PID/FOPID Tuning Rule Based on a Second Order Inverse Response Process Model with Robustness Considerations

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Abstract: This paper deals with the design of a proportional-integral-derivative (PID) and fractional-order proportional-integral-derivative (FOPID) controller tuning rule for second-order controlled processes with inverse response. The design of the rules takes into account specific levels of robustness and performance for both load disturbance rejection and set-point tracking tasks. In the first step, the optimal parameters for both controllers are obtained from optimizations that seek to minimize the error in both control modes while maintaining the robustness constraint. Subsequently, fitting functions to the optimal parameters are sought to capture their behavior with the possibility of finding optimal parameters with the tuning rule designed between certain ranges of the model parameters. For FOPID and PID controllers, the rule IRM-RoT (Inverse Response Model Robust Tuning) have been developed. The performance of the controllers was compared using examples, and the results showed that fractional order controllers provide a novel solution for inverse response dynamics in process control.

Keywords: PID, FOPID, Fractional Control, Tuning Rules, Inverse Response, Robust Control.

1. INTRODUCTION

It is widely known in the literature that processes with inverse response, also known as non-minimum phase systems, are present in a large number of applications in industry. For example, in some chemical processes, the inverse response effect appears, such as reactor drums, boilers, kettles, as studied in Asimbaya et al. (2017). This represents a challenge when it comes to finding a controller that is able to deal with these process dynamics.

In this context, it is important to acknowledge the increasing role of fractional calculus in the field of applied control theory over the past few decades Abdelhalcy et al. (2020), Babu and Chiranjeevi (2016). Its implementation in the industrial setting has shown strong acceptance in the development and implementation of fractional order control systems Tepljakov et al. (2021), including its feasibility in the control of systems with inverse response dynamics has been studied Sir Elkhateem et al. (2021).

Recent work has analyzed the effectiveness of applying the fractional order proportional-integral-derivative (FOPID) control algorithm Meneses et al. (2022), when working in load disturbance rejection and set-point tracking modes, and has also studied the achievement of specific levels of robustness to deal with the nonlinearities encountered in real applications. There are studies that have focused on the applicability and tunability of FOPID controllers for stable first-order plus dead-time process models Padula and Visioli (2011), as well as their feasibility for systems with unstable or integrating dynamics Padula and Visioli (2012). Returning to the dynamics that we want to study in this work, several studies have been presented that analyze from different perspectives the implementation of FOPID controllers Nagarsheth and Sharma (2020), Yadav et al. (2022) and Gutierrez et al. (2023), however, no specific tuning rule has been presented to deal with processes that possess inverse response involving the use of fractional order controllers considering specific levels of robustness.

This paper presents the IRM-RoT (Inverse Response Model Robust Tuning) rule, which is applicable to both PID and FOPID controllers, in order to make a comparison of the performance obtained using both controllers, showing the performance improvement that the use of FOPID provides, even considering the robustness restrictions. In addition, it is intended to compare the performance of the two types of controllers included in the IRM-RoT rule, in order to conclude on the improvement of the implementation of fractional order controllers, through examples using inverse response models proposed in other works.

The organization of the research is detailed below: Section 2 presents the formulation of the control problem and
the corresponding considerations of the proposed control system, Section 3 studies the methodology proposed to find the optimal parameters, describes the formulation of the IRM-RoT rule for PID and FOPID controllers. Subsequently, in Section 4, the IRM-RoT rule is compared by means of specific examples that represent the proposed dynamics, concluding with results that suggest an improvement in the system performance taking into account the robustness achieved when the FOPID controller is implemented. Finally, Section 5 presents the conclusions of this research.

2. PROBLEM FORMULATION

2.1 Control System Configuration

The control system considered is shown in Fig. 1, where \( P(s) \) models the controlled process and \( C(s) \) represents the PID or FOPID controller to be tuned.

![Figure 1. Close-loop control system.](image)

The variables previously described in Fig. 1 are:
- \( y(s) \) is the process output (controlled variable).
- \( r(s) \) is the set-point for the process output.
- \( u(s) \) is the control signal.
- \( d(s) \) is the load-disturbance of the system.

Considering the closed-loop control system of Fig. 1 the process output is described by the following expression:

\[
y(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} r(s) + \frac{P(s)}{1 + C(s)P(s)} d(s)
\]

(1)

in which the output of the process \( y(s) \) is determined by two input signals, \( r(s) \) and \( d(s) \). Depending on these signals, the system can operate in either of its two modes: servo control or regulatory control. The main objective is design a systems capable of a suitable tracking of the reference signal and additionally a good rejection of disturbance using and only tuning of the PID or FOPID controller.

