Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004
WeM03.4

Input Shaping with Coulomb Friction Compensation on a Solder Cell Machine

Dr. Keith Hekman
Department of Mechanical Engineering
The American University in Cairo
113 Kasr El Aini St. PO Box 2511 Cairo, 11511 Egypt
hekman@aucegypt.edu

Dr. William Singhose and Jason Lawrence
George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0405
William.Singhose@me.gatech.edu

***Abstract*—Input shaping is combined with a proportional plus derivative controller to control a solder machine. Input shaping is applied to reduce the vibrations of three different modes of the system, including the mode from the controller. The results are further improved by including the effect of coulomb friction when designing the shaper. Experimental results verify the benefits of the coulomb friction-compensating input shaper over normal input shaping, S-curves and constant velocity commands.**

1. INTRODUCTION

Coulomb friction can be a major detriment to the performance of high accuracy machines. The control of machines with coulomb friction is further complicated if the machine has flexible dynamics that lead to vibration and transient deflection. Command generation schemes have been shown to eliminate many of the problems associated with flexible dynamics and controller oscillations. Recently, command shaping has been applied to systems with coulomb friction. Given the recent successes, command generation techniques for position control in the presence of coulomb friction are further investigated here.

Command generation is the process of determining how and when machines are moved throughout their workspace. For example, Fig. 1 shows the response of a gantry crane to two different reference commands. In Fig. 1a, the operator presses the GO button only one time. The trolley moves four units and comes to rest, but the resulting payload oscillation is large. In Fig. 1b, the operator presses the GO button twice. If the button is pressed and released with correct timing, then the trolley moves the desired distance and the payload has no oscillations after the move is completed.

A command generator can create a command signal so that a machine can be moved any distance without residual vibration. Rather than depending on the skill of the operator to move without vibrations, the command generator takes any arbitrary command and modifies its shape so that it will not cause vibration. One such method, called input shaping, is shown in Fig. 2. In this example, the original step command is convolved with a sequence of impulses, and the result of the convolution is the new shaped command that will move the system without residual vibration.

The success of input shaping depends on the impulse sequence. For a linear system if the impulse amplitudes and time locations are chosen correctly, then any function can be used as the original command and the shaped command will result in no vibration [1,2]. Command generation has had a significant impact in high-tech manufacturing [3,4] and crane control [5,6,7,8]. It has also shown promise in the control of flexible spacecraft [9,10,11,12,13].

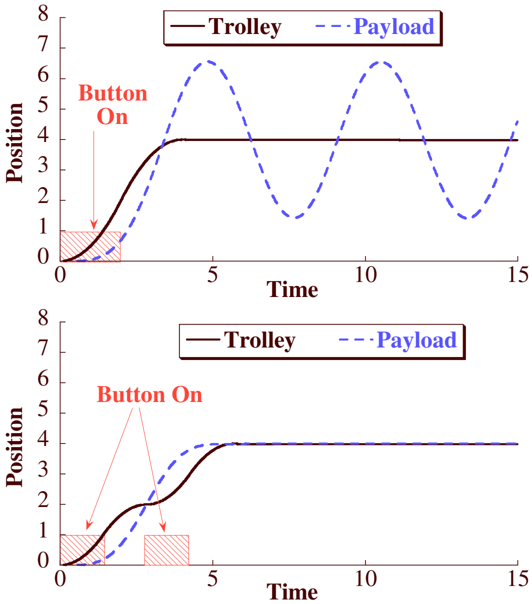

Fig. 1 Crane response to two different commands

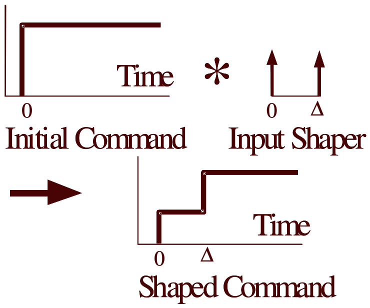

Fig 2 Command shaping process

The next section provides a brief review of input shaping. It also discusses the work done in adapting input shaping to work in the presence of coulomb friction. Section three presents the methodology for generating a multimode shaper designed to compensate for coulomb friction in a system under PD control. Section four presents the experimental development of the shaper parameters. Section five gives an experimental comparison between traditional shapers, coulomb friction-compensating shapers, S-curves and constant velocity

0-7803-8335-4/04/$17.00 ©2004 AACC

trajectory profiles. Finally, section six provides a summary of the work and conclusions.

