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The horizon in predictive energy storage control

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***Abstract*— Energy saving storage control in power conversion systems is considered. Savings are achieved by operating the power converters in efficient operating points, meeting the desired power output with an energy storage device. Predictions of the future desired output and conditions are used, with the largest gain for an infinite prediction horizon. This is not realistic: predictions for the far future are unreliable. So, it is of interest to know the performance degradation due to a finite horizon. This paper presents a guideline for the horizon length, using an interpretation of the characteristics of the optimal solution for a stylized problem, that will give performance close to the infinite horizon one. Characteristics of the active constraints determine the desired horizon. When avoiding active constraints covering the full horizon, the horizon can be reduced to the smallest possible one.**

I. INTRODUCTION

Power conversion systems are abundant. A few examples are power plants, propulsion systems (air, land, marine), solid state power electronics, etc. In all devices energy losses occur during the transformation. The losses depend on the operating conditions, like speed, duty factor, load. Operating the devices in their optimal work point would be best, but supply and demand will not be balanced. A storage device can take up the excess and provide during shortage. Several methods are known for scheduling the storage.

For instance, [1] provides a convex (LP) formulation to obtain optimal conditions for operating the storage, taking into account storage losses. Being LP, the solution is not a continuous function of the parameters, which is not convenient. They assume future information about demand is available, so this result is non-causal and cannot be implemented in real time. In [2] the problem is also cast as a convex one, but in the more convenient QP framework. Here, information about the future is assumed to be available also, but a technique to reduce the requirements on the horizon length, by avoiding constraints to be active that relate the far future to the present [3], is theoretically justified. This technique will also be employed here.

It is unlikely that information about the future is perfect. So, how to choose the horizon? In standard MPC, the choice of horizon length is related to stability or settling times [4], but these are no issues here. This paper presents a guideline for choosing the length of the prediction horizon, based on characteristics that quantify the influence on performance of the horizon length. The guideline is demonstrated with an example.

II. PROBLEM FORMULATION

We formulate the storage control problem in discrete time as the minimization of the energy needed to meet a specified power demand under known operating conditions. Various constraints have to be met. We use a QP setting, so

$$\min\ x'Hx + f'x, \qquad \text{sub } Ax < b.$$

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Here x is the vector of design variables related to the operation of the storage device over the horizon, *i.e.*, the flow into the storage device. The constraints include the system dynamics. The dependency of the future enters via H, f, A, and b, that will depend on future demand and operating conditions. If predictions of these parameters are not available, we cannot do much, if they are of low quality, the results will be suboptimal. A main constraint is the requirement that the level of the storage at the end of the horizon should be equal or higher than a desired one, otherwise the storage device would not be charge retaining, so $\sum x = 0$. This also guarantees stability. With a short horizon this constraint will reduce the performance because it will limit the variation in x. Therefore, a large horizon is best.

III. CHOOSING THE HORIZON

As explained in [2], when the storage control problem is structured so the Hessian H is diagonal, the constraint is of the form $\sum x = 0$, and there are only simple bounds on x, the optimal solution is characterized by equal incremental cost at all time instances. So $x(i)$ will be positive or negative depending on the sign of the difference between the incremental cost at time instance i and the optimal incremental cost. Taking this as a hint, we argue that to obtain effective control the horizon *should encompass intervals where the incremental cost will be both lower and higher than optimal*. It is therefore natural to look at the temporal characteristics of the incremental cost, like its power spectrum. A criterion for choosing a “good” horizon length would then be the time corresponding to the lowest dominant frequency in the power spectral density. Looking at the incremental cost avoids characterizing the temporal characteristics in terms of the demand and operating conditions directly, where we would meet more data to deal with and would be without the clear interpretation of the selection criterion given.

IV. REDUCING THE HORIZON

From the formulation of the problem it was already clear that the charge retaining constraint $\sum x = 0$ relates the (far) future with the present. As proven in [2] for the formulation sketched in the previous section, this constraint can be removed. This allows a very short horizon. It appears that the Lagrange multiplier λ represents the optimal incremental cost. Modifying the objective to

$$\min\ x'Hx + f'x - \lambda \sum x$$

makes the constraint superfluous. However, the optimal λ is a function of future conditions, so is not known very well. When it is chosen incorrectly the storage level will drift out of its desired range, therefore the constraint is needed in practice, necessitating a larger horizon. When other constraints are important a larger horizon may also be required.

