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Tracking or Rejecting Rotational-Angle Dependent Signals Using Time Varying Repetitive Control

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Abstract

Rotational-angle is a natural clock for the internal combustion engine. Many of the engine control problems involve controlling actuators or mechanisms to track or reject signals that are periodic in the rotational-angle domain. However a periodic signal in the rotational-angle domain becomes cyclic but aperiodic in the time domain as the rotational speed changes. This phenomenon poses a great challenge for achieving asymptotic tracking or disturbance rejection. To take advantage of the fact that the signal is periodic in the rotational-angle domain, we can transform the actuator dynamics into the rotational-angle domain and design the controller correspondingly. However the linear time invariant (LTI) actuator model becomes linear time varying (LTV) after the transformation. Time varying repetitive control is then proposed to drive the time varying plant to track or reject periodic signals. The periodic signal internal model is embedded in the time varying feedback loop to ensure asymptotic performance. Simulation results demonstrate the effectiveness of the proposed algorithms.

1. Introduction

Many industrial applications involve controlling actuators or mechanisms to track or reject signals that are periodic in the rotational-angle domain. To name a few, rejection of position-dependent disturbances for DC motors [1], control of radial runout in face milling [2], transmission disturbance rejection for laser printers [3]. If the rotational speed doesn't change, those problems can be solved by using repetitive control [4-6]. However a periodic signal in the rotational-angle domain becomes cyclic but aperiodic in the time domain as the rotational speed changes. This phenomenon poses a great challenge for achieving asymptotic tracking or disturbance rejection.

Among different industrial applications, internal combustion engine presents a very interesting yet challenging problem. On one hand, many of the engine subsystems demonstrate rotational-angle dependent behavior; On the other hand, engine rotational speed changes all the time, and the change is not necessarily small or periodic. Chin and Coats [7] examined the air to torque dynamics, fuel to torque dynamics, engine rotational dynamics and the exhaust gas transport delay in both time domain and rotational-angle domain and concluded that most engine dynamics except the fuel are rotational-angle dependent. So the rotational-angle is indeed a natural clock for the internal combustion engine. Inspired by this intrinsic characteristic, control designs in the rotational-angle domain have been investigated by a number of researchers. However, a linear time invariant actuator model becomes linear time varying when converted from time domain to rotational-angle domain, which greatly complicates the control design process. Yurkovich and Simpson [8,9] compared different linear and nonlinear control designs for engine idle speed control in the rotational-angle domain. The nonlinear controllers outperformed the linear ones due to the intrinsic time varying plant dynamics in the rotational-angle domain. Scotson and Heath [10] developed a rotational-angle based dynamic model for the speed control of diesel engines. As a result, the resonance dynamics become dependent on the rotational speed. A linear time invariant controller was designed in the rotational-angle domain to regulate engine speed subject to load disturbances. However, the control authority is only limited to the low frequency range due to the limitation of the LTI controller. Song and Grigoriadis [11] presented diesel engine speed regulation using linear parameter varying control, where the engine speed control problem was formulated as an L_2 gain optimization problem. To achieve asymptotic tracking or disturbance rejection regardless of the varying rotational speed, we were motivated to develop time varying repetitive control design.

Tsakalis and Ioannou [12] presented the internal model principle based tracking control design for linear time varying systems. Sun and Tsao presented nonlinear internal model principle control and predictive internal model control for linear [13, 14] or nonlinear systems [15] with nonlinear disturbance dynamics, especially chaotic disturbances, in the discrete and continuous time domains respectively. Based on these results, Sun and Tsao [16] presented repetitive control design for linear time varying systems in the continuous-time domain. A constructive algorithm was proposed to embed the periodic signal internal model in the time varying feedback loop. Necessary and sufficient conditions for asymptotic disturbance rejection were then derived. Similar to the LTI repetitive control design, it is shown that asymptotic performance cannot be achieved with a finite dimensional controller in the continuous-time domain. Analytical results on the achievable performance bound with finite dimensional controllers in the continuous-time domain are also presented.

