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Large-Signal Stability in High-Order Switching Converters

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***Abstract*—This article presents the application of a new passivity-based control law that stabilizes the output voltage of a fourth-order DC-DC converter. This control law assures large-signal stability, provides zero steady-state error despite uncertainty in converter parameters and has enough degree of freedom to satisfy the usual transient specifications. This new integral control is derived in three steps. First, a static law is obtained. Second, a positive semi-definite storage function is studied to guarantee zero steady-state error of the output voltage. Finally, the storage functions of the first two steps are combined to derive the new control law for high-order DC-DC converters.**

I. Introduction

Switching converters enable raw energy to be efficiently adapted from a primary source to satisfy load specifications. Basic DC-DC switching-converter topologies, i.e. buck, buck-boost and boost, have second-order dynamics. These second-order circuits have bilinear averaged dynamics. Also, the control signal, or duty cycle, is constrained in the [0,1] interval. However, linear control is mostly used in the power electronics domain, though it can not ensure global stability of these nonlinear systems. Since large-signal stability is not ensured in the regulator design process, a simulation or experimental verification must be done to verify the behavior of the closed loop system is correct when start-up or large perturbations occur. Often, 'ad-hoc' start-up aid circuits are added to overcome these problems. An analytical guarantee that no limit cycles or instabilities will appear in large perturbation situations will therefore improve the reliability of these power processing units. These facts have prompted several authors to apply nonlinear control to DC-DC converters.

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A nonlinear control technique, which can afford the bilinear nature and control saturation, is passivity-based control. Passivity control technique appeared in the 1970s [1-4] and plays a very important role in nonlinear control today [5-7]. Several attempts have been made to apply passivity control to switching regulators. The first attempt was made by Sanders [8] and in the last few years there has been an increasing interest in this control technique applied to DC-DC converters [9-12]. However, several problems still prevent the use of this technique in real situations. One of these is the complexity of the control law, which is strongly related to the question of control law implementation. Some implementation of a passivity-based controller for DC-DC converter [10] has been reported, though the performances are not yet as good as those of a conventional control, at least in small-signal terms. This may be due to the fact that the complexity of the law requires the use of a digital device. The control has to react much faster than time constants and switching frequency in switching converters. Also, the converter time constants and the switching frequency have to be minimized in order to optimize ripple and component sizes. A digital solution therefore needs a very fast device, so this may not be the best technological solution. On the other hand, analog controllers do not need a previous sampling process and are therefore less demanding in bandwidth terms. In this paper, we propose a control law that is simple enough so that it can be easily implemented in an analog design by means of operational amplifiers and an analog multiplier.

Moreover, most of the previously proposed laws have been applied to second order converters. However, to meet EMI specifications filters are usually added to the basic converter topologies [13]. Adding these filters modifies the converter dynamics and the control has to be redesigned. Therefore, there is a need to extend passivity-based control to high order converters without substantially increasing the complexity of the control law so that it still allows an analog implementation. This paper will show passivity-based control design in high order converters, and in the two-inductor buck in particular [14]. The approach can be easily applied to other high order converters such as buck with input filter, Cuk converter and SEPIC.

Other important aims of this control are to ensure zero steady state error and to ensure that the control law is smooth. The proposed law satisfies these requirements.

Finally, it is essential that the law can assure correct performances around the equilibrium point since most of the performance standards [15] in power electronics are specified in frequency domain. This will allow the control law to be easily compared with the usual control laws for DC-DC converters [13]. In the paper, once proved the large signal stability, an analysis in small signal is provided.

The paper is organized as follows. Section 2 describes a dynamic model of two-inductor buck. Section 3 proposes a passivity-based control using a combination of incremental energy and an integral storage function. The behavior of closed loop around the equilibrium point is analyzed in section IV. Finally, some simulations and conclusions are provided.

II. Problem Statement

A. Converter Model

Figure 1 shows a two-inductor buck converter or zero ripple buck [14] whose dynamic behavior during T_{ON} and T_{OFF} can be expressed as

$$\dot{x}_a = A_i\, x_a + b_i \qquad i = 1,2 \tag{1}$$

where
$$A_1 = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{L_1} \\ 0 & 0 & \frac{1}{L_2} & -\frac{1}{L_2} \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ \frac{1}{C_2} & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & -\frac{1}{L_1} \\ 0 & 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ \frac{1}{C_2} & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \quad b_1 = b_2 = \begin{bmatrix} \frac{V_g}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and the state vector corresponds to $x_a = [i_{a1} \;\; i_{a2} \;\; v_{a1} \;\; v_{a2}]^t$, where i_{a1}, i_{a2}, v_{a1} and v_{a2} correspond to input current, L_2 inductor current, C_1 capacitor voltage and output voltage, respectively.

