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# Performance Comparison of Robust PID and FOPID for an Inverse Response Process Model

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Abstract: This paper provides a quantitative comparison between the performance provided by the use of Proportional-Integral-Derivative (PID) and Fractional-Order Proportional-Integral-Derivative (FOPID) controllers for the control of second-order processes with inverse response based on the integrated absolute value of the error (IAE index). These controllers are designed in order to achieve an optimal performance in both, set-point tracking task and regulatory control operation, while robustness considerations are taken into account, and, therefore, also the trade-off between robustness and performance of the control system is considered. The performance comparison and example, show that FOPID controllers achieve higher performance that PID controllers mainly for processes with larger normalized dead times as well when more robustness should be considered in the closed-loop system. These advantages are due to the use of the fractional parameter $\mu$ associated to the derivative part of the FOPID controller.

*Keywords:* FOPID, Fractional Control, Inverse Response, Non-Minimum Phase Systems, PID

## 1. INTRODUCTION

The presence of non-minimum-phase or inverse response dynamics is encountered in several applications, for instance, chemical processes as the level of drum boiler in a distillation column (Seborg et al., 2016), bio-chemical reactions in Continuous Stirred-Tank Reactor (CSTR) (Sree and Chidambaram, 2006) and the aircraft pitching-up action of Boeing 747 aircraft (Sir Elkhatem et al., 2021). This dynamics is characterized by at least one zero located at the right-hand side of the complex plane (Camacho et al., 1999), and this entails a challenge due the decrease in the margin phase on the open-loop system, and even more, if the process has dead-time (Balaguer et al., 2011).

In the last decade, the fractional calculus has risen interest in several stages of the control system design, for instance in the development of process modelling methods (Kothari et al., 2019) and in the study of several fractional-order models (Shah et al., 2019). In the design of the feedback control systems, the application of the fractional calculus has also increased because the fractional-order controller provides more flexibility to achieve suitable performance and robustness requirements due to the use of two additional controller parameters, namely, the fractional orders associated to the integral part and to the derivative part (El-Khazali, 2013).

Despite of the study of the benefits of using fractional-order Proportional-Integral-Derivative controller (FOPID) in the control of self-regulated processes (overdamped and underdamped dynamics) (Meneses et al., 2022b) and integrating process (Meneses et al., 2022a) to achieve particular performance specifications under robustness considerations, there is still no study that quantifies the improvement in the closed-loop systems performance when a FOPID controller is employed instead of a Proportional-Integral-Derivative (PID) controller and minimum relative stability margins are required in order to deal with the non-linearities found in most of the real processes when non-minimum phase dynamics has to be controlled. In this context, this work has the aim to provide this information to the user with the purpose to facilitate the decision to choose between a FOPID controller and a PID controller, knowing the increase in the performance that can be achieved in the control system loop by the fractional-order controller considering both trade-offs: performance against robustness (Garpinger et al., 2014) and servo-control mode against regulatory control operation (Arrieta and Vilanova, 2017) when a process exhibits an inverse response. It is important to remember that one of the main challenges to deal with fractional-order controllers is focused in their complexity to be implemented (El-Khazali, 2013) because its discretization requires a real rational approximation of the fractional term that usually needs a high number of poles and zeros (Oustaloup et al.,

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2000), and therefore, it is important for the user to know if it is really worth designing a more complex controller.

The paper is organized as follows. Section 2 provides a description of the control system configuration with the concepts required to approach the performance and robustness issues. Section 3 provides the results of the closed-loop performance assessment using the integer and fractional order controllers and shows some cases of the behavior of the fractional parameter $\mu$ which has a relevant role in this analysis. Section 4 presents an illustrative example to validate the obtained results. Section 5 ends with the main conclusions of this work.

## 2. PROBLEM FORMULATION

### *2.1 Control System Configuration*

The control system considered is shown in Fig. 1, where $P(s)$ represents the process to be controlled and the controller is decoupled in $C_r(s)$ and $C_y(s)$ in order to apply the derivative mode only to the feedback signal. This avoids extreme changes in the controller output when a step chance in the set-point value happens.

