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PID/FOPID Tuning Rule Based on a Second Order Inverse Response Process Model with Robustness Considerations

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Abstract: This paper deals with the design of a proportional-integral-derivative (PID) and fractional-order proportional-integral-derivative (FOPID) controller tuning rule for second-order controlled processes with inverse response. The design of the rules takes into account specific levels of robustness and performance for both load disturbance rejection and set-point tracking tasks. In the first step, the optimal parameters for both controllers are obtained from optimizations that seek to minimize the error in both control modes while maintaining the robustness constraint. Subsequently, fitting functions to the optimal parameters are sought to capture their behavior with the possibility of finding optimal parameters with the tuning rule designed between certain ranges of the model parameters. For FOPID and PID controllers, the rule IRM-RoT (Inverse Response Model Robust Tuning) have been developed. The performance of the controllers was compared using examples, and the results showed that fractional order controllers provide a novel solution for inverse response dynamics in process control.

Keywords: PID, FOPID, Fractional Control, Tuning Rules, Inverse Response, Robust Control.

1. INTRODUCTION

It is widely known in the literature that processes with inverse response, also known as non-minimum phase systems, are present in a large number of applications in industry. For example, in some chemical processes, the inverse response effect appears, such as reactor drums, boilers, kettles, as studied in Asimbaya et al. (2017). This represents a challenge when it comes to finding a controller that is able to deal with these process dynamics.

In this context, it is important to acknowledge the increasing role of fractional calculus in the field of applied control theory over the past few decades Abdelbaky et al. (2020), Babu and Chiranjeevi (2016). Its implementation in the industrial setting has shown strong acceptance in the development and implementation of fractional order control systems Tepljakov et al. (2021), including its feasibility in the control of systems with inverse response dynamics has been studied Sir Elkhatem et al. (2021).

Recent work has analyzed the effectiveness of applying the fractional order proportional-integral-derivative (FOPID) control algorithm Meneses et al. (2022), when working in load disturbance rejection and set-point tracking modes, and has also studied the achievement of specific levels of robustness to deal with the nonlinearities encountered in real applications. There are studies that have focused on the applicability and tunability of FOPID controllers for stable first-order plus dead-time process models Padula and Visioli (2011), as well as their feasibility for systems with unstable or integrating dynamics Padula and Visioli (2012). Returning to the dynamics that we want to study in this work, several studies have been presented that analyze from different perspectives the implementation of FOPID controllers Nagarsheth and Sharma (2020), Yadav et al. (2022) and Gutiererez et al. (2023), however, no specific tuning rule has been presented to deal with processes that possess inverse response involving the use of fractional order controllers considering specific levels of robustness.

This paper presents the IRM-RoT (Inverse Response Model Robust Tuning) rule, which is applicable to both PID and FOPID controllers, in order to make a comparison of the performance obtained using both controllers, showing the performance improvement that the use of FOPID provides, even considering the robustness restrictions. In addition, it is intended to compare the performance of the two types of controllers included in the IRMRoT rule, in order to conclude on the improvement of the implementation of fractional order controllers, through examples using inverse response models proposed in other works.
The organization of the research is detailed below: Section 2 presents the formulation of the control problem and

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the corresponding considerations of the proposed control system, Section 3 studies the methodology proposed to find the optimal parameters, describes the formulation of the IRM-RoT rule for PID and FOPID controllers. Subsequently, in Section 4, the IRM-RoT rule is compared by means of specific examples that represent the proposed dynamics, concluding with results that suggest an improvement in the system performance taking into account the robustness achieved when the FOPID controller is implemented. Finally, Section 5 presents the conclusions of this research.

2. PROBLEM FORMULATION

2.1 Control System Configuration

The control system considered is shown in Fig. 1, where $P(s)$ models the controlled process and $C(s)$ represents the PID or FOPID controller to be tuned.