Equation (1) can be expressed in compact form as

$$\dot{x}_a = (A_1\, x_a + b_1)u + (A_2\, x_a + b_2)(1 - u) \tag{2}$$

where $u = 1$ during T_{ON} and $u = 0$ during T_{OFF}.

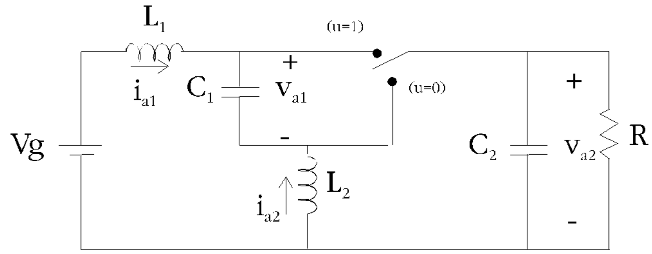

Fig.1. Two-inductor buck converter topology

If the switching frequency is significantly higher than the natural frequencies of the converter, this discontinuous model can be approximated by a continuous averaged model in which a new variable d_a is introduced. In the [0,1] subinterval d_a is a continuous function and constitutes the converter duty cycle.

In the zero ripple buck converter $b_1 = b_2$, which leads to

$$\dot{x}_a = A_2\, x_a + b_2 + (A_1 - A_2)\, x_a d_a \tag{3}$$

Considering that the system variables consist of two components:

$$\begin{aligned} x_a &= x_e + x \\ d_a &= d_e + d \end{aligned} \tag{4}$$

where x_e and d_e represent the equilibrium values and x and d are the perturbed values of state and duty cycle, equation (3) can be written as follows

$$(\dot{x}_e + \dot{x}) = A_2\,(x_e + x) + b_2 + (A_1 - A_2)(x_e + x)(d_e + d) \tag{5}$$

which results in

$$\dot{x} = A\,x + B\,x\,d + b\,d \tag{6}$$

where $A = A_2 + (A_1 - A_2)d_e$, $B = (A_1 - A_2)$ and $b = (A_1 - A_2)x_e$ and vector x_e represents the equilibrium point

$$x_e = \begin{bmatrix} V_g\, d_e^2 / R \\ V_g\, d_e\, (d_e - 1)/R \\ Vg \\ d_e\, Vg \end{bmatrix} \tag{7}$$

The dynamics of the perturbed system are therefore summarized in the following equations:

$$\begin{aligned} \dot{x}_1 &= -\frac{1-d_e}{L_1}x_3 - \frac{x_4}{L_1} + \frac{x_3 + V_g}{L_1}d \\ \dot{x}_2 &= \frac{d_e}{L_2}x_3 - \frac{x_4}{L_2} + \frac{x_3 + V_g}{L_2}d \\ \dot{x}_3 &= \frac{1-d_e}{C_1}x_1 - \frac{d_e}{C_1}x_2 - \left(\frac{1}{C_1}x_1 + \frac{1}{C_1}x_2 + \frac{d_e V_g}{RC_1}\right)d \\ \dot{x}_4 &= \frac{1}{C_2}x_1 + \frac{1}{C_2}x_2 - \frac{1}{RC_2}x_4 \end{aligned} \tag{8}$$

B. Duty Cycle Saturation

Since the control signal in switching converters corresponds to the duty cycle d_a, the perturbed duty cycle d is constrained in the $[-d_e, 1-d_e]$ interval. Considering a nonlinear feedback output $y(x)$, the control law $d=-\phi(y)y$ will be applied, where $\phi(y)>0$ models the saturation effect and corresponds to

$$\phi(y)=\begin{cases} \dfrac{-d_e}{y} & y<\dfrac{-d_e}{\phi_{max}} \\ \phi_{max} & \dfrac{-d_e}{\phi_{max}}\le y\le\dfrac{1-d_e}{\phi_{max}} \\ \dfrac{1-d_e}{y} & y>\dfrac{1-d_e}{\phi_{max}} \end{cases} \quad (9)$$

and ϕ_{max} represents the small signal gain in a linearized model of the closed-loop system.