This paper addresses the problem of controlling actuators or mechanisms to track or reject signals that are periodic in the rotational-angle domain while the rotational speed varies in real-time. To take advantage of the fact that the signals to be tracked or rejected are periodic in the rotational-angle domain, we transform the actuator dynamics into the rotational-angle domain and design the controller correspondingly. However the linear time invariant actuator model becomes linear time varying in the rotational-angle domain. Time varying repetitive control is then applied to drive the time varying plant to track or reject periodic signals. The periodic signal internal model is embedded in the time varying feedback loop to ensure asymptotic performance. Simulation results on engine variable valve actuation control demonstrate the effectiveness of the proposed algorithms.

The rest of this paper is organized as follows. Section 2 describes the control problem; Section 3 addresses the conversion of plant dynamics from time domain to rotational-angle domain; Section 4 presents the time varying repetitive control design; Section 5 presents the simulation results and conclusions are in Section 6.

2. Problem Description

We consider the problem of controlling an actuator to track or reject signals that are periodic in the rotational-angle domain while the rotational speed varies in real-time. The linear time invariant (LTI) plant model is shown as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t) + d(t)$$

where $x(t) \in R^m$, $u(t) \in R$ and $y(t) \in R$ are the state, input and output signals respectively. $d(t) \in R$ is the bounded but unmeasurable disturbance.

The rotational-angle $\theta(t)$ is defined as:

$$\theta(t) = \int_0^t \omega(\tau)d\tau$$

where $\omega(\tau)$ is the rotational speed.

Since the disturbance is periodic in the rotational-angle domain, it satisfies the following model:

$$d(\theta + 2k\pi) = d(\theta) \qquad (2)$$

where k is an integer.

To take advantage of the periodicity of the disturbance, we sample the plant in the rotational-angle domain, i.e. sample the system at $\theta(0)$, $\theta(1)$, $\cdots, \theta(k)\cdots$. Correspondingly, the state, input and output signals become $x(k)$, $u(k)$ and $y(k)$.

The control laws we are considering are output feedback. Detailed control structures will be presented later. The control objective is to achieve *asymptotic disturbance rejection*, i.e. the unforced system is uniformly asymptotic stable and the steady state output of the forced system goes to zero, $\lim_{k\to\infty} y(k) = 0$, for any initial conditions of the plant and disturbance.

Remark: The asymptotic tracking problem $\lim_{k\to\infty} y(k) = r(k)$ can be easily formulated as the asymptotic disturbance rejection problem by letting $d(k) = -r(k)$, where $r(k)$ is the desired reference signal.

3. Formulation of the Linear Time Varying Plant in Rotational-Angle Domain

To design the controller in the rotational-angle domain, we need to convert the continuous-time plant model (1) into the rotational-angle domain first.

Let $t = t_{k+1}$ at $\theta(k+1)$, based on the plant model (1), we have:

$$x(t_{k+1}) = e^{At_{k+1}}x(0) + \int_0^{t_{k+1}} e^{A(t_{k+1}-\tau)}Bu(\tau)d\tau$$

Similarly we can get:

$$x(t_k) = e^{At_k}x(0) + \int_0^{t_k} e^{A(t_k-\tau)}Bu(\tau)d\tau$$

Define $T(k) = t_{k+1} - t_k$ and multiply $e^{AT(k)}$ to both sides of the above equation,

$$e^{AT(k)}x(t_k) = e^{At_{k+1}}x(0) + \int_0^{t_k} e^{A(t_{k+1}-\tau)}Bu(\tau)d\tau$$

Then

$$x(t_{k+1}) = e^{AT(k)}x(t_k) + \int_{t_k}^{t_{k+1}} e^{A(t_{k+1}-\tau)}Bu(\tau)d\tau$$

Let $\lambda = \tau - t_k$ and assume $u(\tau) = u(t_k)$ for $t_k \le \tau < t_{k+1}$, we have:

$$x(t_{k+1}) = e^{AT(k)}x(t_k) + e^{AT(k)}\int_0^{T(k)} e^{-A\lambda}Bu(t_k)d\lambda$$