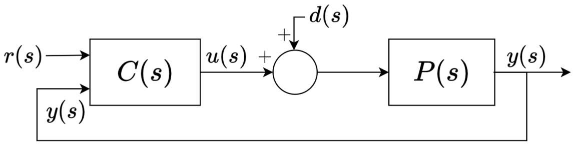

Figure 1. Close-loop control system.

The variables previously described in Fig. 1 are:

- $y(s)$ is the process output (controlled variable).
- $r(s)$ is the set-point for the process output.
- $u(s)$ is the control signal.
- $d(s)$ is the load-disturbance of the system.

Considering the closed-loop control system of Fig. 1 the process output is described by the following expression:

$$y(s) = \underbrace{\frac{C(s)P(s)}{1 + C(s)P(s)} r(s)}_{servo-control} + \underbrace{\frac{P(s)}{1 + C(s)P(s)} d(s)}_{regulatory-control} \quad (1)$$

in which the output of the process $y(s)$ is determined by two input signals, $r(s)$ and $d(s)$. Depending on these signals, the system can operate in either of its two modes: *servo control* or *regulatory control*. The main objective is design a systems capable of a suitable tracking of the reference signal and additionally a good rejection of disturbance using and only tuning of the PID or FOPID controller.

2.2 Controlled Process Model

The controlled process $P(s)$ is modelled by an inverse-response-second-order-plus-dead-time (IRSOPDT) transfer function of the form:

$$P(s) = \frac{K(-bTs + 1)e^{-Ls}}{(Ts + 1)(\alpha Ts + 1)} \quad (2)$$

where K is the process gain, T is the time constant, L is the dead-time, α is the ratio between time constants and b gives the relative position of the right half plane zero respect to the dominant time constant.

In this context, characterizing the process parameters through the normalized dead time $\tau = \frac{L}{T}$ is a common practice Visioli (2006). Performing the transformation $\hat{s} = Ts$, the second-order model with inverse response (2) is expressed as:

$$P(\hat{s}) = \frac{K(-b\hat{s} + 1)e^{-\tau\hat{s}}}{(\hat{s} + 1)(\alpha\hat{s} + 1)} \quad (3)$$

reducing the parameters used to describe the process dynamics to α, b and τ.

The second-order inverse response plus normalized dead time model (3) will be used to perform the optimizations for the IRM-RoT rule.

2.3 1DoF PID and Fractional Order PID Controllers

The control of the process is achieved by implementing one-degree-of-freedom PID and FOPID controllers; in this control scheme, the control signal has the following form Åström and Hägglund (2006):

$$u(s) = K_p \left[\left(1 + \frac{1}{T_i s}\right) e(s) - \left(\frac{T_d s}{\frac{T_d}{\eta} s + 1}\right) y(s) \right] \quad (4)$$

where, K_p refers to the proportional controller gain, T_i is the integral time constant, T_d the derivative time constant and T_d/η the constant of the derivative filter.

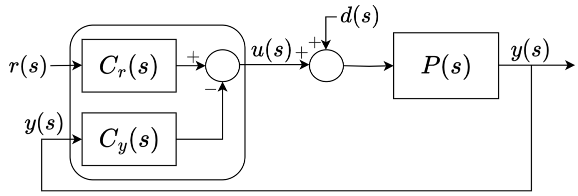

Figure 2. 1DoF Controller close-loop control system

The control scheme implementing 1DoF controllers is described in Fig. 2, where $C_r(s)$ is the transfer function of the set-point controller and $C_y(s)$ of the feedback controller.

$$C_r(s) = K_p \left(1 + \frac{1}{T_i s}\right) \quad (5)$$

$$C_y(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{\eta} s + 1}\right) \quad (6)$$

Considering this, the process output in 1DoF control scheme, $y(s)$, can be expressed in terms of $C_r(s)$ and $C_y(s)$ using the following expression:

$$y(s) = \underbrace{\frac{C_r(s)P(s)}{1 + C_y(s)P(s)}}_{M_{yr}(s)} r(s) + \underbrace{\frac{P(s)}{1 + C_y(s)P(s)}}_{M_{yd}(s)} d(s) \quad (7)$$