III. Passivity-Based Control For High Order Converter

The aim of this section is to derive a smooth control law that assures stability despite large signal perturbation, while taking into account the control signal saturation.

We propose a smooth nonlinear control function for high order converters based on the combination of two storage functions. This control will ensure both stability and transient specifications, namely zero steady-state error of the output voltage and desired damping, since it has enough degree of freedom.

Consider the following 5-order system made up of (8) and a new state variable x_5, which corresponds to the integral of the output voltage error:

$$\begin{aligned} \dot{x}_1 &= -\frac{1-d_e}{L_1}x_3-\frac{x_4}{L_1}+\frac{x_3+V_g}{L_1}d \\ \dot{x}_2 &= \frac{d_e}{L_2}x_3-\frac{x_4}{L_2}+\frac{x_3+V_g}{L_2}d \\ \dot{x}_3 &= \frac{1-d_e}{C_1}x_1-\frac{d_e}{C_1}x_2-\left(\frac{1}{C_1}x_1+\frac{1}{C_1}x_2+\frac{d_eV_g}{RC_1}\right)d \\ \dot{x}_4 &= \frac{1}{C_2}x_1+\frac{1}{C_2}x_2-\frac{1}{RC_2}x_4 \\ \dot{x}_5 &= x_4 \end{aligned} \quad (10)$$

Lemma 1: The nonlinear output

$$y_1=\frac{Vg}{R}\left(R\,x_1+R\,x_2-d_e\,x_3\right) \quad (11)$$

provides a passive relationship for the system (10)

Proof: Consider the positive storage function, which is positive semi-definite,

$$V_1(x)=\frac{1}{2}L_1x_1^2+\frac{1}{2}L_2x_2^2+\frac{1}{2}C_1x_3^2\frac{1}{2}C_2x_4^2 \quad (12)$$

and whose derivative along the state trajectory is

$$\dot{V}_1(x)=-\frac{x_4^2}{R}+\frac{V_g}{R}\left(R\,x_1+R\,x_2-d_e\,x_3\right)d \quad (13)$$

so

$$\dot{V}_1(x)=-\frac{x_4^2}{R}+y_1d\le y_1d \quad (14)$$

which proves Lemma 1

Remark 1: The storage function is positive definite for the system (8)

Lemma 2: The nonlinear output

$$y_2=\left(L_1d_ex_1+L_2(1-d_e)x_2+x_5\right)\left(V_g+x_3\right) \quad (15)$$

provides a lossless passive relationship [7] for the system (10)

Proof: Consider the storage function

$$V_2(x)=\left(d_eL_1x_1+(1-d_e)L_2x_2+x_5\right)^2 \quad (16)$$

whose derivative along the state trajectory is

$$\dot{V}_2(x)=\left(L_1d_ex_1+L_2(1-d_e)x_2+x_5\right)\left(V_g+x_3\right)d \quad (17)$$

so

$$\dot{V}_2(x)=y_2d \quad (18)$$

which proves Lemma 2

Proposition 1: Consider the nonlinear feedback output

$$\begin{aligned} y=y_1+k\,y_2 &= \frac{V_g}{R}\left(R\,x_1+R\,x_2-d_e\,x_3\right) \\ &+k\left(L_1d_ex_1+L_2(1-d_e)x_2+x_5\right)\left(V_g+x_3\right) \end{aligned} \quad (19)$$

where k is a positive constant. Then, the control law

$$d=-\phi(y)y \quad (20)$$

is an integral stabilizing control for the high order converter (10).