Assume the matrix A is non-singular, we get:

$$x(t_{k+1}) = e^{AT(k)}x(t_k) + (e^{AT(k)} - I)A^{-1}Bu(t_k)$$

Assume the rotational speed $\omega(t)$ stays constant during the time interval $[t_k \quad t_{k+1})$ and designate it as $\omega(k)$, then

$$T(k) = \frac{\theta(k+1) - \theta(k)}{\omega(k)} = \frac{\delta_\theta(k)}{\omega(k)} \qquad (3)$$

We can rewrite the above discrete rotational-angle domain state space model as:

$$x(k+1) = F(k)x(k) + G(k)u(k) \tag{4}$$
$$y(k) = Hx(k) + d(k)$$
where $F(k) = e^{AT(k)}$, $G(k) = (e^{AT(k)} - I)A^{-1}B$, $H = C$.

Remark: Changing either the sampling interval $\delta_\theta(k)$ or the speed $\omega(k)$ in real-time will cause the above model to be time varying.

The time varying repetitive control that will be presented in next section is designed using the input-output representation. So we need to convert the state space model (4) into fractional representation. Fractional representation used to describe the I/O properties of discrete LTV systems has been extensively investigated in [17], [18] and [19]. The following definitions on polynomial delay operator (PDO) and polynomial summation operator (PSO) will be used in this paper.

Definition 1: Let q^{-1} represent the one step delay operator. The left polynomial delay operator (PDO) $P(q,k)$ is defined as:

$$P(q,k) = a_0(k) + a_1(k)q^{-1} + \cdots + a_n(k)q^{-n}$$

Similarly, the right PDO is defined as:

$$P(q,k) = a_0(k) + q^{-1}a_1(k) + \cdots + q^{-n}a_n(k)$$

where $a_i(k)$, $i = 0, 1, \cdots, n$ are bounded and $a_n(k) \neq 0$ for some $k \geq 0$. If $a_0(k) = 1$, $\forall k \geq 0$, the above PDO is said to be monic.

Definition 2: A left (right) polynomial summation operator (PSO) $P^{-1}(q,k)$ is defined as the operator that maps the input $u(k)$ to the zero state response of the difference equation $P(q,k)[y] = u$ where $P(q,k)$ is the left (right) monic polynomial delay operator (PDO). More specifically,

$$P^{-1}(q,k)[u](k) = \sum_{j=0}^{k} H(k)\Phi(k, j+1)G(j)u(j)$$

where $\Phi(k, j)$, $G(k)$, $H(k)$ are the state transition matrix, the input and output matrix, respectively, corresponding to the observer (controller) realization of the difference equation.

Definition 3: The PSO $P^{-1}(q,k)$ is said to be uniformly asymptotically stable (UAS) if and only if for any $\varepsilon > 0$ there exists a positive integer n_ε such that the state transition matrix $\Phi(k, j)$, associated with the linear difference equation $P(q,k)[y] = u$, satisfies

$\| \Phi(j+i, j) \| \leq \varepsilon$, for all $j \in Z$ and all $i \geq n_\varepsilon$.

Definition 4: Given the following two PDO's $P(q,k)$ and $Q(q,k)$:

$$P(q,k) = a_0(k) + a_1(k)q^{-1} + \cdots + a_n(k)q^{-n}$$
$$Q(q,k) = b_0(k) + b_1(k)q^{-1} + \cdots + b_n(k)q^{-n}$$

We say $P(q,k) = Q(q,k)$ if and only if $a_i(k) = b_i(k)$ for $i = 0, 1, \cdots, n$.

Now we are ready to transform the plant model (4) into the following fractional representation:

$$y = A_p^{-1}(q,k)B_p(q,k)[u] + d \tag{5}$$

$$A_p^{-1}(q,k)B_p(q,k) = H\{qI - F(k)\}^{-1}G(k) \tag{6}$$

where $B_p(q,k)$ and $A_p^{-1}(q,k)$ are the PDO and PSO as defined in Definitions 1 and 2 respectively.

The periodic disturbance model (2) can be transformed into the following SISO linear time invariant (LTI) dynamic model:

$$\Lambda(q)[d] = 0 \tag{7}$$

where $\Lambda(q) = 1 - q^{-N}$ is a time invariant PDO and N is the period of the signal.

