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Fuzzy weighting function dependent RHC design for TS Fuzzy Systems with input constraints

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Abstract— **In this paper, we present a state-feedback receding horizon control (RHC) for discrete-time fuzzy systems with input constraints. The models of the fuzzy systems are of Takagi-Sugeno type, which is suitable to model a large class of nonlinear systems. To find the control, we first derive an optimization problem involving parameterized linear matrix inequalities (PLMIs) that depend on the current and one-step past information on the time-varying fuzzy weighting functions. Since the PLMIs need to be checked for all values of the weighting functions, for solvability, we convert the PLMIs into a finite number of LMIs by suggesting a special structures for the variables of the PLMIs. Then, it is shown that the closed-loop system with the designed control is stable if the converted optimization problem is feasible at the initial time. A numerical example is presented to illustrate the performance of the controller.**

I. INTRODUCTION

Receding horizon control(RHC), also known as model predictive control(MPC), has received much attention in control societies because of its good tracking performance and many applications to industrial processing systems [1], [2], [3], [4], [5]. The RHC is also a suitable control strategy for time-varying systems including periodic systems and has a merit that constraints may be directly incorporated into the on-line optimization [6], [7], [8].

On the other hand, TS fuzzy model is widely used because it is suitable to model a large class of nonlinear systems and has been studied extensively in the last decades [9], [10], [11], [12]. In the TS fuzzy model, a nonlinear plant is represented by a set of linear models interpolated by membership function (TS fuzzy model) and then a model-based fuzzy controller is developed to stabilize the TS fuzzy model. Therefore, one can utilize a large amount of results for linear systems in solving nonlinear problems. This story is also applicable to the area of RHC.

In this paper, we present a state-feedback receding horizon control (RHC) for discrete-time fuzzy systems with input constraints. The models of the fuzzy systems are of TS type, which is suitable to model a large class of nonlinear systems. To find the control, we first derive an optimization problem involving PLMIs [13], [14] that depend on the current and one-step past information on the time-varying fuzzy weighting functions. Since the PLMIs need to be checked for all values of the weighting functions, for solvability, we convert the PLMIs into a finite number of LMIs by suggesting a special structures for the variables of the PLMIs. Then, it is shown that the closed-loop system with the designed control is stable if the converted optimization problem is feasible at the initial time. A numerical example is presented to illustrate the performance of the controller.

The rest of the paper is organized as follows. Section II states target systems, assumptions, the associated problem. Section III derives an optimization problem involving PLMIs that depend on the current and one-step past information on the time-varying fuzzy weighting functions. For solvability, the PLMIs are converted into a finite number of LMIs by suggesting a special structures for the variables of the PLMIs. Then the closed-loop system stability is showed. Finally, concluding remarks are given in Section V. The notation of this paper is fairly standard. In symmetric block matrices, we use $(*)$ as an ellipsis for terms that are induced by symmetry.

II. PROBLEM STATEMENTS

Consider a general discrete-time TS fuzzy system such as

$$x_{k+1} = \sum_{i=1}^{r} \theta_i(\eta_k)\left[A_i x_k + B_i u_k\right], \tag{1}$$

subject to input constraints

$$-\overline{u} \le u_k \le \overline{u}, \quad k = 0, 1, ..., \infty \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^m$ is the control input, r is the number of system rules, $\eta_k = [\eta_{k1}, \cdots, \eta_{kp}]$ is the premise variable vector that may depend on states in many cases, and $\theta_i(\eta_k)$ denote normalized time-varying fuzzy weighting function (FWF) for each rule at time k. In general, the FWFs $\theta_i(\eta_k)$ have the following conditions for all time k (see [11], [12]) :

$$\begin{gathered} 0 \le \alpha_i \le \theta_i(\eta_k) \le \beta_i \le 1, \quad \text{for } i = 1, 2, \cdots, r \\ |\theta_i(\eta_k) - \theta_i(\eta_{k-1})| \le \delta_i \le 1, \quad \text{for } i = 1, 2, \cdots, r, \\ \sum_{i=1}^{r} \theta_i(\eta_k) = 1. \end{gathered} \tag{3}$$