2. INPUT SHAPING REVIEW

As a first step in understanding how to generate commands that move systems without vibration, it is helpful to start with the simplest such command, an impulse. Fig. 3 shows that the first impulse causes a flexible system to vibrate, but a second properly timed and sized impulse will cancel the vibration induced by the first impulse. If a reasonable estimate of the system's natural frequency, ω_n, and damping ratio, ζ, is available, then the residual vibration that results from a sequence of impulses applied to a second order system can be described by [2]:

$$V(\omega,\zeta)=e^{-\zeta\omega_n t_n}\sqrt{[C(\omega_n,\zeta)]^2+[S(\omega_n,\zeta)]^2}\ , \qquad (1)$$

where

$$\begin{aligned} C(\omega_n,\zeta)&=\sum_{i=1}^{n}A_i e^{\zeta\omega_n t_i}\cos(\omega_d t_i) \\ S(\omega_n,\zeta)&=\sum_{i=1}^{n}A_i e^{\zeta\omega_n t_i}\sin(\omega_d t_i) \end{aligned} \ . \qquad (2)$$

A_i and t_i are the amplitudes and time locations of the impulses, n is the number of impulses in the impulse sequence, and:

$$\omega_d=\omega_n\sqrt{1-\zeta^2}\ . \qquad (3)$$

If (1) is equal to zero, the impulse amplitudes and time locations will lead to zero residual vibration. Additional restrictions of

$$\sum A_i=1 \quad \text{and} \quad A_i>0 \quad i=1,\ldots,n \qquad (4)$$

are placed on the impulses to keep the solution from converging to zero-valued or infinitely-valued impulses.

Note that constraint (4b) can be relaxed and shapers with negative impulses will produce a faster rise time. [3, 14] However, to simplify the investigation, only positive impulses are used in this paper. Without loss of generality, the time of the first impulse is set to t_1=0.

The simplest solution to the problem (n=2) yields the Zero-Vibration (ZV) input shaper shown in Fig. 3 [2,14]:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix}=\begin{bmatrix} 1/(1+K) & K/(1+K) \\ 0 & 0.5T_d \end{bmatrix}, \qquad (5)$$

where,

$$K=e^{-\zeta\pi/\sqrt{1-\zeta^2}} \qquad (6)$$

and T_d is the damped period of vibration.

In general, to generate commands, the impulse sequence that causes no residual vibration is convolved with any desired command signal. The result is then used as the command to the system. This input shaping process was shown in Fig. 2

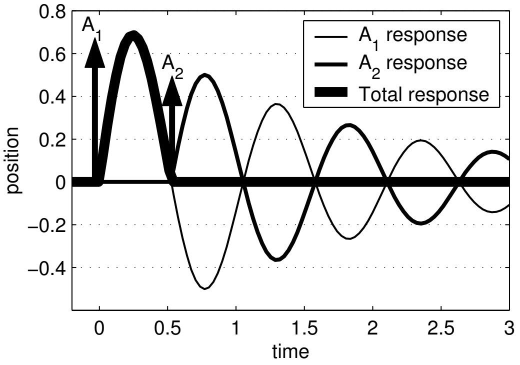

Fig. 3 Two impulses can cancel vibration

2.1 Robustness to Modeling Errors

The amplitudes and time locations of the impulses depend on the system parameters ω_n and ζ. If there are modeling errors in these values (and there always are), then the impulse sequence will not result in zero vibration. In fact, for the two-impulse sequence discussed above, there can be large magnitudes of vibration for relatively small modelling errors. This lack of robustness was a major obstacle for the original formulation of this idea [14, 15].