Remark: The plant model (5) and disturbance model (7) are the input-output representations of the original plant and disturbance models (1) and (2) in the discrete rotational-angle domain.

4. Time Varying Repetitive Control Design

As shown in Figure 1, the output feedback control law is as follows:

$$u = P(q,k)Q^{-1}(q)[u_1 - u_2] \tag{8}$$
$$u_1 = N(q,k)M^{-1}(q,k)[-y]$$
$$u_2 = A_p^{-1}(q,k)B_p(q,k)[u]$$

where $P(q,k)$, $N(q,k)$ are PDO's as defined in Definition 1. $M^{-1}(q,k)$ is a PSO as defined in Definition 2. $Q^{-1}(q)$ is a time invariant UAS PSO as defined in Definition 3. The motivation behind control structure (8) is that we need to embed a self-excitation mechanism in the feedback loop so that it will drive the plant to cancel out the persistent but bounded disturbance once the output goes to zero. The fundamental reason that we include the plant model $A_p^{-1}(q,k)B_p(q,k)$ in the feedback structure (8) is due to the non-commutative properties of the time varying operators. More detailed explanations can be found in [16], where the necessary and sufficient conditions for achieving asymptotic performance were derived in the continuous-time domain.

The following theorem provide the sufficient conditions for achieving asymptotic performance:

Theorem: Consider the plant model (5), disturbance model (7) and the control law (8), if $P(q,k)Q^{-1}(q)$ and $N(q,k)M^{-1}(q,k)$ satisfy the following conditions, asymptotic disturbance rejection can be achieved:

$$A_p(q,k)Q(q) + B_p(q,k)P(q,k) = X(q,k)\Lambda(q) \quad (9)$$

$$A_p(q,k)Q(q)\tilde{M}(q,k) + B_p(q,k)P(q,k)\tilde{M}(q,k) + B_p(q,k)P(q,k)\tilde{N}(q,k) = A_*(q,k) \quad (10)$$

where $A_*^{-1}(q,k)$ is an UAS PSO, $M = Q\tilde{M}$, $N = Q\tilde{N}$, $X(q,k)$, $\tilde{M}(q,k)$ and $\tilde{N}(q,k)$ are PDO's as defined in Definition 1.

Proof: As shown in Figure 1, the output of the system is:

$$y = A_p^{-1}B_p[u] + d$$

From (8), we have

$$(1 + PQ^{-1}A_p^{-1}B_p)[u] = PQ^{-1}[u_1]$$

So $y = d - A_p^{-1}B_p(1 + PQ^{-1}A_p^{-1}B_p)^{-1}PQ^{-1}NM^{-1}[y]$

$$y = [1 + (B_p^{-1}A_p + PQ^{-1})^{-1}PQ^{-1}NM^{-1}]^{-1}[d]$$

$$y = M[M + Q(A_pQ + B_pP)^{-1}B_pP\tilde{N}]^{-1}[d]$$

By condition (9), we get

$$y = M[M + Q(X\Lambda)^{-1}B_pP\tilde{N}]^{-1}[d]$$

$$y = M[X\Lambda Q^{-1}M + B_pP\tilde{N}]^{-1}X\Lambda Q^{-1}[d]$$

$$y = M[X\Lambda\tilde{M} + B_pP\tilde{N}]^{-1}XQ^{-1}\Lambda[d]$$

From the disturbance model (7), we have $\Lambda[d] = 0$, together with condition (10) we conclude $\lim_{k\to\infty} y(k) = 0$.

It is well known from the internal model principle [20-22] that the disturbance model need to be included in the feedback loop to ensure asymptotic performance. Condition (9) is designed to embed the periodic disturbance model in the time varying feedback loop. Condition (10) is to ensure the asymptotic stability of the closed loop system.

5. Simulation Results

In this section, we simulate the variable valve actuation control for the internal combustion engine using the proposed schemes. As we know, engine valve motion is periodic in the rotational-angle domain and a simplified valve profile is shown in Figure 2. There are three key parameters for the valve profile: lift, phase and duration. However, the valve motion becomes cyclic but aperiodic in time domain as rotational speed changes. We apply the time varying repetitive control to achieve asymptotic performance.