Since $\theta_i(\eta_k)$ are measurable in current time k, the system (1) belongs to a special class of LPV systems whose state-space matrices are assumed to depend on a time-varying parameter vector. Hence, we can rewrite the discrete-time TS fuzzy system (1) as

$$x_{k+1} = A(\Theta_k)x_k + B(\Theta_k)u_k, \tag{4}$$

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where

$$[A(\Theta_k)|B(\Theta_k)] =\triangleq \sum_{i=1}^{r} \theta_i(\eta_k)[A_i|B_i], \quad (5)$$

and $\Theta_k \in \mathbb{R}^r$ denotes a time-varying FWF vector consisting of the time-varying FWFs $\theta_i(\eta_k)$.

The goal of this paper is to find a new state-feedback RHC for fuzzy systems with input constraints, which depends on both the current-time FWF vector Θ_k and the one-step-past FWF vector Θ_{k-1} at time k (see [15]):

$$u_k = K(\Theta_k, \Theta_{k-1})x_k, \quad (6)$$

where $K(\Theta_k, \Theta_{k-1})$ is dependent on Θ_k and Θ_{k-1}. To find the state-feedback RHC of (6) for system (4), at each sampling time k, we consider the following cost function

$$\begin{aligned} J_{0,\infty}(k) &= x_{k|k}^T Q x_{k|k} + u_{k|k}^T R u_{k|k} + \sum_{j=1}^{\infty}(x_{k+j|k}^T \\ &\quad \times Q x_{k+j|k} + u_{k+j|k}^T R u_{k+j|k}) \\ &= x_{k|k}^T Q x_{k|k} + u_{k|k}^T R u_{k|k} + J_{1,\infty}(k), \end{aligned} \quad (7)$$

and the corresponding optimization problem

$$\min_{u_{k+j|k},\ j\in[0,\infty)} \ \max_{[A(\Theta_{k+j})|B(\Theta_{k+j})]} J_{0,\infty}(k), \quad (8)$$

subject to

$$x_{k+j+1|k} = A(\Theta_{k+j})x_{k+j|k} + B(\Theta_{k+j})u_{k+j|k}, \quad (9)$$

$$-\overline{u} \le u_{k+j|k} \le \overline{u}, \quad (10)$$

for $j = 0, ..., \infty$, where Q, R are positive definite symmetric weighting matrices for all admissible Θ_k. We denote that $x_{k+j|k}$ and $u_{k+j|k}$ are predicted variables of state and input at the time k respectively, with $x_{k|k} = x_k$.

III. RHC DESIGN FOR TS FUZZY SYSTEMS

Following the approach given in [8] where the quasi-min-max MPC algorithms for LPV systems are presented, it is easy to derive an upper bound on the worst value of the cost function $J_{0,\infty}(k)$. Consider an FWF dependent quadratic function [15]) $V(x,\Theta) = x^T P(\Theta)x,\ P(\Theta) > 0$ of the state with $V(0,\Theta) = 0$. At sampling time k, suppose V satisfies the following inequalities for all $x_{k+j|k}$, $u_{k+j|k}$, $j \ge 1$ satisfying (10), and for any Θ_{k+j-1} and Θ_{k+j} satisfying (2):

$$\begin{aligned} &V(x_{k+j+1|k}, \Theta_{k+j}) - V(x_{k+j|k}, \Theta_{k+j-1}) \\ &\quad \le -[x_{k+j|k}^T Q x_{k+j|k} + u_{k+j|k}^T R u_{k+j|k}]. \end{aligned} \quad (11)$$