2.2 Controlled Process Model

The controlled process \( P(s) \) is modelled by an inverse-response-second-order-plus-dead-time (IRSOPDT) transfer function of the form:

\[
P(s) = \frac{K(-bTs + 1)e^{-Ls}}{(Ts + 1)(\alpha Ts + 1)}
\]

(2)

where \( K \) is the process gain, \( T \) is the time constant, \( L \) is the dead-time, \( \alpha \) is the ratio between time constants and \( b \) gives the relative position of the right half plane zero respect to the dominant time constant.

In this context, characterizing the process parameters through the normalized dead time \( \tau = \frac{\tau}{T} \) is a common practice Visioli (2006). Performing the transformation \( \hat{s} = Ts \), the second-order model with inverse response (2) is expressed as:

\[
P(\hat{s}) = \frac{K(-b\hat{s} + 1)e^{-\hat{s}}}{(\hat{s} + 1)(\alpha \hat{s} + 1)}
\]

reducing the parameters used to describe the process dynamics to \( \alpha, b \) and \( \tau \).

The second-order inverse response plus normalized dead time model (3) will be used to perform the optimizations for the IRM-RoT rule.

2.3 1DoF PID and Fractional Order PID Controllers

The control of the process is achieved by implementing one-degree-of-freedom PID and FOPID controllers; in this control scheme, the control signal has the following form Åström and Hägglund (2006):

\[
u(s) = K_p \left[ \left( 1 + \frac{1}{T_i s} \right) c(s) - \left( \frac{T_d s}{\eta s + 1} \right) y(s) \right]
\]

(4)

where, \( K_p \) refers to the proportional controller gain, \( T_i \) is the integral time constant, \( T_d \) the derivative time constant and \( T_d / \eta \) the constant of the derivative filter.

![Figure 2. 1DoF Controller close-loop control system.](image)

The control scheme implementing 1DoF controllers is described in Fig. 2, where \( C_r(s) \) is the transfer function of the set-point controller and \( C_y(s) \) of the feedback controller.

\[
C_r(s) = K_p \left( 1 + \frac{1}{T_i s} \right)
\]

(5)

\[
C_y(s) = K_p \left( 1 + \frac{1}{T_i s} + \frac{T_d s}{\eta s + 1} \right)
\]

(6)

Considering this, the process output in 1DoF control scheme, \( y(s) \), can be expressed in terms of \( C_r(s) \) and \( C_y(s) \) using the following expression:

\[
y(s) = \frac{C_r(s)P(s)}{1 + C_r(s)P(s)} r(s) + \frac{P(s)}{1 + C_y(s)P(s)} d(s)
\]

(7)

Everything above is applicable when implementing a Fractional Order PID, except for the feedback controller transfer function, \( C_y(s) \), which has the fractional order of the derivative action \( \mu \).

\[
C_y(s) = K_p \left( 1 + \frac{1}{T_i s} + \frac{T_d s^\mu}{\eta s + 1} \right)
\]

(8)

For all scenarios, the constant for the derivative filter can be defined by (9), where \( \mu \) equals 1 in the PID controller.

\[
\eta = 10^\frac{\mu - 1}{\mu}
\]

(9)
When employing a fractional order controller, the integer order approximation of Oustaloup et al. (2000) is used to implement the controller. This involves applying the following recursive approximation based on a product of poles and zeros:

\[ s^n_{\omega_l, \omega_h} \cong C_n \prod_{k=1}^{N} \frac{1 + \frac{\omega}{\omega_p}}{1 + \frac{\omega}{\omega_p}}, \quad \mu > 0 \] (10)

The valid frequency range for the approximation is set to \( \{\omega_l, \omega_h\} = \{0.001, 1000\} \), the \( C_n \) term is set in way that the approximation has unity gain at the crossover frequency, and the parameter \( N \), which refers to the number of poles and zeros for the real-rational transfer function approximating of the fractional term, \( s^n \), is set to \( N = 8 \). By selecting these parameters, it is possible to obtain a good approximation of the fractional term without compromising the processing speed.

3. IRM-ROT RULES DEVELOPMENT

The Inverse Response Model Robust Tuning rule was developed by optimizing the appropriate parameters of the PID or FOPID controller. In the optimization process, \( \theta = \{K_p, T_i, T_d\} \) and \( \theta_{fc} = \{K_p, T_i, T_d, \mu\} \) are the PID and FOPID parameters to be optimized, respectively.