Consider the following linear time invariant plant model:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (11)$$

$$y(t) = Cx(t) + d(t)$$

where $u(t) \in R$, $y(t) \in R$ and $x(t) \in R$ are the input, output and state signals respectively. $A = -1$, $B = 1$, $C = 10$.

Sampling the above system in the rotational-angle domain, we get the following linear time varying plant model:

$$x(k+1) = F(k)x(k) + G(k)u(k) \quad (12)$$

$$y(k) = Hx(k) + d(k)$$

where

$$F(k) = e^{AT(k)} = e^{-T(k)}, G(k) = [e^{AT(k)} - 1]A^{-1}B = 1 - e^{-T(k)}$$

and $H = C$. $T(k)$ is defined in (3).

We can then calculate the input-output representation of the system using (6):

$$y = A_p^{-1}(q,k)B_p(q,k)[u] + d \quad (13)$$

where $A_p(q,k) = 1 + q^{-1}a_1(k)$, $a_1(k) = -e^{-T(k)}$

$$B_p(q,k) = q^{-1}b_1(k),\ b_1(k) = 10(1 - e^{-T(k)})$$

The control objective is to achieve asymptotic tracking, i.e. $\lim_{k\to\infty} y(k) = r(k)$, where $r(k)$ is the desired reference signal shown in Figure 2. As we mentioned before, the asymptotic tracking problem can be easily transformed into asymptotic disturbance rejection problem by letting $d(k) = -r(k)$. Since the reference signal is a square wave in the rotational-angle domain, we only need to sample it twice every engine cycle, i.e. one sample per 360 degrees ($\delta_\theta(k) = 360$). The corresponding disturbance model is as follows:

$$\Lambda(q) = 1 - q^{-2} \quad (14)$$

Now we are ready to solve for the linear time varying controllers to satisfy the sufficient conditions (9) and (10).

Step 1: Solve $P(q,k)Q^{-1}(q)$ to satisfy condition (9):

Let $X = 1$, $Q = 1 - 0.5q^{-1}$ and $P = p_0(k) + q^{-1}p_1(k)$, plug them into (9):

$$(1 + q^{-1}a_1(k))(1 - 0.5q^{-1}) + q^{-1}b_1(k)(p_0(k) + q^{-1}p_1(k)) = 1 - q^{-2}$$

Solve the above time varying polynomial equation, we get:

$$-0.5 + a_1(k) + b_1(k)p_0(k) = 0$$

$$-0.5a_1(k+1) + b_1(k+1)p_1(k) = -1$$

So $$p_0(k) = \frac{0.5 - a_1(k)}{b_1(k)} = \frac{e^{-T(k)} + 0.5}{10(1 - e^{-T(k)})} \quad (15)$$

$$p_1(k) = \frac{0.5a_1(k+1) - 1}{b_1(k+1)} = \frac{-0.5e^{-T(k+1)} - 1}{10(1 - e^{-T(k+1)})} \quad (16)$$

Step 2: Solve $\tilde{N}(q,k)\tilde{M}^{-1}(q,k)$ to satisfy condition (10):

Choose $\tilde{N}(q,k) = n_0(k) + q^{-1}n_1(k)$, $\tilde{M}(q,k) = 1 + q^{-1}m_1(k)$, $A_*(q,k) = 1 + q^{-1} + q^{-2} + 0.5q^{-3}$ and plug them into (10):

$$(1 - q^{-2})(1 + q^{-1}m_1(k)) + (q^{-1}b_1(k)p_0(k) + q^{-1}b_1(k)q^{-1}p_1(k))(n_0(k) + q^{-1}n_1(k)) = 1 + q^{-1} + q^{-2} + 0.5q^{-3}$$