This robustness problem can be visualized by plotting a sensitivity curve that shows the amplitude of residual vibration as a function of the system frequency or damping ratio. One such sensitivity curve for the zero-vibration shaper is shown in Fig. 4 with the normalized frequency on the horizontal axis and the percent vibration on the vertical axis. Note that the vibration increases rapidly as the actual frequency deviates from the modelling frequency. Singer and Seering [2] developed the first robust shaper in the late 1980's. They designed the shaper by requiring the derivative of the vibration, with respect to the frequency, to be equal to zero at the modelling frequency. Including this constraint has the effect of keeping the vibration near zero as the actual frequency starts to deviate from the modelling frequency. The resulting shaper is

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix}=\begin{bmatrix} 1/(1+K)^2 & 1/(1+K) & K^2/(1+K)^2 \\ 0 & 0.5T_d & T_d \end{bmatrix}. \qquad (7)$$

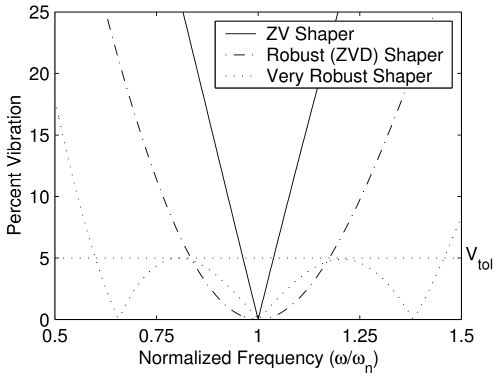

Fig. 4 Sensitivity curves for various shapers

The sensitivity curve for their zero vibration and derivative (ZVD) shaper is also shown in Fig. 4. Additional types of robust shapers have been developed that allow the robustness to be set to any desired level [16]. The sensitivity curve for one such very robust shaper is also shown in Fig. 4.

2.2 Input Shaping with Coulomb Friction

The majority of work in input shaping has been done with linear systems. In this case, the shaper design is independent of the motion. Hekman *et al*. [17] showed that input shaping is effective in reducing vibration levels in position control under the effects of coulomb friction. However, they noted that with coulomb friction, the shaper given by (5) does not result in zero residual vibration. Further work by Hekman *et al.* [18] showed that the shaper could be modified to result in zero residual vibration for a PD controlled system. Lawrence *et al.* [19] showed a simplified way to generate the command signal with the assumption that the sign of the friction force did not change direction during the motion.

3. THEORETICAL SHAPER DEVELOPMENT

The approach for shaper design of Lawrence *et al.* [19] for a second order system can be seen in Fig. 5. In this approach, a shaper (denoted ZV_S in the figure) is designed for the system as if coulomb friction was not present. Since the input to the system is a step, the resulting output will be a staircase type command. Then, an additional quantity ($\mu_k N/K_p$) is added to all of the steps except the final one in the staircase. The additional quantity compensates for the coulomb friction. The shaper modification is denoted ZV_C in the figure. This addition has the effect of linearizing the system, as long as the motion is only in one direction, causing friction to keep the same sign throughout the motion. This is true if the zero from the derivative control is negligible. Although their shaper addressed only a second order system, the linearizing effect of the additional quantity should still be effective for multiple mode shapers, as long as the movement is only in one direction, keeping sign of the friction constant. With these assumptions, it is possible to use any shaper design with the linearizer element to produce a friction compensating shaper.

4. EXPERIMENTAL SHAPER DESIGN

The input shaping based control scheme was implemented on a solder cell machine. Linear motors were used to control the position of the machine. In designing the shaper, the largest challenge was the cogging of the linear motors. This resulted in different responses for different starting and ending positions. The magnet spacing was six centimeters; so several trials were done at even intervals along the magnet spacing to examine the robustness in the controller response.

Previous research by Hekman *et al.* [18] and Lawrence *et al.* [19] have shown the effectiveness of PD control with input shaping in cases of coulomb friction. A large proportional gain PD controller was implemented on the system to reduce the steady state error. The derivative gain did not need to be large, as the input shaping would remove system oscillations. The initial controller command was a single step, which excited the natural frequency of the controller of approximately 20Hz, as seen in Fig. 6. From previous experimentation it was observed that the natural frequency of the controller changed depending on starting location. Because of this, a ZVD shaper was used to eliminate the vibrations in the region of the natural frequency of the controller. Recall that the ZVD shaper has an increased robustness over a ZV shaper.