Everything above is applicable when implementing a Fractional Order PID, except for the feedback controller transfer function, $C_y(s)$, which has the fractional order of the derivative action μ.

$$C_y(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s^{\mu}}{\frac{T_d}{\eta} s + 1}\right) \quad (8)$$

For all scenarios, the constant for the derivative filter can be defined by (9), where μ equals 1 in the PID controller.

$$\eta = 10 T_d^{\frac{\mu - 1}{\mu}} \quad (9)$$

When employing a fractional order controller, the integer order approximation of Oustaloup et al. (2000) is used to implement the controller. This involves applying the following recursive approximation based on a product of poles and zeros:

$$s^{\mu}_{[\omega l, \omega h]} \cong C_o \prod_{k=1}^{N} \frac{1+\frac{s}{\omega_{z,k}}}{1+\frac{s}{\omega_{p,k}}},\ \mu > 0 \tag{10}$$

The valid frequency range for the approximation is set to $\{\omega l, \omega h\} = \{0.001, 1000\}$, the C_o term is set in way that the approximation has unity gain at the crossover frequency, and the parameter N, which refers to the number of poles and zeros for the real-rational transfer function approximating of the fractional term, s^{μ}, is set to $N = 8$. By selecting these parameters, it is possible to obtain a good approximation of the fractional term without compromising the processing speed.

3. IRM-ROT RULES DEVELOPMENT

The Inverse Response Model Robust Tuning rule was developed by optimizing the appropriate parameters of the PID or FOPID controller. In the optimization process, $\theta_c = \{K_p, T_i, T_d\}$ and $\theta_{fc} = \{K_p, T_i, T_d, \mu\}$ are the PID and FOPID parameters to be optimized, respectively.

To measure the system performance, the integrated absolute value of the error (IAE), is defined according to the following equation:

$$IAE = \int_0^{\infty} |e(t)|\, dt = \int_0^{\infty} |r(t) - y(t)|\, dt \tag{11}$$

In this work, the IAE is considered both J_{sp} for the set-point tracking task and J_{ld} for the load disturbance rejection case, since at the time of defining the cost function this will be the sum of both:

$$J_{erd} = \underbrace{\int_0^{\infty} |r(t) - y(t)|\, dt}_{J_{sp} \to d(t)=0} + \underbrace{\int_0^{\infty} |-y(t)|\, dt}_{J_{ld} \to r(t)=0} \tag{12}$$

by considering the total sum of both errors without any weighting, it can be ensured that a good performance of the tuning rule is achieved for both control modes, servo-control, and regulatory-control.

The closed-loop control system robustness is considered in terms of constraints. This is because the design procedure for the controller typically utilizes a reduced-order linear model at a specific operating point. This model effectively characterizes process dynamics. However, since real processes involve nonlinearities, it is crucial to consider a margin of stability or robustness (relative stability) for the control system.

To assess the relative stability of the system, the Sensitivity Function's peak value will serve as an indicator. After acquiring both optimal sets of θ_c and θ_{fc}, the control systems maximum sensitivity (M_s), will be used to measure the closed-loop control system's robustness according to the following formula:

$$M_s \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C(j\omega)P(j\omega)|} \tag{13}$$

It is common to find Ms values ranging from 1.40 to 2.00 Åström and Hägglund (2006). Where $M_s = 2.00$ represents the minimum robustness required to obtain gain and phase margins that guarantee stability, while for $M_s = 1.40$ the system is considered quite robust and stable since the margins obtained are higher. Optimizations were performed considering the trade-off between system performance and robustness for both controllers, considering the extreme levels of $M_s = \{1.40, 2.00\}$ and evaluate both the robustness achieved and the performance in all cases.