For the performance function to be finite, we must have $x_{\infty|k} = 0$, and hence $V(x_{\infty|k}, \Theta_{k+j-1}) = 0$. Summing (11) from $j = 1$ to $j = \infty$, we get

$$-V(x_{k+1|k}, \Theta_k) \le -J_{1,\infty}(k). \quad (12)$$

Thus,

$$\begin{aligned} &\max_{[A(\Theta_{k+j})|B(\Theta_{k+j})]} J_{0,\infty}(k) \\ &\quad \le x_{k|k}^T Q x_{k|k} + u_{k|k}^T R u_{k|k} + V(x_{k+1|k}, \Theta_k) \\ &\quad = x_{k|k}^T Q x_{k|k} + u_{k|k}^T R u_{k|k} + x_{k+1|k}^T P(\Theta_k) x_{k+1|k}. \end{aligned} \quad (13)$$

This gives an upper bound on the cost. Thus the goal has been redefined to design, at each sampling time k, an FWF dependent state-feedback control law (6) to minimize this upper bound.

A. Unconstrained FWF dependent RHC design

First, we present an optimization problem from which we may obtain an FWF dependent state-feedback control law for unconstrained system. The problem is same as that in Theorem 1 of [8] except that $p(k), A_i, B_i, X(k), Y(k)$ are replaced by $\Theta_k, A(\Theta_{k+j}), B(\Theta_{k+j}, X(\Theta_{k+j-1}),$ $Y(\Theta_{k+j}, \Theta_{k+j-1})$ respectively, hence the derivation is omitted for clarity. We shall omit the derivation procedure for clarity.

$$\underset{\gamma, u_{k|k}, X(\Theta_{k+j}), X(\Theta_{k+j-1}), X(\Theta_k), Y(\Theta_{k+j}, \Theta_{k+j-1}), j\ge 0}{min} \gamma \quad (14)$$

subject to

$$\begin{bmatrix} -1 & (*) & (*) & (*) \\ M_{21} & -X(\Theta_k) & (*) & (*) \\ Q^{1/2}x_{k|k} & 0 & -\gamma I & (*) \\ R^{1/2}u_{k|k} & 0 & 0 & -\gamma I \end{bmatrix} \le 0, \quad (15)$$

$$\begin{bmatrix} -X(\Theta_{k+j-1}) & (*) & (*) & (*) \\ Q^{1/2}X(\Theta_{k+j-1}) & -\gamma I & (*) & (*) \\ M_{31} & 0 & -\gamma I & (*) \\ M_{41} & 0 & 0 & -X(\Theta_{k+j}) \end{bmatrix} \le 0, \quad (16)$$

where $j \in [1, \infty]$, and

$$M_{21} = A(\Theta_k)x_{k|k} + B(\Theta_k)u_{k|k}, \quad (17)$$

$$M_{31} = R^{1/2}Y(\Theta_{k+j}, \Theta_{k+j-1}), \quad (18)$$

$$\begin{aligned} M_{41} &\triangleq A(\Theta_{k+j})X(\Theta_{k+j-1}) \\ &\quad + B(\Theta_{k+j})Y(\Theta_{k+j}, \Theta_{k+j-1}), \end{aligned} \quad (19)$$

$$X(\Theta_{k+j-1}) = \gamma P^{-1}(\Theta_{k+j-1}), \quad (20)$$

$$K(\Theta_{k+j}, \Theta_{k+j-1}) = Y(\Theta_{k+j}, \Theta_{k+j-1})X^{-1}(\Theta_{k+j-1}). \quad (21)$$

If this problem is solvable and feasible, the control input sequence $\mathcal{U}_k$ is given by

$$\mathcal{U}_k = \{u_{k|k}, u_{k+j|k} = K(\Theta_{k+j}, \Theta_{k+j-1})x_{k+j|k}, j \ge 1\}. \quad (22)$$

Unfortunately, the above optimization problem cannot be solved because of unknown Θ_{k+j} and Θ_{k+j-1} contained in the infinite number of LMIs in (15) and (16). Therefore, it is important to develop a finite number of solvable conditions from (15) and (16).