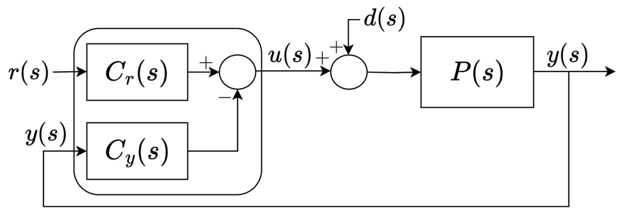


Figure 1. 2DoF Controller close-loop control system

The signals shown in Fig. 1 are:

- $y(s)$ is the feedback signal (controlled variable).
- $r(s)$ is the set-point value for the process output.
- $u(s)$ is the controller output.
- $d(s)$ represents load-disturbances that change the controlled variable.

Using the closed-loop control system of Fig. 1, the feedback signal is described by the following expression:

$$y(s) = \underbrace{\frac{C_r(s)P(s)}{1+C_y(s)P(s)}r(s)}_{servo-control} + \underbrace{\frac{P(s)}{1+C_y(s)P(s)}d(s)}_{regulatory-control} \quad (1)$$

where $C_r(s)$ is the transfer function of the set-point controller and $C_y(s)$ is the transfer function of the feedback controller which are defined in (2) and (3), respectively.

$$C_r(s) = K_p\left(1+\frac{1}{T_i s^{\lambda}}\right) \quad (2)$$

$$C_y(s) = K_p\left(1+\frac{1}{T_i s^{\lambda}}+\frac{T_d s^{\mu}}{\frac{T_d}{\eta}s+1}\right) \quad (3)$$

where $K_p$ is the controller proportional gain, $T_i$ is the integral time constant and $T_d$ is the derivative time constant, $\frac{T_d}{\eta}$ represents the filter constant and $\lambda$ and $\mu$ are the fractional-order associated to the integral part and derivative part, respectively. Note that $\eta$ is defined as:

$$\eta = 10T_d^{\frac{\mu-1}{\mu}} \quad (4)$$

in order to locate the cut-off frequency of the pole one decade above the zero frequency of the derivative part which is a common practice as it is mentioned in Visioli (2006).

Note that when $\lambda = 1$ and $\mu = 1$ the integer-order PID controller is obtained, and when either of these parameters are different from zero, a fractional-order PID controller, known as FOPID controller is obtained. On this way, the control signal $u(s)$ will have the following form (Astrom and Hagglund (2006)):

$$u(s) = K_p\left[\left(1+\frac{1}{T_i s^{\lambda}}\right)e(s) - \left(\frac{T_d s^{\mu}}{\frac{T_d}{\eta}s+1}\right)y(s)\right] \quad (5)$$

In order to implement a fractional order controller, the integer order approximation of (Oustaloup et al. (2000)) is used. This is based on a recursive approximation composed by a product of poles and zeros as it is defined in (6).

$$s^{\mu}_{[\omega l,\omega h]} \cong C_o \prod_{k=1}^{N} \frac{1+\frac{s}{\omega_{z,k}}}{1+\frac{s}{\omega_{p,k}}},\ \mu > 0 \quad (6)$$

In this work, this approximation is defined in the frequency range $\{\omega l, \omega h\} = \{0.001, 1000\}$. Additionally, the $C_o$ term is defined in such a way that, the approximation has unity gain at the gain crossover frequency. Finally, the parameter $N$, which refers to the number of poles and zeros for the real-rational transfer function approximation of the fractional terms $s^{\lambda}$ and $s^{\mu}$, is set to $N = 8$ according to the recommendation given by (Oustaloup et al., 2000).