To evaluate the improvement obtained by implementing fractional order controllers in the control system with respect to a PID controller, an improvement index is defined in (14), which can be used for each control mode separately (for J_{sp} and J_{ld}) or as a combined performance index considering J_{erd}.

$$\Upsilon = \left(1 - \frac{J_{FOPID}}{J_{PID}}\right) \cdot 100\% \tag{14}$$

3.1 Tuning methodology

The IRM-RoT rule was developed using specific values for normalized dead time τ, the relative position of the right half plane zero b, and ratio of time constants α. In this context, the MATLAB® *fminimax* solver has been used for the optimization procedure to minimize J_{erd}, considering the following values:

$$\alpha = \{0.10, 0.25, 0.50, 0.75, 1.00\} \tag{15}$$

as fixed values to find the optimal parameters. For intermediate α values, a linear interpolation is suggested.

Regarding b, the typical values considered were:

$$b = \{0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5\} \tag{16}$$

However, in some cases, intermediate values had to be considered for FOPID controllers in order to improve the performance and robustness results obtained in linear interpolation.

Finally, for the normalized dead time, the value is taken for the ranges $0.1 \leq \tau \leq 2$ as is commonly specified.

3.2 PID and FOPID tuning equations

The optimal parameters θ_c^{opt} and θ_{fc}^{opt} obtained from the optimization procedure were used to adjust the PID and FOPID controller parameter equations; in the case of integer controller parameters, equations with a maximum of four constants were used, and in the case of fractional order controller, an additional constant was added to the proposed equations.

For both types of controllers, it is assumed that

$$\kappa_p = K_p K, \quad \tau_i = \frac{T_i}{T}, \quad \tau_d = \frac{T_d}{T^{\mu}} \tag{17}$$

PID tuning: As stated previously, a maximum of four constants are used for the PID controller parameters.

$$\kappa_p = \frac{a_1\tau^2 + a_2\tau + a_3}{\tau + a_4} \tag{18}$$

$$\tau_i = k_1\tau^3 + k_2\tau^2 + k_3\tau + k_4 \tag{19}$$

$$\tau_d = c_1\tau^3 + c_2\tau^2 + c_3\tau + c_4 \tag{20}$$

In all cases, *cftool* has been used to find equations for the controller parameters that provide a low level of SSE (Sum Squared Error) with respect to the actual optimal values.

To implement the IRM-RoT rule for PID controllers, the constants a_i, k_i and c_i for $i = \{1, 2, 3, 4\}$ are used, whose optimal values are shown in tables (1) and (2) given below:

Table 1. PID parameter constants for $\alpha = 0.75$ and a target $M_s = 1.40$.

b	0, 25	0, 50	0, 75	1.00	1, 25	1, 50	1, 75	2.00
a_1	-0.0159	-0.0162	-0.0399	-0.0406	0.0077	0.2531	0.0247	0.1314
a_2	0.2630	0.2923	0.3436	0.2948	0.1014	-0.7611	0.0717	-0.3134
a_3	0.4257	0.3437	0.2109	0.2062	0.5457	2.9362	0.4872	2.1339
a_4	0.2052	0.3459	0.3234	0.4331	1.4796	9.8750	1.9536	9.9994
k_1	0.0786	-0.0160	-0.1282	-0.0504	-0.0589	0.0074	-0.1039	0.0165
k_2	-0.2081	0.1513	0.4623	0.1668	0.1908	0.0319	0.3453	-0.0121
k_3	0.2873	-0.0355	-0.1337	0.1411	0.0936	0.1818	-0.0211	0.2513
k_4	1.4663	1.4626	1.4023	1.3728	1.3729	1.3434	1.3516	1.3166
c_1	-0.0191	-0.0332	0.0244	0.0698	0.1083	0.0935	0.1621	0.1024
c_2	0.0663	0.0845	-0.1658	-0.2771	-0.3792	-0.3455	-0.5338	-0.3456
c_3	0.2571	0.2892	0.4853	0.4811	0.5053	0.4468	0.5315	0.3504
c_4	0.3204	0.3570	0.3743	0.4063	0.4313	0.4642	0.4787	0.5136

Table 2. PID parameter constants for $\alpha = 1.00$ and a target $M_s = 1.40$.