B. Relaxation of the proposed controller using LMIs

From this point on, we shall develop a relaxed LMI conditions for (15) and (16), based on the polynomial dependency of the FWFs for the discrete-time TS fuzzy system, using the the convex relaxation techniques of general fuzzy systems. To this ends, let us first select the structures of the variables in (15) and (16) as follows:

$$\begin{aligned} X(\Theta_{k+j}) &= \sum_{i=1}^{r} \theta_i(\eta_{k+j}) X_i, \\ X(\Theta_{k+j-1}) &= \sum_{i=1}^{r} \theta_i(\eta_{k+j-1}) X_i, \\ Y(\Theta_{k+j}, \Theta_{k+j-1}) &= \sum_{i=1}^{r} \theta_i(\eta_{k+j}) Y_{i1} + \sum_{i=1}^{r} \theta_i(\eta_{k+j-1}) Y_{i2}, \end{aligned} \tag{23}$$

where X_i, Y_{i1}, and Y_{i2} are constant matrices, that is, $X(\Theta_{k+i})$, $X(\Theta_{k+i-1})$, and $Y(\Theta_{k+i}, \Theta_{k+i-1})$ are polynomially dependent on Θ_{k+i} and Θ_{k+i-1}. Furthermore, by using constraint-elimination methods for the conditions (2) of the current-time FWF $\theta_i(\eta_{k+N})$ and the one-step-past FWF $\theta_i(\eta_{k+N-1})$ with the $\mathcal{S}$-procedure, we relax the conservatism of the proposed controller (6).

Theorem 3.1: The optimization problem (14) subject to (15) and (16) is feasible if the following optimization problem involving a finite number of LMIs is feasible

$$\min_{\gamma, u_{k|k}, X_i, Y_{i1}, Y_{i2}, \Lambda_i, \Sigma_i, \Xi_i, \Lambda, \Sigma} \gamma, \tag{24}$$

subject to

$$\begin{bmatrix} -1 & (*) & (*) & (*) \\ M_{21} & -X_i & (*) & (*) \\ Q^{1/2} x_{k|k} & 0 & -\gamma I & (*) \\ R^{1/2} u_{k|k} & 0 & 0 & -\gamma I \end{bmatrix} \leq 0, \quad i = 1, ..., r, \tag{25}$$

$$\begin{bmatrix} \Upsilon & (*) & \cdots & (*) & (*) & \cdots & (*) \\ \Gamma_1 & \Delta_{11} & \cdots & (*) & (*) & \cdots & (*) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ \Gamma_r & \Delta_{r1} & \cdots & \Delta_{rr} & (*) & \cdots & (*) \\ \Omega_1 & \Pi_{11} & \cdots & \Pi_{1r} & \Psi_{11} & \cdots & (*) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & (*) \\ \Omega_r & \Pi_{r1} & \cdots & \Pi_{rr} & \Psi_{r1} & \cdots & \Psi_{rr} \end{bmatrix} \leq 0, \tag{26}$$

$$X_i \geq 0, \quad \gamma > 0, \tag{27}$$

$$\Lambda_i + \Lambda_i^T \geq 0, \quad \Sigma_i + \Sigma_i^T \geq 0, \quad \Xi_i + \Xi_i^T \geq 0, \tag{28}$$