### *2.2 Process Model*

The process $P(s)$ to be controlled is modelled by an inverse-response-second-order-plus-dead-time (IRSOPDT) transfer function defined as:

$$P(s) = \frac{K(-bTs+1)e^{-Ls}}{(Ts+1)(\alpha Ts+1)} \quad (7)$$

where $K$ is the process static gain, $T$ is the dominant time constant, $L$ is the dead-time, $\alpha$ is the ratio between time constants and $b$ is the parameter that defines the position of the right half plane zero in relation with the dominant time constant $T$, and therefore, it quantifies the inverse response magnitude.

In order to reduce the parameters used to describe the process dynamics to $\alpha$, $b$, and $\tau_0 = \frac{L}{T}$, and to make the design of the control system easier for different inverse-response processes, it is useful to employ the normalization transformation $\hat{s} = Ts$. On this way, the normalized second-order model with inverse response (7) is defined as:

$$P(\hat{s}) = \frac{K(-b\hat{s}+1)e^{-\tau_0\hat{s}}}{(\hat{s}+1)(\alpha\hat{s}+1)} \quad (8)$$

### *2.3 Performance criteria*

With the purpose to measure the performance of the closed-loop system, the integrated absolute value of the error (IAE) is employed. This index is defined as:

$$IAE = \int_0^{\infty} |e(t)|\ dt = \int_0^{\infty} |r(t)-y(t)|\ dt \quad (9)$$

The IAE is considered for both set-point tracking task $J_{sp}$ and for regulatory control mode $J_{ld}$ at the same time, therefore, the optimal parameters for the integer and fractional-order PID controllers will be obtained optimizing the following cost function:

$$J_{rd} = J_{sp} + J_{ld} \tag{10}$$

in order to achieve a good performance of the closed-loop system for both control modes: servo-control and regulatory control.

To quantify the improvement in the performance due to the use of FOPID controllers in comparison to PID controllers, the $J_\eta$ index defined in (11) is used.

$$J_\eta = \frac{J_{rd-FOPID}}{J_{rd-PID}} \tag{11}$$

Because the FOPID controller is a generalization of the PID controller, it is expected that $J_\eta$ index will be always less than or equal to one.

### 2.4 Robustness criteria

In order to consider relative stability margins in the control system design due to the non-linear characteristics of most of real processes, the maximum value of the magnitude of Sensitivity Function will be used. Therefore, the closed-loop control system's robustness will be measured according to the following expression:

$$M_S \doteq \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + C(j\omega)P(j\omega)|} \tag{12}$$

For self-regulated processes, it is recommended to design the controller to get $M_S$ values from 1.4 to 2.0 (Astrom and Hagglund (2006)). When $M_S = 1.4$ is selected a smoother controller output is achieved while when $M_S = 2.0$ a more aggressive and faster controller output is achieved.
Therefore, an optimization procedure has been used to solve the optimization problem of minimizing the cost function (10) subject to the constraint defined by $M_S = 1.4$ or $M_S = 2.0$.

## 3. PERFORMANCE ASSESSMENT

This section provides a comparison between the optimal values of the cost function (10) obtained for both FOPID and PID controllers. In all cases $\tau_0$ ranges from 0.1 to 2.0 in steps of 0.1, $a \in \{0.10, 0.25, 0.50, 0.75, 1.0\}$ and $b$ ranges from 0.25 to 2.0 when the robustness constraint is defined by $M_S = 1.4$ and from 0.25 to 2.5 when $M_S$ is set to 2.0, in steps of 0.25 for both $M_S$ values. It is important to mention that in all cases the optimal value for the $\lambda$ controller parameter was always one, and therefore, this section only provides for the case of $M_S = 1.4$ (for brevity purposes) the values of the derivative fractional-order controller parameter $\mu$ which is responsible of the performance improvement in the closed-loop system.

### 3.1 Robustness given by $M_S = 2.0$

First, the case when $M_S = 2.0$ is analyzed. The $J_\eta$ index for $a = 0.1$ is shown in Fig. 2. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 3 %) for low values of $b$, namely, $b \in \{0.25, 0.50, 0.75, 1.0\}$ and for $\tau_0 > 0.7$.