Solve the above time varying polynomial equation:

$$\begin{bmatrix} 1 & b_1(k)p_0(k) & 0 \\ 0 & b_1(k+1)p_1(k) & b_1(k+1)p_0(k+1) \\ -1 & 0 & b_1(k+2)p_1(k+1) \end{bmatrix} \begin{bmatrix} m_1(k) \\ n_0(k) \\ n_1(k) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0.5 \end{bmatrix}$$

$$m_1(k) = \frac{8.5 + 9e^{-T(k)} + 3e^{-T(k+1)} + 4e^{-T(k+2)} + 2e^{-T(k)}e^{-T(k+1)} + e^{-T(k+1)}e^{-T(k+2)} + 4e^{-T(k)}e^{-T(k+2)}}{3 - 2e^{-T(k)} + 2e^{-T(k+2)} - 4e^{-T(k)}e^{-T(k+1)} + e^{-T(k+1)}e^{-T(k+2)}} \tag{17}$$

$$n_0(k) = \frac{-6e^{-T(k+1)} - 4e^{-T(k+2)} - 11}{3 - 2e^{-T(k)} + 2e^{-T(k+2)} - 4e^{-T(k)}e^{-T(k+1)} + e^{-T(k+1)}e^{-T(k+2)}}$$

$$n_1(k) = \frac{-3e^{-T(k+1)} - 8e^{-T(k)} - 10}{3 - 2e^{-T(k)} + 2e^{-T(k+2)} - 4e^{-T(k)}e^{-T(k+1)} + e^{-T(k+1)}e^{-T(k+2)}}$$

It is worth to point out that $p_1(k)$, $m_1(k)$, $n_0(k)$ and $n_1(k)$ all require certain length of preview of the speed signal $\omega(k)$. The reference signal and the rotational speed in both rotational-angle domain and continuous-time domain are shown in Figures 3 and 4 respectively. As it is shown, the rotational speed surges at about 70 second and ramps up quickly after that. The reference signal remains periodic in the rotational-angle domain regardless of the varying rotational speed, but it becomes aperiodic in the time domain when rotational speed changes. Figures 5 shows the tracking error and the control signal. Obviously asymptotic tracking has been achieved. Figures 6 shows the parameters of the time varying controllers.

6. Conclusions

This paper presents the discrete time-varying repetitive control design and its application for tracking or rejecting rotational-angle dependent signals. To take advantage of the fact that the signal is periodic in the rotational-angle domain, we first convert the linear time invariant plant model into the rotational-angle domain. However, the LTI plant model becomes linear time varying after the conversion. A periodic signal internal model is then embedded in the feedback loop to ensure asymptotic tracking or disturbance rejection. Sufficient conditions for achieving asymptotic performance are presented in the form of two time varying Diophantine equations. Simulation results on engine variable valve actuation control show the effectiveness of the proposed scheme.

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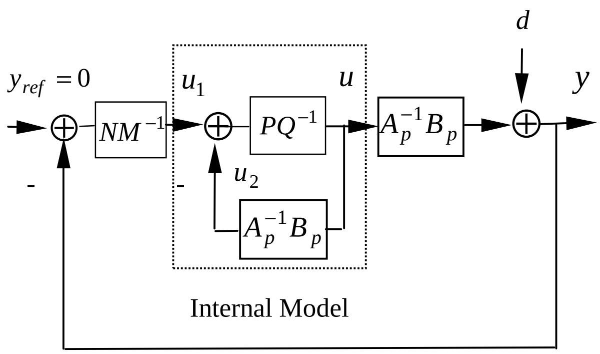