where

$$\begin{aligned} M_{21} &= A_i x_{k|k} + B_i u_{k|k}, \\ \Upsilon &\triangleq \Upsilon_{sub} - \sum_{i=1}^{r} \alpha_i \beta_i \{(\Lambda_i + \Lambda_i^T) + (\Sigma_i + \Sigma_i^T)\} \\ &\quad - (\Lambda + \Lambda^T) - (\Sigma + \Sigma^T) + \sum_{i=1}^{r} \delta_i^2 (\Xi_i + \Xi_i^T), \\ \Gamma_i &\triangleq \Gamma_{i_{sub}} + (\alpha_i + \beta_i) \Lambda_i + (\Lambda + \Lambda^T), \\ \Delta_{ii} &\triangleq D_{i_{sub}} - (\Lambda_i + \Lambda_i^T) - (\Lambda + \Lambda^T) - (\Xi_i + \Xi_i^T), \\ \Delta_{hi} &\triangleq \Delta_{hi_{sub}} - (\Lambda + \Lambda^T), \quad h \neq i, \\ \Omega_i &\triangleq \Omega_{i_{sub}} + (\alpha_i + \beta_i) \Sigma_i + (\Sigma + \Sigma^T), \\ \Pi_{hi} &\triangleq \begin{cases} \Pi_{hi_{sub}} + (\Xi_i + \Xi_i^T) & i = h \\ \Pi_{hi_{sub}} & i \neq h \end{cases}, \\ \Psi_{ii} &\triangleq -(\Sigma_i + \Sigma_i^T) - (\Sigma + \Sigma^T) - (\Xi_i + \Xi_i^T), \\ \Psi_{hi} &\triangleq -(\Sigma + \Sigma^T), \quad h \neq i, \\ \Upsilon_{sub} &\triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma I & 0 & 0 \\ 0 & 0 & -\gamma I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Gamma_{i_{sub}} &\triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R^{\frac{1}{2}} Y_{i1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} X_i \end{bmatrix}, \\ D_{i_{sub}} &\triangleq \begin{bmatrix} 0 & 0 & 0 & Y_{i1}^T B_i^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ B_i Y_{i1} & 0 & 0 & 0 \end{bmatrix}, \\ \Delta_{hi_{sub}} &\triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ B_h Y_{i1} + B_i Y_{h1} & 0 & 0 & 0 \end{bmatrix}, \\ \Omega_{i_{sub}} &\triangleq \begin{bmatrix} -\frac{1}{2} X_i & 0 & 0 & 0 \\ Q^{\frac{1}{2}} X_i & 0 & 0 & 0 \\ R^{\frac{1}{2}} Y_{i2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Pi_{hi_{sub}} &\triangleq \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_i X_h + B_i Y_{h2} & 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

Proof: The proof for a similar problem to (26)-(28) can be found in [15], [16], so not discussed any more here. And we can obtain (25) from (15) that includes $X(\Theta_k)$ since Θ_k is known at time k. ∎

C. Constrained FWF dependent RHC design

Input constraints can be expressed as LMIs and therefore included in the RHC problem. The input constraints (10) is satisfied if the following conditions are satisfied

$$\begin{bmatrix} -X(\Theta_{k+j-1}) & Y(\Theta_{k+j}, \Theta_{k+j-1}) \\ Y^T(\Theta_{k+j}, \Theta_{k+j-1}) & -Z \end{bmatrix} \leq 0, \tag{29}$$

$$Z_{ii} \leq \overline{u}_i^2, \quad i = 1, 2, ..., m,$$

where $\overline{u}_i$ is defined by $\overline{u}_i = \min(-\underline{u}_i, \overline{u}_i)$ and $\underline{u}_i$ and $\overline{u}_i$ are the ith elements of $\underline{u}$ and $\overline{u}$, respectively. It is a simple extension of the result in [17], hence the derivation process from (10) to (29) is omitted.

By the relaxation technique used in the previous section, the inequality (29) can also be relaxed to the following