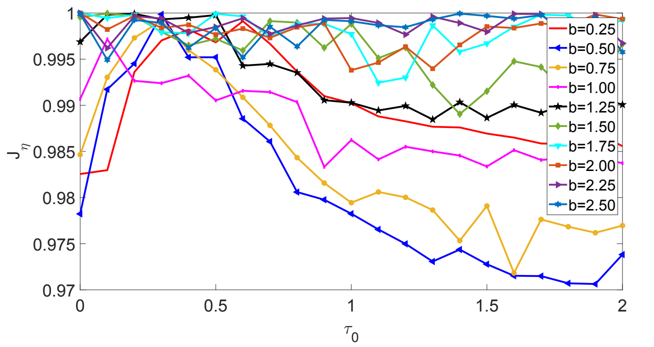


Figure 2. $J_\eta$ index for $a = 0.10$ and $M_S = 2.0$

The $J_\eta$ index for $a = 0.25$ is shown in Fig. 3. From this figure it can be seen that the improvement in the control system performance achieved by the use of FOPID controllers over PID controllers is greater (up to 4 %) for a low value of $b$ ($b = 0.25$) in the whole range of the normalized dead time $\tau_0$. For the rest of the cases, the performance improvement is marginal. The $J_\eta$ index

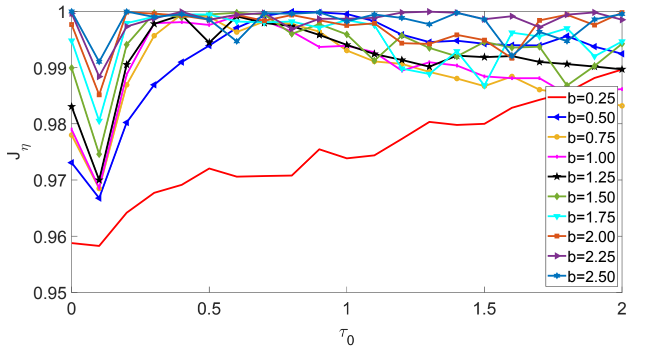


Figure 3. $J_\eta$ index for $a = 0.25$ and $M_S = 2.0$

for $a = 0.50$ is shown in Fig. 4. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 4.5 %) for low values of $b$, namely, $b \in \{0.25, 0.50\}$ and for the whole range of the normalize dead time $\tau_0$. The $J_\eta$ index for $a = 0.75$ is

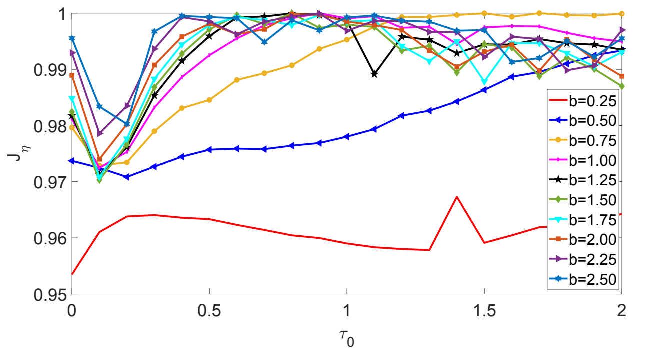


Figure 4. $J_\eta$ index for $a = 0.50$ and $M_S = 2.0$

shown in Fig. 5. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 5 %) for low values of $b$, namely, $b \in \{0.25, 0.50, 0.75\}$ and for the whole range of the normalize dead time $\tau_0$. The $J_\eta$ index for $a = 1.0$ is shown in Fig.