Fig. 7 shows the results of the implementation of the ZVD shaper. With the majority of the oscillations at the controller frequency eliminated, one can now see from the response that there are other modes of vibration remaining after the shaped command is completed. A Fourier transform was calculated for the system response for the time after the completion of the shaper (Fig 8). The plot shows that the two predominate frequencies of vibration are at 11Hz and 76Hz. The lower frequency corresponds to the structural oscillations of the motor supports. This frequency was confirmed by measuring the structural vibrations of the machine with an accelerometer. The low frequency vibrations would affect the positioning accuracy of the system, as they move the reference frame of the linear encoders.

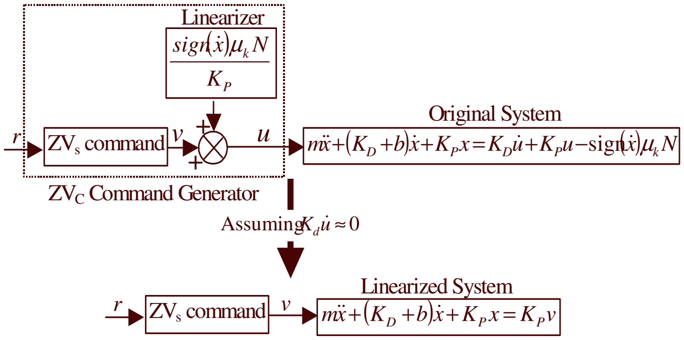

Fig. 5 Forming the ZV_C command from the ZV_S Command [19]