Proof: The proof is based on the combination of the previous storage functions (12) and (16)

$$V(x)=V_1(x)+k\,V_2(x) \quad (21)$$

whose derivative along the state trajectory is

$$\dot{V}(x)=\dot{V}_1(x)+k\,\dot{V}_2(x)\le\left(y_1+k\,y_2\right)d=yd \quad (22)$$

By applying control law $d=-\phi(y)y$, it is obtained

$$\dot{V}(x)=-\frac{x_4^2}{R}-\phi^2y^2\le 0 \quad (23)$$

We will now prove the stability of the closed loop system. As $\dot{V}(x)$ is a negative semi-definite function, we need to use LaSalle Theorem to demonstrate the asymptotic stability. In other words, we must prove Zero State Detectability or Zero State Observability [7]. Therefore, to prove stability, we need to study dynamics of (10) when $\dot{V}(x)=0$. The fact that $\dot{V}(x)=-\frac{x_4^2}{R}-\phi^2 y^2=0$ implies that $x_4=0$ and $y=0$ since $\phi>0$. On the other hand, if $y=0$, then $d=0$ and x_5 is constant. Then the asymptotically stability of system (8) when $\dot{V}(x)=0$ must be analyzed.

It is important to note that if $y=0$ and $d=0$, then variables x_1 and x_2 are linearly dependent. Also, if $y=0$ then $x_2=-x_1-\frac{d_e}{R}x_3$. Then the zero dynamics corresponds to

$$\begin{aligned}\dot{x}_1&=-\frac{1-d_e}{L_1}x_3\\ \dot{x}_3&=\frac{1}{C_1}x_1-\frac{1}{C_1}\left(1-\frac{d_e^2}{R}\right)x_3\end{aligned} \tag{24}$$

As $0<d_e<1$, then trajectory of the system constrained to $\dot{V}(x)=0$ is linear and strictly asymptotically stable. Hence, it is zero state detectable and it has been demonstrated by LaSalle Theorem that this law stabilizes the high order converter asymptotically.

Remark 2: Function $V_2(x)$ has been obtained by means of a more general function

$$V_c(x)=\frac{1}{2}(c_1x_1+c_2x_2+x_5)^2 \tag{25}$$

by choosing the coefficients such that the terms not multiplied by the control are negative or zero. The following values of coefficients adapt this storage function to the two-inductor buck converter

$$\begin{aligned}c_1&=d_eL_1\\ c_2&=(1-d_e)L_2\\ c_3&=2(1-d_e)\end{aligned} \tag{26}$$

Such a process can easily be applied to many different topologies of switching converters.

Remark 3: A control law based only on Lemma1, i.e. $d=-\phi(y_1)y_1$, is linear and can assure large signal stability. However, it is not an integral law and can not guarantee zero steady-state in small signal.

Remark 4: A control law based only on Lemma2, i.e. $d=-\phi(y_2)y_2$, cannot improve the damping degree due to its lossless nature.

IV. Small Signal Requirements

Once the nonlinear control law has been established, the closed loop system around equilibrium may be linearized to ensure that the behavior of the switching regulator in small-signal operation is satisfactory.

We can consider the following linear representation of the switching regulator

$$\begin{aligned}\dot{x}&=A\,x+B\,d_{lin}\\ y&=C\,x\end{aligned} \tag{27}$$

where, from (10),

$$A=\begin{bmatrix}0 & 0 & (d_e-1)/L_1 & -1/L & 0\\ 0 & 0 & d_e/L_2 & -1/L_2 & 0\\ -(d_e-1)/C & -d_e/C_1 & 0 & 0 & 0\\ d_e/C_2 & d_e/C_{21} & 0 & -1/R\,C_2 & 0\\ 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

$$B=\begin{bmatrix}V_g/L_1\\ V_g/L_2\\ -d_e\,V_g/(R\,C_1)\\ 0\\ 0\end{bmatrix} \tag{28}$$

According to expressions (9) and (19)-(20), the corresponding linear law is

$$d_{lin}=-\phi_{max}\begin{pmatrix}\frac{V_g}{R}(R\,x_1+R\,x_2-d_e\,x_3)\\ +kV_g(L_1d_ex_1+L_2(1-d_e)x_2+x_5)\end{pmatrix} \tag{29}$$

The design parameters $\phi_{\max}$ and k may be selected from a linear loop gain analysis, where the corresponding loop gain, derived from (27), (28) and (29), corresponds to

$$T(s)=-\phi_{max}\begin{pmatrix}\frac{V_g}{R}\left(R\frac{X_1}{D_{lin}}+R\frac{X_2}{D_{lin}}-d_e\frac{X_3}{D_{lin}}\right)\\ +kV_g\left(L_1d_e\frac{X_1}{D_{lin}}+L_2(1-d_e)\frac{X_2}{D_{lin}}+\frac{X_5}{D_{lin}}\right)\end{pmatrix} \tag{30}$$

The expressions of $\frac{X_1}{D_{lin}},\frac{X_2}{D_{lin}},\frac{X_3}{D_{lin}}$ and $\frac{X_5}{D_{lin}}$ are transfers functions and can be derived from a linear model of the system (28).