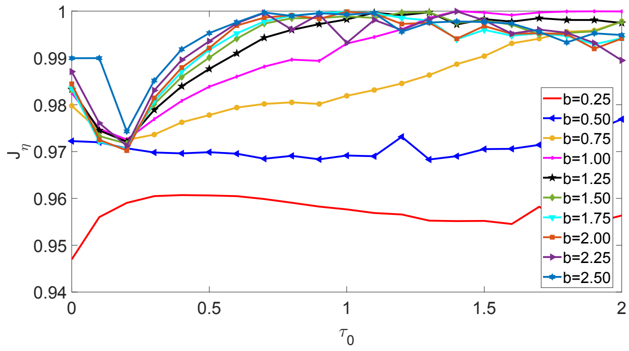


Figure 5. $J_\eta$ index for $a = 0.75$ and $M_S = 2.0$

6. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 6 %) for low values of $b$, namely, $b \in \{0.25, 0.50, 0.75, 1.0\}$ and for the whole range of the normalize dead time $\tau_0$.

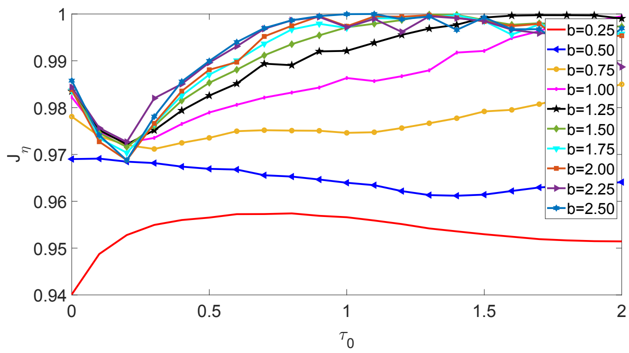


Figure 6. $J_\eta$ index for $a = 1.0$ and $M_S = 2.0$

*3.2 Robustness given by $M_S = 1.4$*

When $M_S = 1.4$ the following results are obtained:
The $J_\eta$ index for $a = 0.1$ is shown in Fig. 7. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 10 %) for higher values of $b$, namely, $b \in \{0.50, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0\}$ and for the whole range of the normalize dead time $\tau_0$. This suggests that when a more robust closed-loop system is required and the process has a larger inverse response, the FOPID controller provides a higher performance than a PID controller. The $J_\eta$ index for $a = 0.25$ is shown

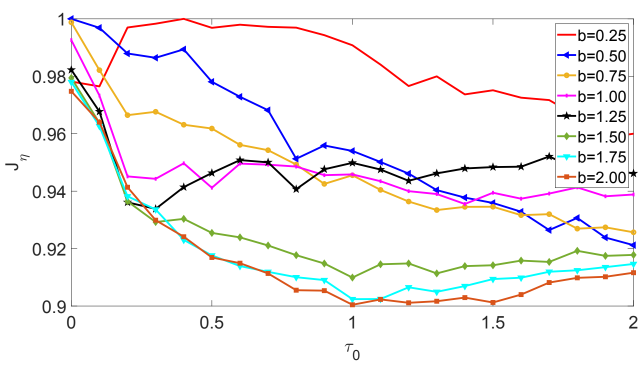


Figure 7. $J_\eta$ index for $a = 0.10$ and $M_S = 1.4$

in Fig. 8. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 9.5 %) for higher values of $b$, namely, $b \in \{0.75, 1.0, 1.25, 1.5, 1.75, 2.0\}$ and specially for the range of the normalize dead time $\tau_0 \geq 0.5$. This suggests that when a more robust closed-loop system is required and the process has a larger inverse response as well as dead time, the FOPID controller provides a higher performance than a PID controller. This performance improvement is

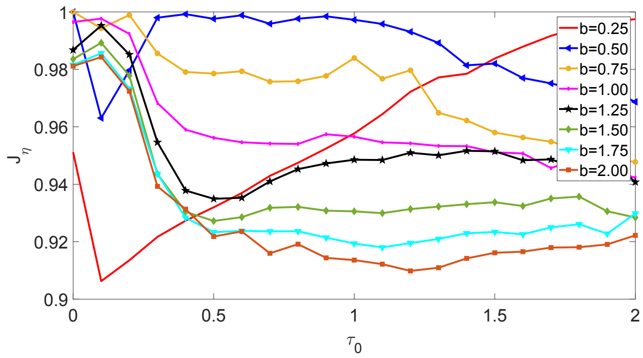


Figure 8. $J_\eta$ index for $a = 0.25$ and $M_S = 1.4$

mainly due of the fractional parameter associated to the derivative part $\mu$. Its behavior is presented in Fig. 9. The