b	0, 25	0, 50	0, 75	1.00	1, 25	1, 50	1, 75	2.00
a_1	-0.0134	-0.0217	-0.0157	-0.0434	-0.0401	0.1315	0.1502	0.1368
a_2	0.2511	0.3031	0.3038	0.3463	0.2881	-0.6682	-0.5404	-0.3983
a_3	0.5015	0.4001	0.3219	0.1843	0.2131	3.3674	2.8648	2.4641
a_4	0.2094	0.3503	0.4304	0.3325	0.4966	9.7851	9.9990	9.9975
k_1	0.1279	-0.0326	-0.0274	-0.1331	-0.0648	0.0075	-0.0113	-0.0160
k_2	-0.3993	0.1843	0.2000	0.4663	0.2017	-0.0514	0.0538	0.0899
k_3	0.4768	-0.0903	-0.0298	-0.1339	0.1057	0.3008	0.1884	0.1515
k_4	1.6472	1.7154	1.6006	1.6061	1.5663	1.5374	1.5142	1.5137
c_1	-0.0038	-0.0122	-0.0549	0.0507	0.0902	0.0864	0.1251	0.1464
c_2	0.0223	0.0343	0.1148	-0.2493	-0.3443	-0.3075	-0.4424	-0.5115
c_3	0.2837	0.3170	0.2878	0.5360	0.5385	0.4560	0.5317	0.5517
c_4	0.3650	0.3931	0.4467	0.4478	0.4803	0.5164	0.5452	0.5626

FOPID tuning: In this case, the derivative fractional order needs adjustment.

$$\tau_i = k_1\tau^4 + k_2\tau^3 + k_3\tau^2 + k_4\tau + k_5 \tag{21}$$

$$\tau_d = c_1\tau^4 + c_2\tau^3 + c_3\tau^2 + c_4\tau + c_5 \tag{22}$$

$$\mu = d_1\tau^4 + d_2\tau^3 + d_3\tau^2 + d_4\tau + d_5 \tag{23}$$

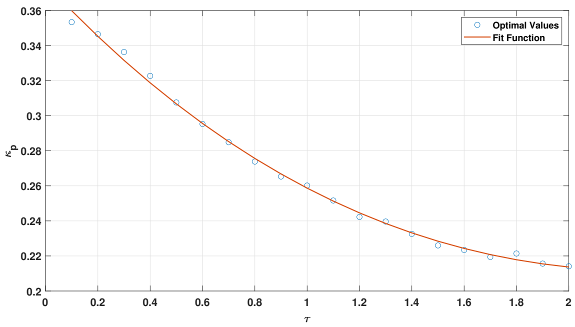

Figure 3. κ_p parameter of the FOPID for $\alpha = 0.75$ and $b = 1.25$ with target $M_s = 1.40$.

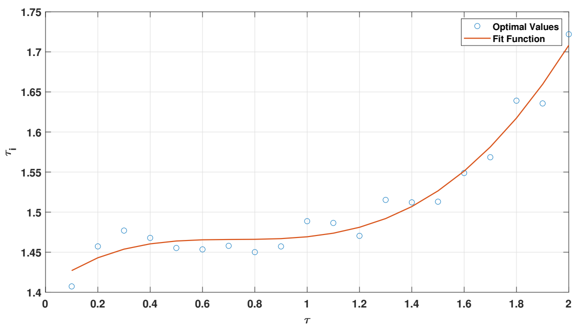

Figure 4. τ_i parameter of the FOPID for $\alpha = 0.75$ and $b = 1.25$ with target $M_s = 1.40$.

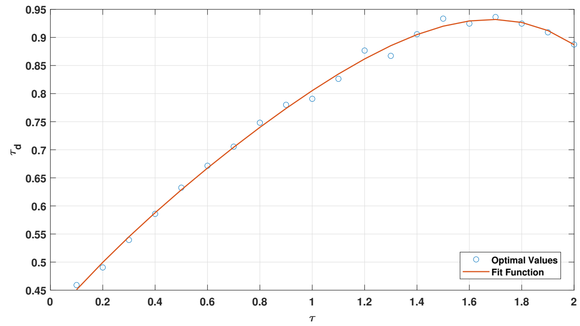

Figure 5. τ_d parameter of the FOPID for $\alpha = 0.75$ and $b = 1.25$ with target $M_s = 1.40$.