Next, Figure 2 shows the root locus corresponding to gain loop $\phi_{\max}$ in (30) when the converter parameters are $L_1 = 30\mu H$, $L_2 = 500\mu H$, $C_1 = 10\mu F$, $C_2 = 200\mu F$, $R = 10\,\Omega$, $V_g = 20V$, $d_e = 0.5$. The chosen value of k is k=1000. It is beyond the scope of this paper to find a systematic procedure for simultaneously optimizing the design parameters $\phi_{\max}$ and k in this nonlinear control law.

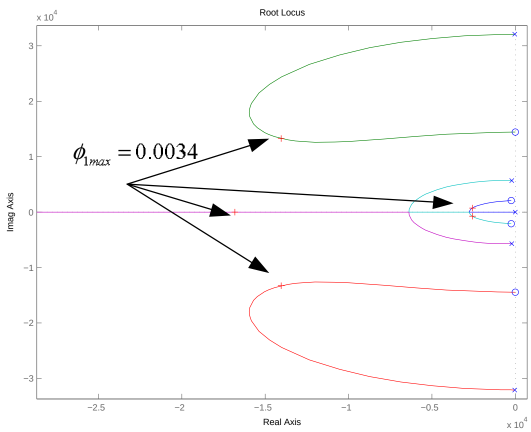

Fig.2. Root Locus for gain loop in law (34).

Therefore, a value $\phi_{1max} = 0.0034$ is chosen in order to minimize the settle time.

Remark 4: The next figure shows the root locus for a control law based on feedback of only y_2, i.e. $d = -\phi(y_2)y_2$. Damping coefficient do not change when We can see that in order to improve the damping coefficient ϕ_{max} varies. The law does not have any degree of freedom to improve damping. This behavior can be expected from its lossless nature, as shown in Lemma 2.

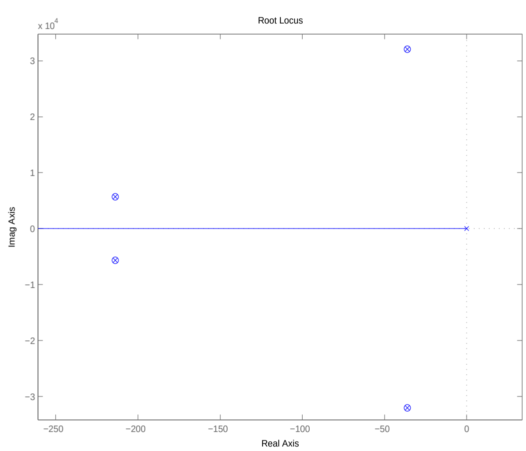

Fig.3. Root Locus for lossless control, i.e. $d = -\phi(y_2)y_2$.

V. Simulation Results

The following figures show the transient behavior of a two-inductor buck with the proposed control law (19)-(20). The switching period is $T_s = 10\mu s$. The figures show the start-up process, a variation of the real load, a variation of the feed voltage and a change in the desired output voltage. Load changes its real value from *10Ω* to *5Ω* at time t = 10*ms*. The value of the feed voltage changes from *20V* to *30V* at time t = *20ms*. There is also a change in the reference voltage from *10V* to *20V* at *100ms*. However, the nominal values of the controller parameters remain unchanged including nominal equilibrium state. Figures 4 to 9 depict current i_{1a}, current i_{2a}, voltage v_{1a}, voltage v_{2a}, the error integral of the output voltage and the duty cycle, respectively.