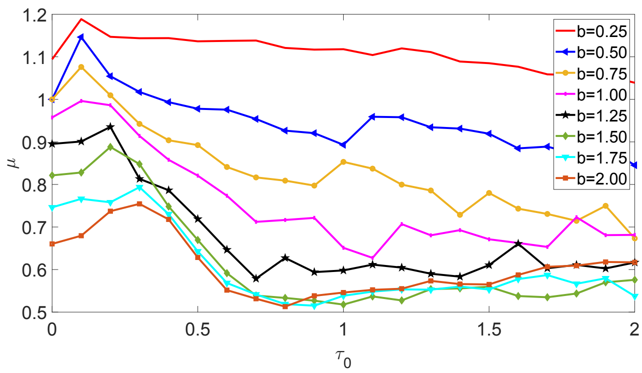


Figure 9. $\mu$ for $M_S = 1.4$ and $a = 0.25$

$J_\eta$ index for $a = 0.50$ is shown in Fig. 10. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 10 %) in general, for higher values of $b$, namely, $b \in \{1.0, 1.25, 1.5, 1.75, 2.0\}$ and specially for the range of the normalize dead time $\tau_0 \geq 0.5$. This suggests that when a more robust closed-loop system is required and the process has a larger inverse response as well as dead time, the FOPID controller provides a higher performance than a PID controller. This performance improvement is mainly due of the fractional parameter associated to the derivative part $\mu$. Its behavior is presented in Fig. 11. The $J_\eta$ index for $a = 0.75$ is shown in Fig. 12. From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 13 %) in general, for the whole range of $b$ values except for $b = 0.75$ and $b = 1.0$, and for the range of the normalize dead time $\tau_0 \geq 0.5$. This performance improvement is mainly due of the fractional parameter associated to the derivative part $\mu$. Its behavior is presented