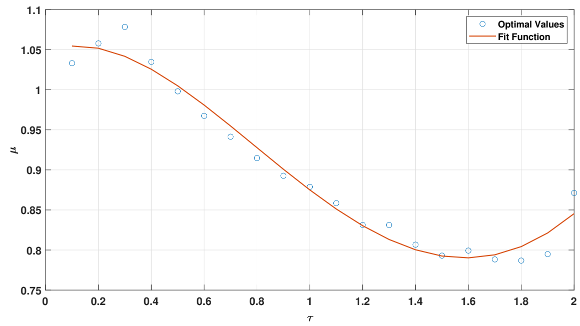

Figure 6. μ parameter of the FOPID for $\alpha = 0.75$ and $b = 1.25$ with target $M_s = 1.40$.

Table 3. FOPID parameter constants $\alpha = 0.75$ with target $M_s = 1.40$.

b	0, 25	0, 50	0, 75	1.00	1, 25	1, 50	1, 75	2.00
a_1	-0.0573	0.0033	0.0325	0.1147	0.3024	0.2241	0.1205	-0.0706
a_2	0.3404	0.1784	0.0360	-0.2914	-1.1746	-0.9007	-0.5570	0.2221
a_3	0.7900	0.6861	0.7650	1.4255	3.6364	3.0371	2.4511	0.0593
a_4	0.3270	0.6087	1.0860	2.9263	9.6852	9.9995	9.9852	0.3160
k_1	0.0533	0.1184	-0.0264	0.0100	-0.0225	-0.1552	-0.2804	-0.1372
k_2	-0.1758	-0.5279	0.1159	-0.0514	0.2090	0.8733	1.3827	0.6198
k_3	0.0741	0.7775	-0.1009	0.1786	-0.3823	-1.4731	-2.1784	-1.1286
k_4	0.4645	-0.1739	0.1006	-0.0380	0.2604	0.8476	1.2282	0.7380
k_5	1.6837	1.6915	1.5916	1.4918	1.4046	1.2801	1.1555	1.1903
c_1	-0.0022	-0.0558	-0.0080	-0.0291	-0.0515	0.0608	0.1805	0.4861
c_2	-0.0125	0.2478	0.0260	0.1545	0.1433	-0.4519	-0.9061	-1.5808
c_3	0.0438	-0.3850	-0.0824	-0.3841	-0.2320	0.7529	1.2708	1.7999
c_4	0.3304	0.5484	0.4261	0.6302	0.5472	0.0361	-0.1460	-0.2894
c_5	0.2422	0.2734	0.3255	0.3511	0.3983	0.4965	0.5651	0.6001
d_1	-0.0627	0.0023	-0.0244	-0.1281	-0.0306	-0.1763	-0.2382	-0.1266
d_2	0.3455	-0.0090	0.1053	0.5552	0.2704	0.8937	1.1985	0.7532
d_3	-0.7061	-0.0043	-0.1146	-0.7226	-0.5251	-1.4319	-1.9449	-1.4183
d_4	0.5827	-0.0176	-0.0954	0.1764	0.1119	0.5761	0.8987	0.6979
d_5	1.1217	1.1668	1.1545	1.0659	1.0484	0.9476	0.8509	0.8365

Table 4. FOPID parameter constants $\alpha = 1.00$ with target $M_s = 1.40$.