To measure the system performance, the integrated absolute value of the error (IAE), is defined according to the following equation:

\[ IAE = \int_0^\infty |e(t)| \, dt = \int_0^\infty |r(t) - y(t)| \, dt \] (11)

In this work, the IAE is considered both \( J_{sp} \) for the set-point tracking task and \( J_{ld} \) for the load disturbance rejection case, since at the time of defining the cost function this will be the sum of both:

\[ J_{erd} = \int_0^\infty |r(t) - y(t)| \, dt + \int_0^\infty | - y(t)| \, dt \] (12)

by considering the total sum of both errors without any weighting, it can be ensured that a good performance of the tuning rule is achieved for both control modes, servo-control, and regulatory-control.

The closed-loop control system robustness is considered in terms of constraints. This is because the design procedure for the controller typically utilizes a reduced-order linear model at a specific operating point. This model effectively characterizes process dynamics. However, since real processes involve nonlinearities, it is crucial to consider a margin of stability or robustness (relative stability) for the control system.

To assess the relative stability of the system, the Sensitivity Function’s peak value will serve as an indicator. After acquiring both optimal sets of \( \theta_c \) and \( \theta_{fc} \), the control system maximum sensitivity \( (M_s) \), will be used to measure the closed-loop control system’s robustness according to the following formula:

\[ M_s = \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C(j\omega)P(j\omega)|} \] (13)

It is common to find \( M_s \) values ranging from 1.40 to 2.00 Åström and Hägglund (2006). Where \( M_s = 2.00 \)

represents the minimum robustness required to obtain gain and phase margins that guarantee stability, while for \( M_s = 1.40 \) the system is considered quite robust and stable since the margins obtained are higher. Optimizations were performed considering the trade-off between system performance and robustness for both controllers, considering the extreme levels of \( M_s = \{1.40, 2.00\} \) and evaluate both the robustness achieved and the performance in all cases.

To evaluate the improvement obtained by implementing fractional order controllers in the control system with respect to a PID controller, an improvement index is defined in (14), which can be used for each control mode separately (for \( J_{sp} \) and \( J_{ld} \)) or as a combined performance index considering \( J_{erd} \):

\[ \Upsilon = \left(1 - \frac{J_{FOPID}}{J_{PID}}\right) \cdot 100\% \] (14)

3.1 Tuning methodology

The IRM-RoT rule was developed using specific values for normalized dead time \( \tau \), the relative position of the right half plane zero \( b \), and ratio of time constants \( \alpha \). In this context, the MATLAB\textsuperscript{®} fminimax solver has been used for the optimization procedure to minimize \( J_{erd} \), considering the following values:

\[ \alpha = \{0.10, 0.25, 0.50, 0.75, 1.00\} \] (15)

as fixed values to find the optimal parameters. For intermediate \( \alpha \) values, a linear interpolation is suggested.

Regarding \( b \), the typical values considered were:

\[ b = \{0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5\} \] (16)

However, in some cases, intermediate values had to be considered for FOPID controllers in order to improve the performance and robustness results obtained in linear interpolation.

Finally, for the normalized dead time, the value is taken for the ranges \( 0.1 \leq \tau \leq 2 \) as is commonly specified.

3.2 PID and FOPID tuning equations

The optimal parameters \( \theta_{opt} \) and \( \theta_{fc} \) obtained from the optimization procedure were used to adjust the PID and FOPID controller parameter equations; in the case of integer controller parameters, equations with a maximum of four constants were used, and in the case of fractional order controller, an additional constant was added to the proposed equations.

For both types of controllers, it is assumed that

\[ \kappa_p = K_pK_i, \quad \tau_i = \frac{T_i}{T}, \quad \tau_d = \frac{T_d}{T} \] (17)

PID tuning: As stated previously, a maximum of four constants are used for the PID controller parameters.

\[ \kappa_p = \frac{a_1\tau^2 + a_2\tau + a_3}{\tau + a_4} \] (18)

\[ \tau_i = k_1\tau^3 + k_2\tau^2 + k_3\tau + k_4 \] (19)

\[ \tau_d = c_1\tau^3 + c_2\tau^2 + c_3\tau + c_4 \] (20)

In all cases, cftool has been used to find equations for the controller parameters that provide a low level of SSE (Sum Squared Error) with respect to the actual optimal values.
To implement the IRM-RoT rule for PID controllers, the constants \( a_i, k_i \) and \( c_i \) for \( i = \{1, 2, 3, 4\} \) are used, whose optimal values are shown in tables (1) and (2) given below:

### Table 1. PID parameter constants for \( \alpha = 0.75 \) and a target \( M_s = 1.40 \).

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( k_i )</th>
<th>( c_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0534</td>
<td>0.0392</td>
<td>0.0342</td>
</tr>
<tr>
<td>0.2630</td>
<td>0.2824</td>
<td>0.3426</td>
</tr>
<tr>
<td>0.6277</td>
<td>0.2437</td>
<td>0.2199</td>
</tr>
<tr>
<td>0.1675</td>
<td>0.0294</td>
<td>0.0351</td>
</tr>
</tbody>
</table>

### Table 2. PID parameter constants for \( \alpha = 1.00 \) and a target \( M_s = 1.40 \).

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( k_i )</th>
<th>( c_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0534</td>
<td>0.0392</td>
<td>0.0342</td>
</tr>
<tr>
<td>0.2630</td>
<td>0.2824</td>
<td>0.3426</td>
</tr>
<tr>
<td>0.6277</td>
<td>0.2437</td>
<td>0.2199</td>
</tr>
<tr>
<td>0.1675</td>
<td>0.0294</td>
<td>0.0351</td>
</tr>
</tbody>
</table>

### FOPID tuning: In this case, the derivative fractional order needs adjustment.

\[
\tau_i = k_i \tau^3 + k_2 \tau^3 + k_3 \tau^2 + k_4 \tau + k_5
\]

\[
\tau_d = c_1 \tau^4 + c_2 \tau^3 + c_3 \tau^2 + c_4 \tau + c_5
\]

\[
\mu = d_1 \tau^3 + d_2 \tau^3 + d_3 \tau^2 + d_4 \tau + d_5
\]

### Figure 3. \( \kappa_p \) parameter of the FOPID for \( \alpha = 0.75 \) and \( b = 1.25 \) with target \( M_s = 1.40 \).

### Figure 4. \( \tau_r \) parameter of the FOPID for \( \alpha = 0.75 \) and \( b = 1.25 \) with target \( M_s = 1.40 \).

### Figure 5. \( \tau_d \) parameter of the FOPID for \( \alpha = 0.75 \) and \( b = 1.25 \) with target \( M_s = 1.40 \).

### Figure 6. \( \mu \) parameter of the FOPID for \( \alpha = 0.75 \) and \( b = 1.25 \) with target \( M_s = 1.40 \).

### Figures (3), (4), (5), and (6), shown arbitrary examples of how the proposed equations fit to the optimal parameters.
4. EXAMPLES

4.1 Example 1.

In Luyben (2000) analysis of the IRSOPDT process, the process was characterized with a high value of dead time and a ratio between time constants $\alpha = 1.00$. However, for the purpose of this analysis, the range of $\alpha$ specified in (15) is considered, and the performance and robustness obtained for the PID and FOPID controllers are analyzed with $M_r = 1.40$ as the desired robustness. The inverse response model $P_i(s)$ is defined by the following expression:

$$P_i(s) = \frac{(-1.6s + 1)e^{-1.6s}}{(s + 1)(\alpha s + 1)}$$

In Table (5), the integral values of the absolute error for the set-point tracking and load-disturbance rejection tasks are presented. In each case, the analysis is performed for a step setpoint response of magnitude 1 at $t = 0$ and a negative step disturbance change of 50% of the set-point at $t = 40s$. As demonstrated, the $IAE$ values decrease with an increasing time constant ratio in both modes of control. However, it is clear that there is an improvement when implementing the fractional order controller for both, servo-control and regulatory-control in all cases. With regard to robustness, both the PID and FOPID IRM-RoT controllers achieve an optimal desired robustness value at the target value as shown.

Table 5. Performance indices to compare PID and FOPID IRM-RoT for Example 1.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$J_{sp}$</th>
<th>$J_{sd}$</th>
<th>$M_r$</th>
<th>$J_{sp}$</th>
<th>$J_{sd}$</th>
<th>$M_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>5.0593</td>
<td>5.2648</td>
<td>2.998</td>
<td>8.8119</td>
<td>4.8020</td>
<td>1.9568</td>
</tr>
<tr>
<td>0.25</td>
<td>5.0593</td>
<td>5.1527</td>
<td>1.4035</td>
<td>8.8141</td>
<td>4.7755</td>
<td>1.4058</td>
</tr>
<tr>
<td>0.50</td>
<td>5.0393</td>
<td>5.0503</td>
<td>1.3975</td>
<td>8.7166</td>
<td>4.8101</td>
<td>1.3962</td>
</tr>
<tr>
<td>0.75</td>
<td>8.8563</td>
<td>4.9097</td>
<td>1.3997</td>
<td>8.4116</td>
<td>4.5990</td>
<td>1.4120</td>
</tr>
<tr>
<td>1.00</td>
<td>8.6647</td>
<td>4.7631</td>
<td>1.4022</td>
<td>8.3479</td>
<td>4.5084</td>
<td>1.4097</td>
</tr>
</tbody>
</table>