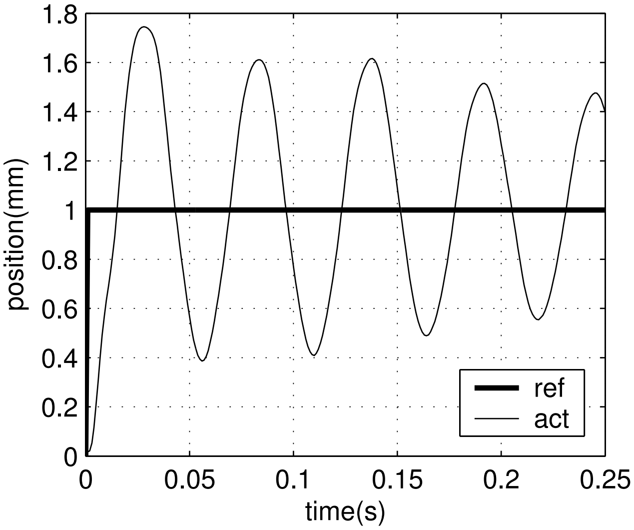

Fig. 6 Step response of the system

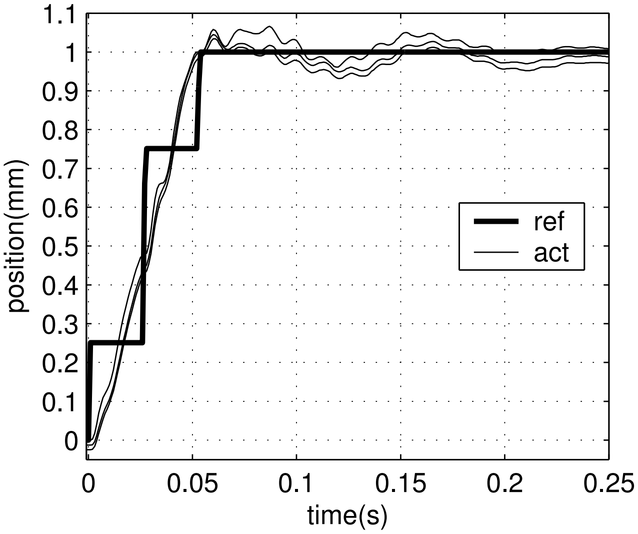

Fig. 7 System response with ZVD shaper

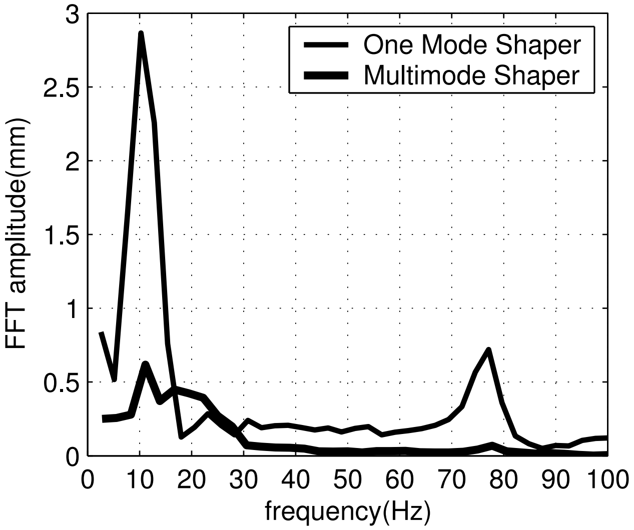

Fig. 8 Fourier transform plot of residual vibration for ZVD and multimode shapers.

Because of the residual vibration at several different frequencies, additional ZV shapers were designed at these frequencies. These shapers were convolved with the ZVD shaper to give the multimode shaper. The drawback of shaping multiple frequencies is that the duration of the shaper increases, and consequently the rise time of the system. ZV shapers were applied for modes at 11hz and 76hz. Since the oscillations were slow to die out, a damping ratio of 0.01 was assumed for each of these modes. Fig. 8 also shows the Fourier transform of the residual vibrations for the multimode shaper. This shows that the shaping significantly reduced the magnitude of the vibration at the problematic frequencies.

With this additional oscillation removed, the ZVD shaper could be fine tuned to produce little steady state error. This was done through a trial and error approach, trying to minimize the velocity and offset at the end of the shaped command. The trial and error approach is was necessary because there is not direct relationship between residual vibration and the natural frequency and damping ratio of the shaper.

Following the design of the multimode shaper, a similar trial and error approach was used to complete the design of the friction-compensating shaper, as described in Fig. 5. Both shapers were designed using a step size of two millimeters, to minimize the affects of friction.

5. EXPERIMENTAL SHAPER VERIFICATION

Once the shapers were designed, it was desired to explore their effectiveness over a range of conditions. This was done by commanding different move sizes and observing the machine response. For a further basis for comparison, moves were also performed with a constant velocity command and with an S-curve. All commands were designed to have the same duration.

Fig. 9a through 9d show the resulting position versus time plots. From the figures one can clearly see the benefits of input shaping (Figs. 9a,b) over the constant velocity motion (Fig. 9d). The constant velocity motion has significantly larger residual vibration than the other profiles. Using an S-curve (Fig. 9c) (i.e. constant acceleration) the residual vibration levels were reduced, but they were still larger than with either type of input shaping.

Based on Hekman *et al.* [18] the performance of the input shaping without coulomb friction compensation should degrade as the step size decreases. The degradation is not as large with the multi-mode shaper because the shaper does not end at the peak of the controller response, unlike the ZV shaper they studied. To see if the decline in performance was observed for the multi-mode shaper, the position error at the end of the shaper was plotted versus step size. (Fig. 10) The average of the errors is also plotted, since there was a distribution due to the different starting locations. From the plot of the averages, one can see the trend that the final position is getting lower as the step size gets smaller. This trend is not as pronounced in the shaper with friction compensation.

In addition to the visual comparison, the 95 percent confidence interval estimate on the difference in the means of the errors extends from 0.00304mm to 0.0300mm. It is therefore concluded that adding the friction compensation offset to the input shaper is effective in improving the response of the system over a larger range of step sizes. These results may be able to be improved with better shaper parameters, as it was difficult to determine the exact value of coulomb friction and damping ratios of each of the mode shapes.

6. CONCLUSION

This paper discussed the application of a multimode input shaper on a solder cell machine. The design of the shaper was discussed, and additional compensation was included to compensate for coulomb friction. Experimental results showed that the multi mode input shaping produced better results than either an S-curve profile or constant velocity motion in terms