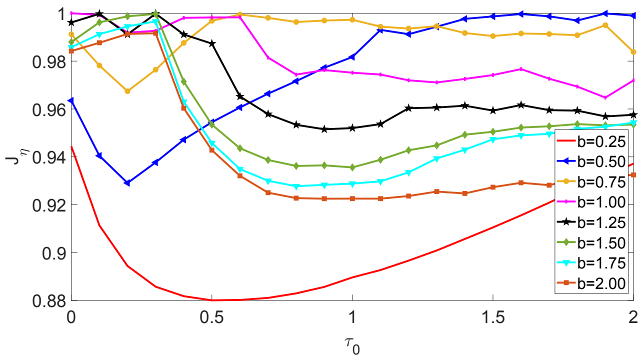


Figure 10. $J_\eta$ index for $a = 0.50$ and $M_S = 1.4$

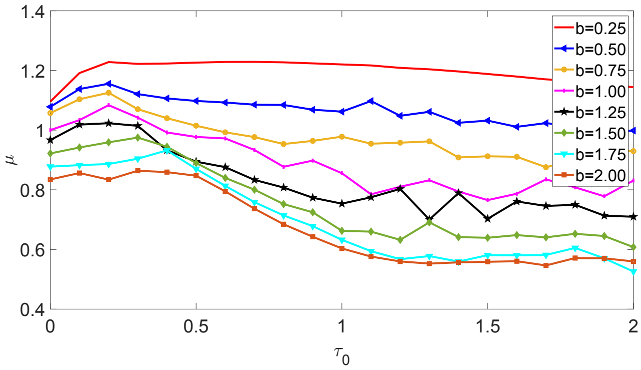


Figure 11. $\mu$ for $M_S = 1.4$ and $a = 0.50$

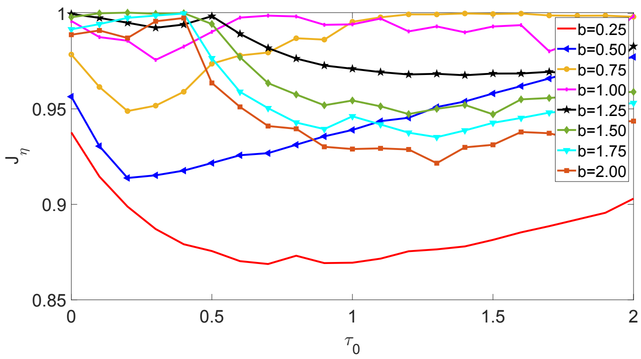


Figure 12. $J_\eta$ index for $a = 0.75$ and $M_S = 1.4$

in Fig. 13. The $J_\eta$ index for $a = 1.0$ is shown in Fig. 14.

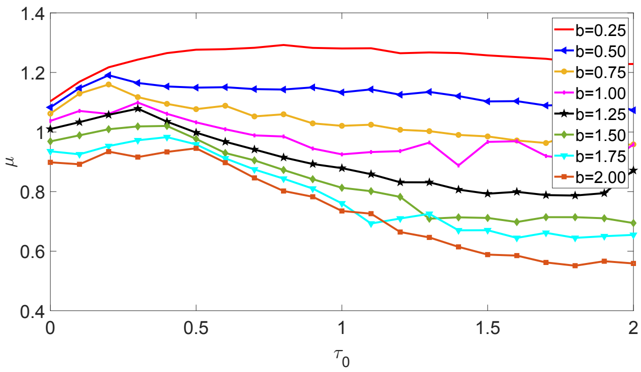


Figure 13. $\mu$ for $M_S = 1.4$ and $a = 0.75$

From this figure it can be seen that the improvement in the control system performance provided by the use of FOPID controllers over PID controllers is greater (up to 13 %) in general, for the whole range of $b$ values except for $b = 1.0$, and for the range of the normalize dead time $\tau_0 \geq 0.5$. This

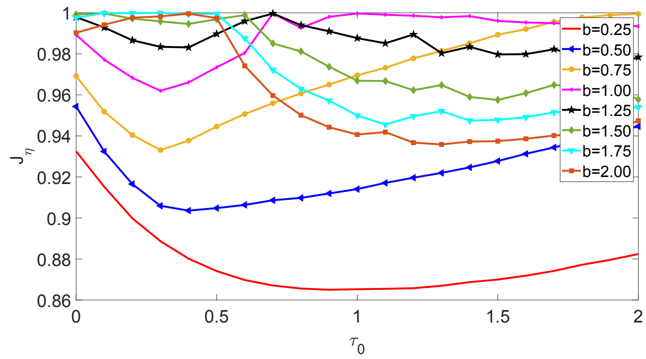


Figure 14. $J_\eta$ index for $a = 1.0$ and $M_S = 1.4$

performance improvement is mainly due of the fractional parameter associated to the derivative part $\mu$. Its behavior is presented in Fig. 15.

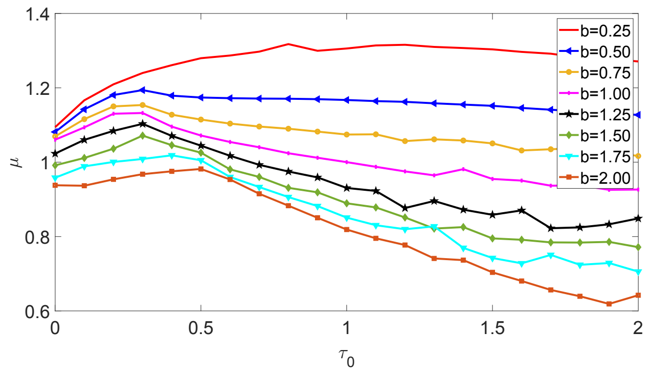


Figure 15. $\mu$ for $M_S = 1.4$ and $a = 1.0$

## 4. SIMULATION EXAMPLE

As an illustrative example consider the IRSOPDT process defined by the transfer function (13) which has a large dead time.