Table 6. Improvement index values for the Example 1.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>4.935</td>
</tr>
<tr>
<td>0.25</td>
<td>2.880</td>
</tr>
<tr>
<td>0.50</td>
<td>3.570</td>
</tr>
<tr>
<td>0.75</td>
<td>5.024</td>
</tr>
<tr>
<td>1.00</td>
<td>3.279</td>
</tr>
</tbody>
</table>

Table 7. Performance and robustness values evaluation for the Example 2.

<table>
<thead>
<tr>
<th>Tuning Rule</th>
<th>$J_{sp}$</th>
<th>$J_{sd}$</th>
<th>$J_{rd}$</th>
<th>$M_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRM-RoT PID</td>
<td>6.8564</td>
<td>6.7890</td>
<td>13.6244</td>
<td>1.4037</td>
</tr>
<tr>
<td>Kaya-Cengiz PID</td>
<td>6.3528</td>
<td>6.4535</td>
<td>12.8363</td>
<td>1.4033</td>
</tr>
</tbody>
</table>

Fig. (7) presented the response of the close-loop control system under the conditions described in the $\alpha = 0.75$ case. The controller parameters for this case are $K_p = 0.1986$, $T_1 = 1.7582s$, $T_d = 0.6571s$ for the PID and $K_p = 0.1767$, $T_1 = 1.4030s$, $T_d = 1.040s$, $\mu = 0.68848$ for FOPID.

As anticipated, the response of the system improves with the fractional order controller, as shown in the graphical analysis and confirmed in 6, where the improvement indices are shown, even obtaining a $\%$ improvement with respect to the PID controller, considering the errors of both operating modes simultaneously.

4.2 Example 2.

Another example is proposed to evaluate the performance of IRM-RoT controllers with respect to the tuning rule proposed in Kaya and Cengiz (2017). In this instance, the controlled process model of (25) is proposed, where interpolated parameters of the IRM-RoT rule are employed.

$$P_2(s) = \frac{(-0.8s + 1)e^{-0.6s}}{2(s + 1)}$$

For this model, the IRM-RoT PID controller parameters are $K_p = 0.5407$, $T_1 = 3.2160s$, $T_d = 1.1020s$ and $K_p = 0.6222$, $T_1 = 3.6979s$, $T_d = 1.1139s$, $\mu = 1.1298$ for the FOPID, $K_p = 1.2160$, $T_1 = 3.9983s$, $T_d = 1.2262s$ in the case of the Kaya-Cengiz PID.

Table (7) presents the performance and robustness metrics for the different tuning rules and Fig. (8) shows the close-loop system response implementing the three controllers.
Table 8. Improvement index with Kaya-Cengiz PID as base controller.

<table>
<thead>
<tr>
<th>Tuning Rule</th>
<th>$%$</th>
<th>$%$</th>
<th>$%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRM-RoT PID</td>
<td>31.21</td>
<td>1.435</td>
<td>19.130</td>
</tr>
<tr>
<td>IRM-RoT FOPID</td>
<td>36.243</td>
<td>5.957</td>
<td>23.808</td>
</tr>
</tbody>
</table>

Furthermore, when implementing the Kaya-Cengiz rule for IRSOPDT models, it is restricted to using only the time constants ratio $\alpha = 1$. This results in the rule being highly impractical for inverse response processes, unlike what is proposed by IRM-RoT rule.

5. CONCLUSIONS AND FUTURE WORK

The present work proposes the IRM-RoT rule for PID and FOPID controllers, considering a trade-off between the performance and robustness of the control system by implementing an IRSOPDT model to determine the optimal parameters. The study validates the enhancement obtained by implementing a IRM-RoT controller for inverse-response process control by means of concrete examples. Given the complexity of controlling processes with inverse response, controller parameters are obtained by fitting equations with either 4 or 5 constants. This results in a substantial amount of tables needed to implement the rule. Therefore, future work proposes to develop an auto-tuning tool for the IRM-RoT rule.

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