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V. EXAMPLE

We consider a storage control problem in an aerial propulsion system, where most of the power is used for propulsion and the remainder for electric devices. Only the electrical energy is assumed to be stored. Data is present for 1800 [s]. The model consists of an integrator for the storage and uses quadratic functions to describe the power conversions. The control input x is the storage power flow. The storage level is measured. For more details see [3]. This example does not meet the requirements mentioned in Section III. It is still possible to use the techniques considered, as shown below.

The power spectral density of the incremental cost is given in Fig.1. Note that the lowest dominant (density $>$ 10)

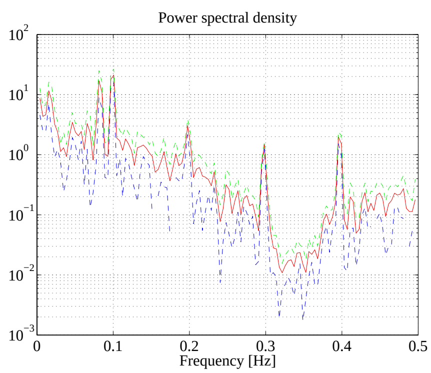

Fig. 1. Power spectral density incremental cost

frequency is $f_m = 1/300$ [Hz], with $1/60$, $1/12$, and $1/10$ next. To illustrate the influence of the lowest dominant frequency on the desired horizon length, the conditions and demand were modified to remove the $1/300$ component, then the lowest dominant frequency is $f_m = 1/60$ [Hz], and then also removing this component to get $f_m = 1/12$ [Hz]. This gives 3 different sequences of demand and operating conditions with different spectral densities for the incremental cost. Using a receding horizon to implement predictive control with different horizon lengths leads to the results in Fig. 2. As can be seen, the horizon length where the

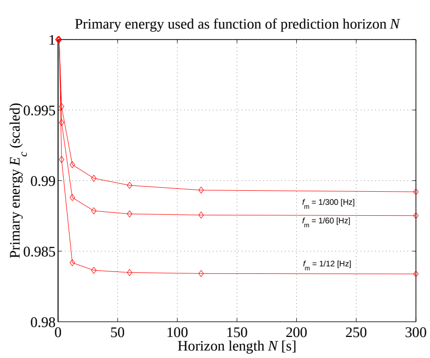

Fig. 2. Influence of dominant frequency f_m on desired horizon length

"infinite" horizon performance is recovered is close to the one derived from the guideline. Accidentally, this figure also shows that variations in demand and operating conditions lead to different opportunities for optimization.

Next we study the influence of the use of λ. It appears that for the complete set of data used in the example, the value of λ that guarantees the level of the storage to be the same at begin and end of the data set can be computed, and is called $\bar{\lambda}$. The influence of horizon length on performance is in Fig. 3, using values of λ equal to 0, $.8 \cdot \bar{\lambda}$, $.9 \cdot \bar{\lambda}$, $.975 \cdot \bar{\lambda}$, and $\bar{\lambda}$. For all values of λ the charge retaining constraint was included. It can be seen that an error of 20% in the

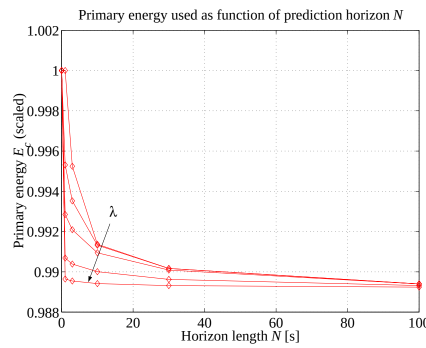

Fig. 3. Influence of λ on desired horizon length

prediction of $\bar{\lambda}$ still leads to better performance, but only for the smaller lengths. This is because for larger horizons the predicted level of the storage will go down, so the constraint will be active, limiting the freedom in choosing x.

The results in the figure should be interpreted with care, because they do not represent a repetitive behavior, in the sense that the storage level at the end may be larger than at the start. With $\lambda = 0$ the storage level will be pressed against the constraint, but with $\lambda \approx \bar{\lambda}$ the level will hover slightly above the constraint. This allows for more freedom in the choice of x. It takes some time to reach this level, so the performance for $\lambda = \bar{\lambda}$ for short horizons is slightly worse than for the "infinite" one.

When $\bar{\lambda}$ is not known accurately, a simple technique for implementation would be to set a zone objective [4] for the storage level, enforced by upper and lower bounds. If the level hits the upper bound often, the value of λ can be reduced, and vice versa. Other techniques are also possible.

VI. CONCLUSION

A guideline for choosing and a technique for reducing the desired horizon length for predictive storage control were presented. An example showed this to be effective.

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