LMIs

$$\begin{bmatrix} \bar{\Upsilon} & (*) & \cdots & (*) & (*) & \cdots & (*) \\ \bar{\Gamma}_1 & \bar{\Delta}_{11} & \cdots & (*) & (*) & \cdots & (*) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ \bar{\Gamma}_r & \bar{\Delta}_{r1} & \ldots & \bar{\Delta}_{rr} & \cdots & \cdots & (*) \\ \bar{\Omega}_1 & \bar{\Pi}_{11} & \cdots & \bar{\Pi}_{1r} & \bar{\Psi}_1 & \cdots & (*) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{\Omega}_r & \bar{\Pi}_{r1} & \cdots & \bar{\Pi}_{rr} & \bar{\Psi}_{r1} & \cdots & \bar{\Psi}_r \end{bmatrix} \leq 0, \quad (30)$$

where

$$\begin{aligned} \bar{\Upsilon} &\triangleq \bar{\Upsilon}_{sub} - \sum_{i=1}^{r} \alpha_i \beta_i \{(\bar{\Lambda}_i + \bar{\Lambda}_i^T) + (\bar{\Sigma}_i + \bar{\Sigma}_i^T)\} \\ &\quad - (\bar{\Lambda} + \bar{\Lambda}^T) - (\bar{\Sigma} + \bar{\Sigma}^T) + \sum_{i=1}^{r} \delta_i^2 (\bar{\Xi}_i + \bar{\Xi}_i^T), \\ \bar{\Gamma}_i &\triangleq \bar{\Gamma}_{i_{sub}} + (\alpha_i + \beta_i)\bar{\Lambda}_i + (\bar{\Lambda} + \bar{\Lambda}^T), \\ \bar{\Delta}_{ii} &\triangleq -(\bar{\Lambda}_i + \bar{\Lambda}_i^T) - (\bar{\Lambda} + \bar{\Lambda}^T) - (\bar{\Xi}_i + \bar{\Xi}_i^T), \\ \bar{\Delta}_{hi} &\triangleq -(\bar{\Lambda} + \bar{\Lambda}^T), \\ \bar{\Omega}_i &\triangleq \bar{\Omega}_{i_{sub}} + (\alpha_i + \beta_i)\bar{\Sigma}_i + (\bar{\Sigma} + \bar{\Sigma}^T), \\ \bar{\Pi}_{hi} &\triangleq \begin{cases} \bar{\Pi}_{hi_{sub}} + (\bar{\Xi}_i + \bar{\Xi}_i^T) & i = h \\ \bar{\Pi}_{hi_{sub}} & i \neq h \end{cases}, \\ \bar{\Psi}_{ii} &\triangleq -(\bar{\Sigma}_i + \bar{\Sigma}_i^T) - (\bar{\Sigma} + \bar{\Sigma}^T) - (\bar{\Xi}_i + \bar{\Xi}_i^T), \\ \bar{\Psi}_{hi} &\triangleq -(\bar{\Sigma} + \bar{\Sigma}^T), \\ \bar{\Upsilon}_{sub} &\triangleq \begin{bmatrix} 0 & 0 \\ 0 & -Z \end{bmatrix}, \quad \bar{\Gamma}_{i_{sub}} \triangleq \begin{bmatrix} 0 & 0 \\ Y_{i1} & 0 \end{bmatrix}, \\ \bar{\Omega}_{i_{sub}} &\triangleq \begin{bmatrix} -\frac{1}{2}X_i & 0 \\ Y_{i2} & 0 \end{bmatrix}, \\ \bar{\Lambda}_i &+ \bar{\Lambda}_i^T \geq 0, \quad \bar{\Sigma}_i + \bar{\Sigma}_i^T \geq 0, \quad \bar{\Xi}_i + \bar{\Xi}_i^T \geq 0. \end{aligned}$$

We present the following theorem without proof, which yields an optimized constrained input sequence at certain time k.

Theorem 3.2: The optimization problem (14) subject to (10), (15) and (16) is feasible if the following optimization problem involving a finite number of LMIs is feasible

$$\min_{\gamma, u_{k|k}, X_i, Y_{i1}, Y_{i2}, \Lambda_i, \Sigma_i, \Xi_i, \Lambda, \Sigma, \bar{\Lambda}_i, \bar{\Sigma}_i, \bar{\Xi}_i, \bar{\Lambda}, \bar{\Sigma}} \gamma, \quad (31)$$

subject to (25)-(28), (30).