Figure 1. Time Varying Repetitive Control Block Diagram

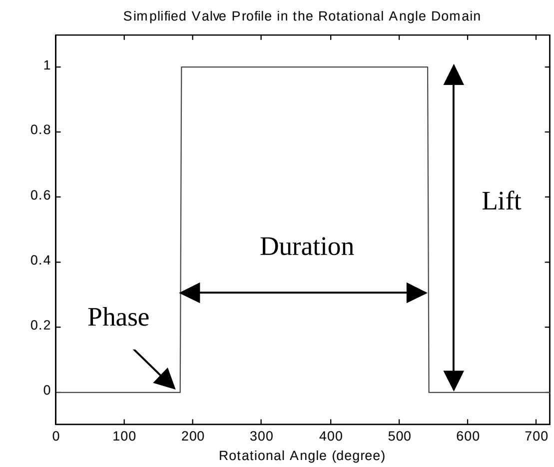

Figure 2. Engine Valve Profile in the Rotational-Angle Domain

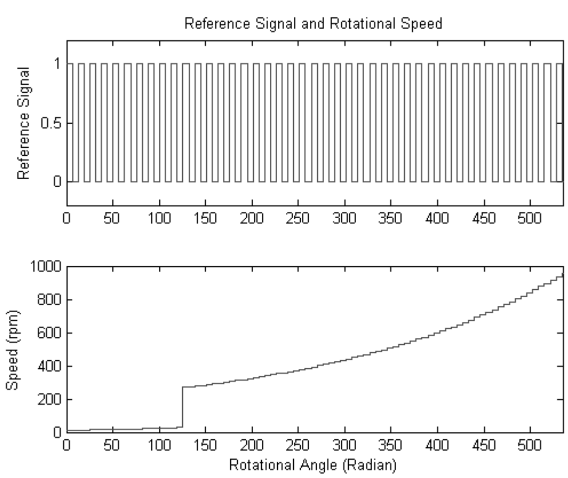

Figure 3. Reference Signal and Rotational Speed in Rotational-Angle Domain

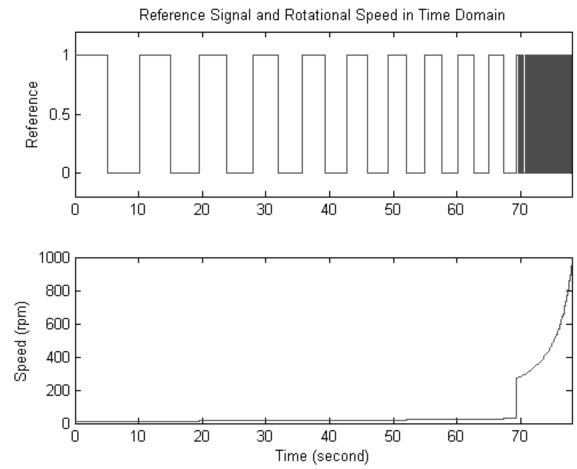

Figure 4. Reference Signal and Rotational Speed in Continuous Time Domain

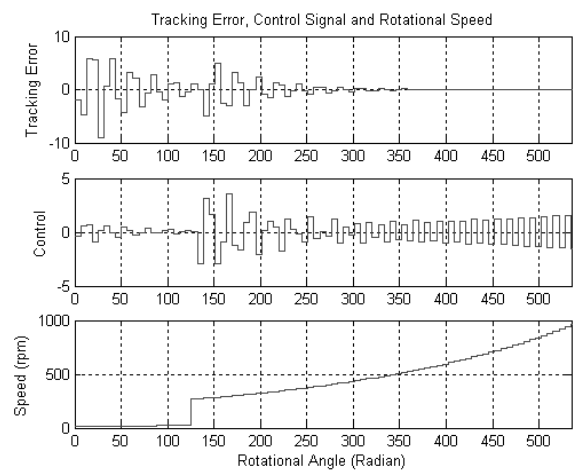

Figure 5. Tracking Error, Control Signal and Rotational Speed

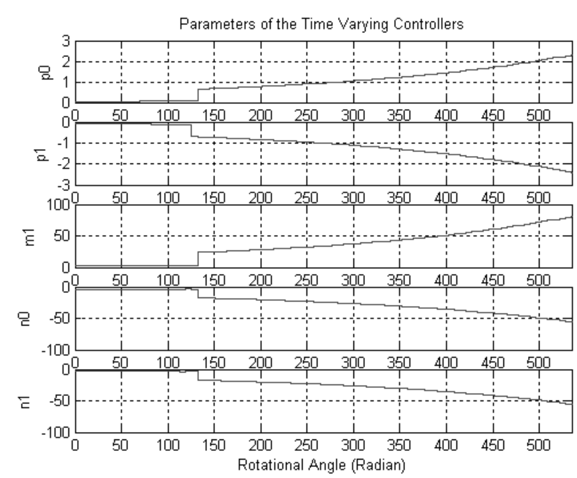

Figure 6. Parameters of the Time Varying Controllers