coarse feedback control a spatially nonuniform coarse stationary state of a kinetic theory based LB "fine" model motivated by the FHN PDE in one dimension.

V. ACKNOWLEDGEMENTS

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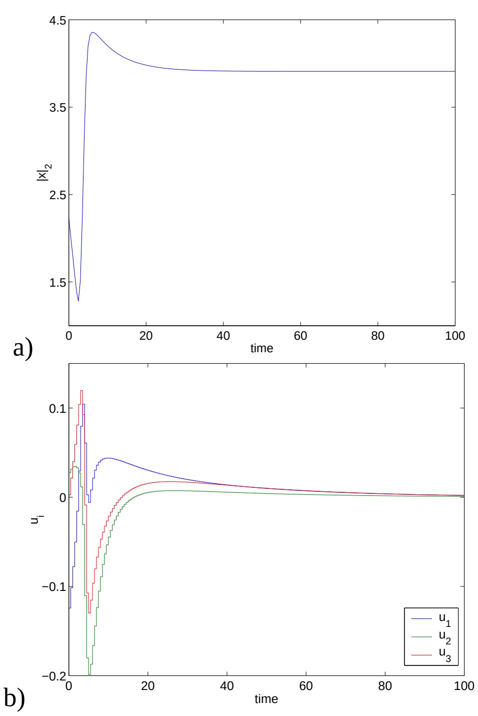

Fig. 5. Time profiles: (a) L_2 norm of closed-loop LB-BGK FHN under 3:GT-RPM LQR, (b) control action of 3:GT-RPM LQR ($v_0 = 0.5cos(\pi z/L)$, $w_0 = 0.5cos(\pi z/L)$)

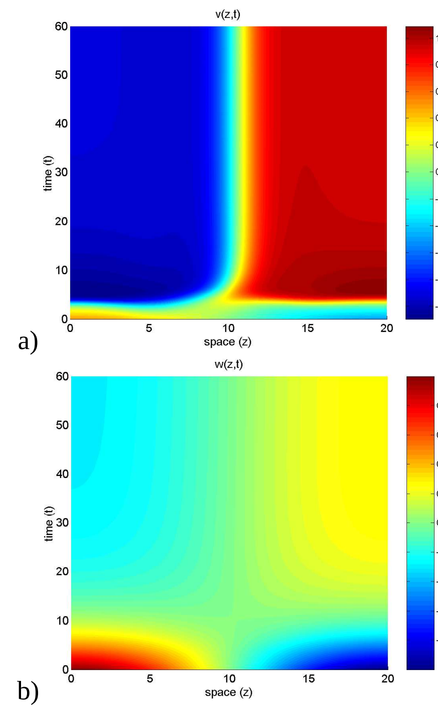

Fig. 6. Closed-loop LB-BGK FHN evolution under 3:GT-RPM LQR ($v_0 = 0.5cos(\pi z/L)$, $w_0 = 0.5cos(\pi z/L)$). (a) v and (b) w.

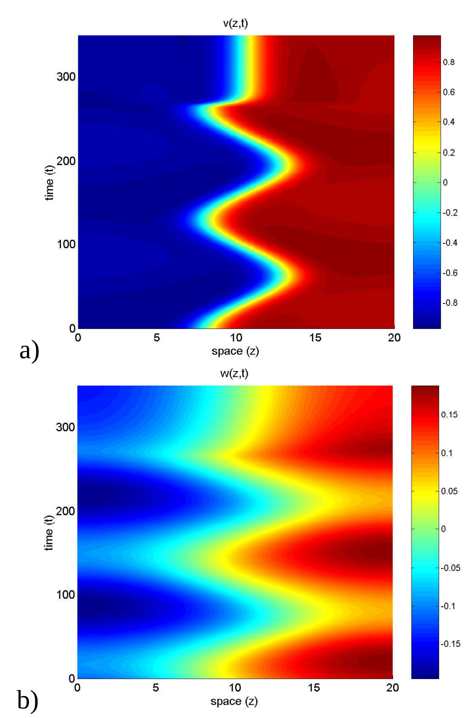

Fig. 7. Closed-loop LB-BGK FHN evolution (a) v and (b) w, under 3:GT-RPM LQR ($t_{act} = 265$; initial condition on periodic orbit).