If the above problem is feasible, the constrained input sequence is given by

$$\mathcal{U}_k = \{u_{k|k}, u_{k+i|k} = K(\Theta_{k+j}, \Theta_{k+j-1}) x_{k+i|k}\}, \quad (32)$$

where

$$K(\Theta_{k+j}, \Theta_{k+j-1}) = \left\{ \sum_{i=1}^{r} \theta_i(\eta_{k+i}) Y_{i1} + \sum_{i=1}^{r} \theta_i(\eta_{k+i-1}) Y_{i2} \right\} \left\{ \sum_{i=1}^{r} \theta_i(\eta_{k+i-1}) X_i \right\}^{-1}. \quad (33)$$

By the RHC strategy, only the first control $u_{k|k}$ among the control sequence $\mathcal{U}_k$ from the optimization problem in Theorem 3.2 is applied to the system at each time k. Then the procedure is repeated at the next time $k+1$. In the following theorem, we show that the closed-loop system with the control is stable.

Theorem 3.3: If the control input from the optimization problem in Theorem 3.2 is applied to the TS fuzzy system (1) with input constraints (2) according to the RHC strategy, the closed-loop system is asymptotically stable.

Proof: The proof is omitted since a similar proof can be seen in [8].

Remark 1: The proposed RHC scheme is similar to that of [8] in many respects. However, there are also some differences. The most important differences are that: 1) while [8] adopted a parameter dependent state-feedback control law, this paper adopt both the current and one-step past FWF dependent state-feedback control law. 2) while we adopt both the time and parameter-dependent quadratic function of state to bound the infinite horizon cost function, but [8] adopted only a time-dependent quadratic function. As a result, the conservatism of [8] is much more enhanced in this paper. ∎

IV. NUMERICAL EXAMPLE

Consider the system (1) with the following matrices

$$A_1 = \begin{bmatrix} 1 & 0.1 \\ -0.5 & 0.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.49 & 0.1 \\ -0.4 & 0.5 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.50.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix} \quad (34)$$

Simulation parameters are sa follows: $x_0 = [1.2 \ \ -0.2]^T$, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, $R = 1e^{-9}$, $|u(k)| \leq 3$. Fuzzy weighting functions are given as $\theta_1(\eta_k) = 1 - \frac{2*abs(xk(1))}{\pi}$ and $\theta_2(\eta_k) = \frac{2*abs(xk(1))}{\pi}$ where $abs(A)$ means absolute value of A. Figure 2 shows simulation results, where we can see that the proposed RHC stabilizes the closed-loop system.

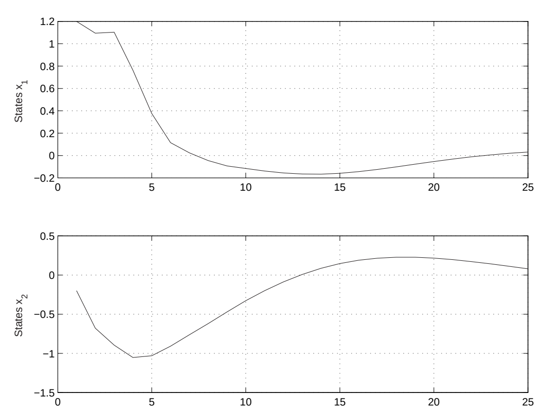

Fig. 1. State trajectories.

V. CONCLUDING REMARKS

In this paper, we presented a state-feedback RHC for discrete-time TS fuzzy systems with input constraints. We

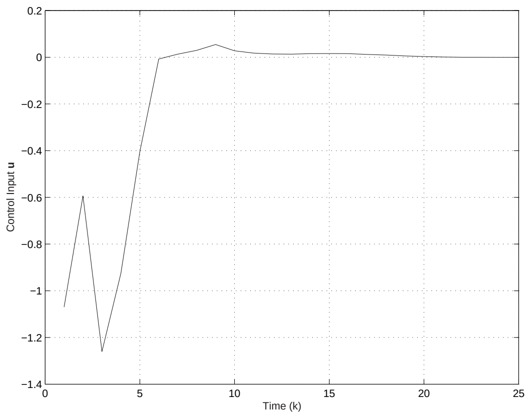

Fig. 2. Input trajectory.