$$P_1(s) = \frac{(-0.25s + 1)e^{-1.6s}}{(s + 1)^2} \tag{13}$$

The optimal parameters for the PID and FOPID controller were obtained for both $M_S = 1.4$ and $M_S = 2.0$ and they are shown in Table 1. This Table also shows the $J_\eta$ index in order to quantify the improvement in the performance provided by the use of FOPID controllers in comparison with the PID controllers for both $M_S = 1.4$ and $M_S = 2.0$. The Table (1) shows that the performance

Table 1. Controller Parameters and $J_\eta$ index

| | PID $M_S = 1.4$ | FOPID $M_S = 1.4$ | PID $M_S = 2.0$ | FOPID $M_S = 2.0$ |
|---|---|---|---|---|
| $K_p$ | 0.479 | 0.702 | 0.865 | 1.059 |
| $T_i$ | 1.914 | 2.582 | 2.365 | 2.770 |
| $T_d$ | 0.868 | 0.891 | 0.747 | 0.856 |
| $\lambda$ | — | 1.000 | — | 1.000 |
| $\mu$ | — | 1.298 | — | 1.190 |
| $J_\eta$ | 0.885 | | 0.945 | |

of the FOPID controllers is better than the PID controllers in a 11.5% for a robustness defined by $M_S = 1.4$ and in a 5.5% for a robustness defined by $M_S = 2.0$. These results as well as the ones presented in Section 3 indicate that FOPID controllers allow a remarkable performance improvement in the closed-loop system when higher robustness is required. Moreover, this Table presents that for $M_S = 1.4$ the fractional-order parameter associated to the derivative part $\mu$ is further away from one than the case for $M_S = 2.0$, and this confirms the effect of this parameter when more robustness is required in the feedback system. Fig. (16) presents the closed-loop system response when

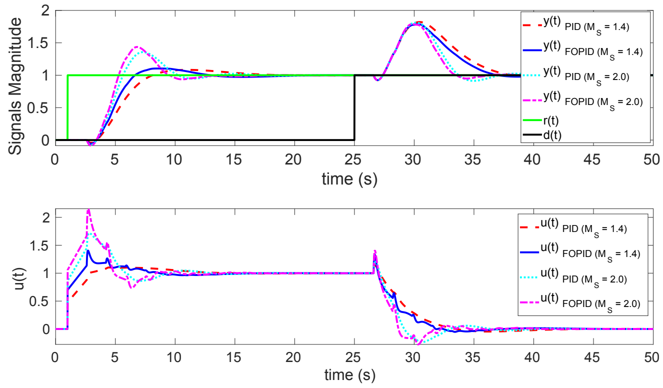


Figure 16. Set-Point and Load-Disturbance response for $P_1(s)$ $\alpha = 0.75$ process example.

a unit step change in the set-point value is applied in $t = 1$ $s$ and when a unit load-disturbance is applied in $t = 25$ $s$. This figure also shows also the controller output which is more aggressive for fractional-order controllers that their integer counterpart and therefore, this allows a better performance in the closed-loop system using this kind of controllers.

## 5. CONCLUSIONS

The present work proposes a quantitative evaluation of the performance provided by the use of fractional-order and integer-order PID controllers when robustness considerations should be considered for the control of non-minimum-phase systems with or without dead-time. The performance comparison between both types of control algorithms proves the advantages of using an additional parameter in the controller structure given by the fractional-order parameter $\mu$. The results show that for higher inverse responses defined by larger values of $b$ and for larger normalized dead-times $\tau_0$, the performance provided by the use of FOPID controllers is most prominent and even more if more robustness is required in the closed-loop systems as happens when $M_S$ is set to 1.4. It is expected that this works contribute to those researchers that require to design a feedback-loop for an application based on a non-minimum-phase system and fractional-order PID controllers.

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