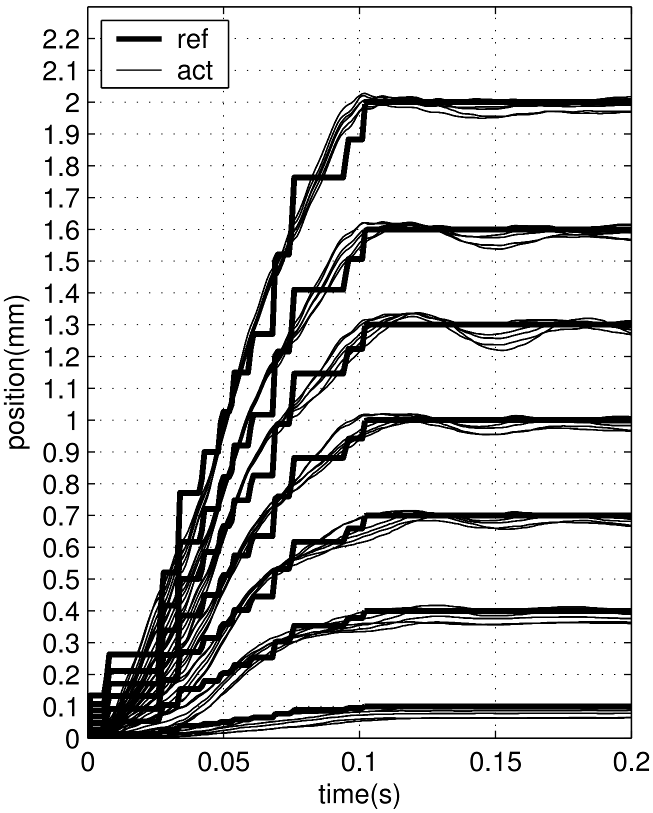

Fig. 9a Shaped motion

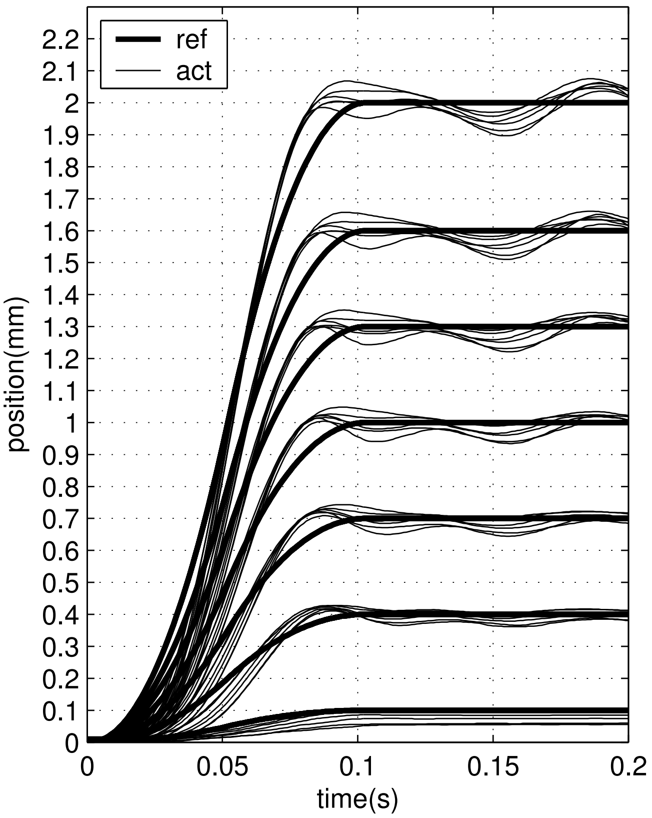

Fig. 9c S-Curve motion

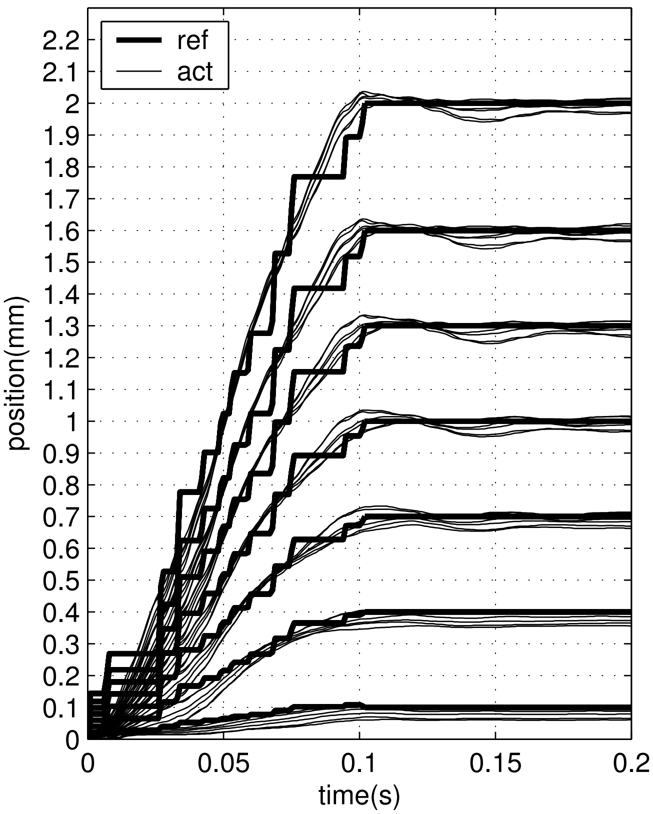

Fig. 9b Shaped motion with Coulomb compensation

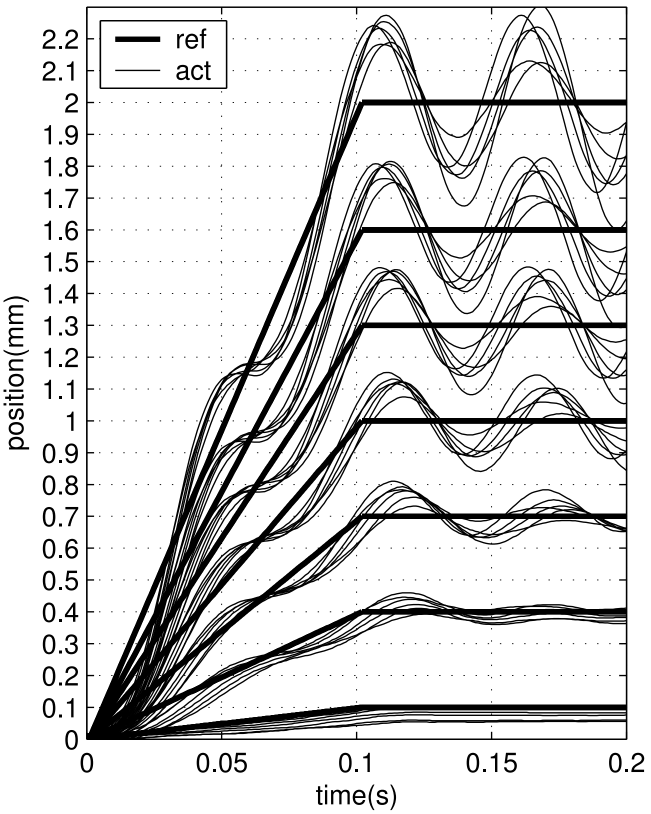

Fig 9d Constant velocity response

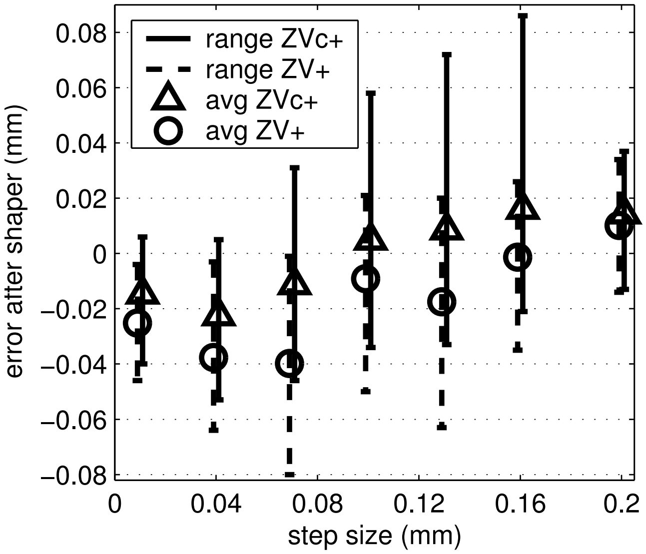

Fig. 10 Residual error after shaper completion

of residual vibrations. Furthermore, the friction-compensating shaper was shown to be more robust than the non-friction-compensating shaper in terms of move size.

Some difficulty was experienced in the determination of the parameters of the system for the shaper design. Therefore, future work could include better system identification methods. Also, future work could include friction compensated shaper design if the velocity changes sign.