first derived an optimization problem involving PLMIs that depend on the current and one-step past information on the time-varying fuzzy weighting functions. Since the PLMIs need to be checked for all values of the weighting functions, for solvability, we converted the PLMIs into a finite number of LMIs by suggesting a special structures for the variables of the PLMIs. Then, it was shown that the closed-loop system with the designed control is stable if the converted optimization problem is feasible at the initial time.

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REFERENCES

[1] W. H. Kwon and A. E. Peason, "A modified quadratic cost problem and feedback stabilization of a linear system," *IEEE Trans. Automatic Control*, vol. 22, pp. 838–842, May 1977.

[2] C. E. Garcia, D. M. Prett, and M. Morari, "Model predictive control: Theory and practice-a survey," *Automatica*, vol. 25, pp. 335–348, 1991.

[3] H. Michalska and D. Q. Mayne, "Robust receding horizon control of constrained nonlinear systems," *IEEE Trans. Automatic Control*, vol. 38, pp. 1623–1633, 1993.

[4] J. B. Rawlings and K. R. Muske, "The stability of constrained receding horizon control," *IEEE Trans. Automatic Control*, vol. 38, no. 10, pp. 1512–1516, 1993.

[5] A. Zheng and M. Morari, "Stability of model predictive control with mixed constraints," *IEEE Trans. Automatic Control*, vol. 40, no. 10, pp. 1818–1823, 1995.

[6] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.

[7] J. W. Lee, W. H. Kwon, and J. H. Choi, "On stability of constrained receding horizon control with finite terminal weighting matrix," *Automatica*, vol. 34, pp. 1607–1612, December 1998.

[8] Y. Lu and Y. Arkun, "Quasi-min-max MPC algorithms for LPV systems," *Automatica*, vol. 36, no. 4, pp. 527–540, 2000.

[9] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man., Cybern.*, vol. 15, pp. 116–132, 1985.

[10] H. Wang, K. Tanaka, and M. Griffin, "An approach to fuzzy control of nonlinear systems : stability and design issues," *IEEE Trans. Fuzzy Systems*, vol. 4, pp. 14–23, 1996.

[11] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers : relaxed stability conditions and lmi-based designs," *IEEE Trans. Fuzzy Systems*, vol. 6, pp. 250–265, 1998.

[12] N. R. S.-G. Cao and G. Feng, "Analysis and design of fuzzy control systems using dynamic fuzzy-state space models," *IEEE Trans. Fuzzy Systems*, vol. 7, pp. 192–200, 1999.

[13] M. Johansson, K.-E. Arzen, and A. Rantzer, "Piecewise quadratic stability of fuzzy systems," *IEEE Trans. Fuzzy Systems*, vol. 7, pp. 713–722, 1999.

[14] H. Tuan, P. Apkarian, T. Narikiyo, and Y. Yamamoto, "Parameterized linear matrix inequality techniques in fuzzy control system design," *IEEE Trans. Fuzzy Systems*, vol. 9, pp. 324–332, 2001.

[15] D. H. Choi and P. Park, "$h - \infty$ state-feedback controller design for discrete-time fuzzy systems using fuzzy weighting-dependent lyapunov functions," *IEEE Trans. Fuzzy Systems*, vol. 11, no. 2, pp. 271–278, 2003.

[16] P. Park and D. J. Choi, "LPV controller design for the nonlinear rtac system," *Int. J. Robust and Nonlinear Control*, vol. 11, pp. 1343–1363, 2001.

[17] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishanan, *Linear Martix Inequalities in System and Control Theory*, vol. 15. Philadelphia: SIAM, June 1994.