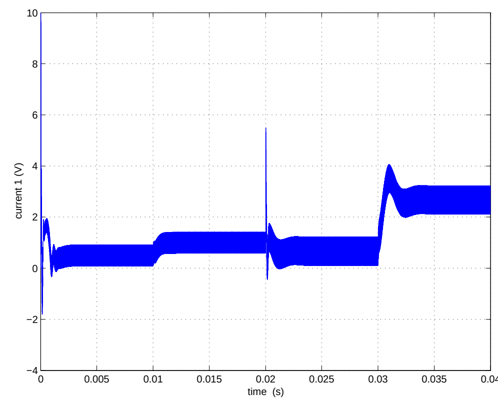

Fig 4. L_1 inductor current

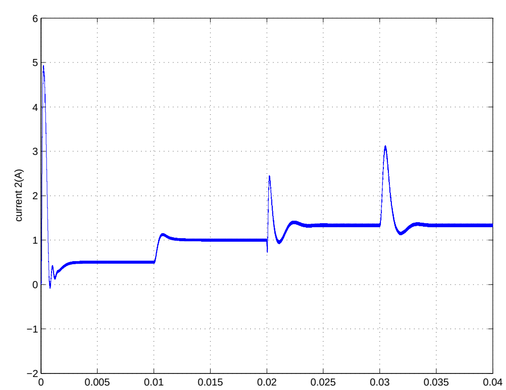

Fig 5. L_2 inductor current.

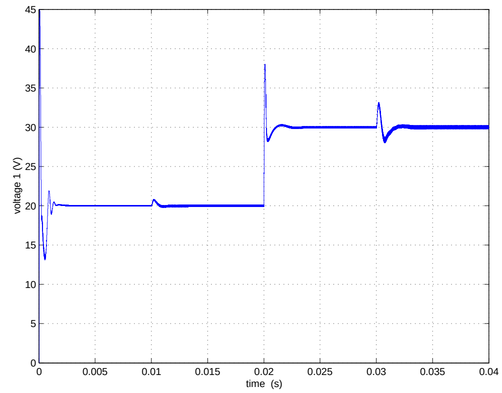

Fig 6. C_1 capacitor voltage.

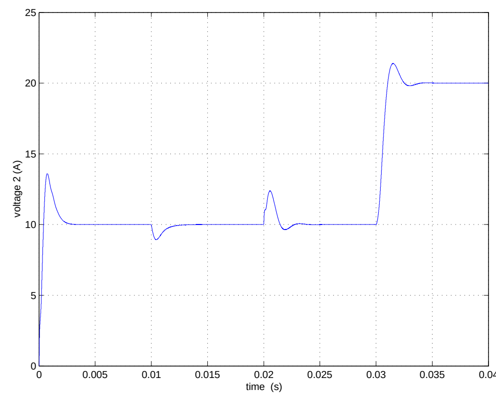

Fig 7. C_2 capacitor voltage or regulated output.

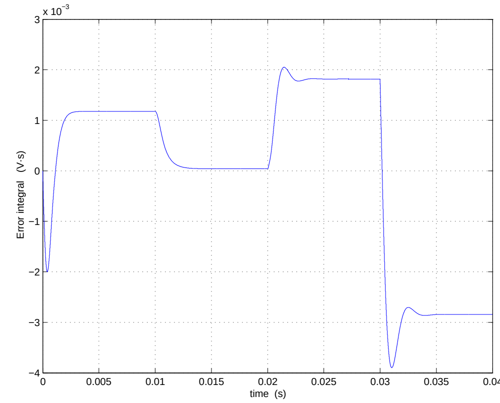

Fig 8. Output error integral.

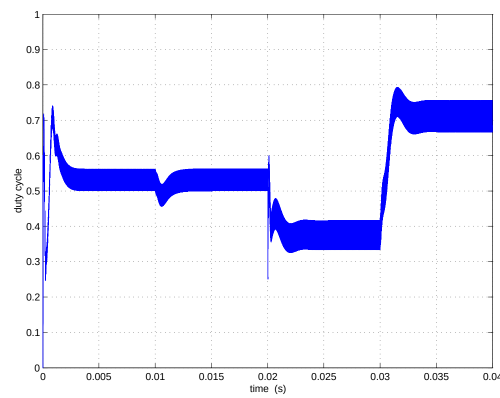

Fig 9. Duty cycle.

VI. Conclusion

In this paper we have shown that a passivity-based integral control is an effective solution for high-order DC-DC converters because it can ensures large-signal stability despite the bilinear nature of their dynamics and duty-cycle saturation. This control law ensures perfect regulation despite uncertainty in parameters. Once designed, it can be tuned by using a small-signal method to satisfy the usual specifications for switching regulators. It is smooth and can be easily implemented using an analog multiplier, operational amplifiers and a pulse width modulator.

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