6. REFERENCES

[1] Bhat, S. P. and Miu, D. K., “Precise Point-to-Point Positioning Control of Flexible Structures,” J. of Dynamic Sys., Meas., and Control, vol. 112(4), 1990, pp. 667-674.

[2] Singer, N. C. and Seering, W. P., “Preshaping Command Inputs to Reduce System Vibration,” J. of Dynamic Sys., Measurement, and Control, vol. 112(March), 1990, pp. 76-82.

[3] Singhose, W., Singer, N. and Seering, W., “Time-Optimal Negative Input Shapers,” J. of Dynamic Systems, Measurement, and Control, vol. 119(June), 1997, pp. 198-205.

[4] deRoover, D., Sperling, F. B. and Bosgra, O. H., "Point-to-Point Control of a MIMO Servomechanism," in American Control Conference, Philadelphia, PA. 1998

[5] Starr, G. P., “Swing-Free Transport of Suspended Objects With a Path-Controlled Robot Manipulator,” J. of Dynamic Systems, Measurement and Control, vol. 107, 1985, pp. 97-100.

[6] Singer, N., Singhose, W. and Kriikku, E., "An Input Shaping Controller Enabling Cranes to Move without Sway," ANS 7th Topical Meeting on Robotics and Remote Systems, Augusta, GA. 1997

[7] Feddema, J., Petterson, B. and Robinett, R., Operator Control Systems and Methods for Swing-Free Gantry-Style Cranes. U. S. Patent 5,785,191. 1998

[8] Singhose, W., Porter, L., Kenison, M. and Kriikku, E., “Effects of Hoisting on the Input Shaping Control of Gantry Cranes,” Control Engineering Practice, vol. 8(10), 2000, pp. 1159-1165.

[9] Banerjee, A. K., “Dynamics and Control of the Wisp Shuttle-Antennae system,” J. of Astronautical Sciences, vol. 41(1), 1993, pp. 73-90.

[10] Singh, T. and Vadali, S. R., "Robust Time-Optimal Control: A Frequency Domain Approach," *J. of Guidance, Control and Dynamics*, vol. 17, 1994, pp. 346-353.

[11] Singhose, W., Bohlke, K. and Seering, W., “Fuel-Efficient Pulse Command Profiles for Flexible Spacecraft,” AIAA J. of Guidance, Control, and Dyn., vol. 19(4), 1996, pp. 954-960

[12] Tuttle, T. and Seering, W., “Experimental Verification of Vibration Reduction in Flexible Spacecraft Using Input Shaping,” J. of Guidance, Control, and Dynamics, vol. 20(4), 1997, pp. 658-664.

[13] Banerjee, A., Pedreiro, N. and Singhose, W., “Vibration Reduction for Flexible Spacecraft Following Momentum Dumping with/without Slewing,” AIAA J. of Guidance, Control, and Dynamics, vol. 24(2). 2001

[14] Smith, O. J. M., *Feedback Control Systems*, New York: McGraw-Hill Book Co., Inc., 1958, pp. 331-345.

[15] Tallman, G. H. and Smith, O. J. M., "Analog Study of Dead-Beat Posicast Control," *IRE Transactions on Automatic Control*, 1958, pp. 14-21

[16] Singhose, W. E., Seering, W. P. and Singer, N. C., "Input Shaping for Vibration Reduction with Specified Insensitivity to Modeling Errors," Japan-USA Sym. on Flexible Automation, Boston, MfA. 1996

[17] Hekman, K., Lawrence, J. and Singhose, W. “Use of Input Shaping to Decrease the Effects of Stiction,” in 5th IFAC Symposium on Nonlinear Control Systems, St. Petersburg, Russia, 2001

[18] Hekman, K., Lawrence, J., Singhose, W., “Input Shaping for a PD Position Controller Under Coulomb Friction” in *the XV IFAC World Congress*, Barcelona, Spain, July 21-26, 2002

[19] Lawrence, J., Singhose, W., Hekman, K., “An Analytical Solution for a Zero Vibration Input Shaper for Systems with Coulomb Friction.” in *the 2002 American Control Conference*, (CDROM) Anchorage, Alaska, USA, May 8-10, 2002