b	0, 25	0, 50	0, 75	1.00	1, 25	1, 50	1, 75	2.00
a_1	-0.0782	0.0403	0.0824	0.1382	0.3746	0.1845	0.0298	-0.0491
a_2	0.4330	0.0539	-0.1391	-0.4090	-1.5169	-0.9012	-0.3486	0.2438
a_3	0.8360	1.0435	1.2470	1.7757	4.4568	3.5317	2.5213	0.0032
a_4	0.2912	0.8237	1.5533	3.0737	9.9995	9.9999	8.8950	0.0189
k_1	0.1177	-0.0339	-0.1224	-0.1300	-0.0897	-0.2128	-0.0830	0.0417
k_2	-0.5244	0.1537	0.5262	0.5734	0.5489	0.9933	0.4861	0.0022
k_3	0.6747	-0.2377	-0.7016	-0.7614	-0.9578	-1.4856	-0.9104	-0.3854
k_4	0.1600	0.4181	0.4883	0.4241	0.5723	0.8113	0.6683	0.4917
k_5	1.9695	1.8389	1.7678	1.7142	1.6243	1.5236	1.4434	1.4079
c_1	-0.0064	-0.0094	0.0072	0.0166	-0.0354	0.0816	-0.0155	-0.1813
c_2	0.0121	0.0473	-0.0180	-0.0564	0.0577	-0.3729	0.0173	0.6916
c_3	0.0038	-0.1062	-0.0432	-0.0246	-0.0392	0.4385	-0.0614	-0.8954
c_4	0.3746	0.4366	0.4116	0.4427	0.4175	0.2476	0.4479	0.8246
c_5	0.2746	0.3341	0.3816	0.4130	0.4565	0.5147	0.5475	0.5395
d_1	-0.0399	-0.0524	-0.0706	-0.1004	-0.0864	-0.1482	-0.1224	-0.0975
d_2	0.2250	0.2412	0.3117	0.4464	0.4559	0.7455	0.6349	0.5715
d_3	-0.5114	-0.3921	-0.4471	-0.6243	-0.7509	-1.2123	-1.0767	-1.1057
d_4	0.5150	0.2321	0.1600	0.1744	0.2593	0.5240	0.4718	0.5751
d_5	1.1229	1.1363	1.1195	1.1011	1.0554	0.9792	0.9511	0.8864

Figures (3), (4), (5), and (6), shown arbitrary examples of how the proposed equations fit to the optimal parameters.

4. EXAMPLES

4.1 Example 1.

In Luyben (2000) analysis of the IRSOPDT process, the process was characterized with a high value of dead time and a ratio between time constants $\alpha = 1.00$. However, for the purpose of this analysis, the range of α specified in (15) is considered, and the performance and robustness obtained for the PID and FOPID controllers are analyzed with $M_s = 1.40$ as the desired robustness.
The inverse response model $P_1(s)$ is defined by the following expression.

$$P_1(s) = \frac{(-1.6s+1)e^{-1.6s}}{(s+1)(\alpha s+1)} \tag{24}$$

In table (5), the integral values of the absolute error for the set-point tracking and load-disturbance rejection tasks are presented. In each case, the analysis is performed for a step setpoint response of magnitude 1 at $t = 0$ and a negative step disturbance change of 50% of the set-point at $t = 40s$. As demonstrated, the IAE values decrease with an increasing time constant ratio in both modes of control. However, it is clear that there is an improvement when implementing the fractional order controller for both, servo-control and regulatory-control in all cases. With regard to robustness, both the PID and FOPID IRM-RoT controllers achieve an optimal desired robustness value at the target value as shown.

Table 5. Performance indices to compare PID and FOPID IRMRoT for Example 1.

α	PID $M_s = 1.40$			FOPID $M_s = 1.40$		
	J_{sp}	J_{ld}	M_s	J_{sp}	J_{ld}	M_s
0.10	9.2693	5.2648	1.3988	8.8119	4.8020	1.3868
0.25	9.0961	5.1527	1.4035	8.8341	4.7755	1.4088
0.50	9.0393	5.0593	1.3975	8.7166	4.8101	1.3962
0.75	8.8563	4.9097	1.3997	8.4116	4.5590	1.4120
1.00	8.6617	4.7631	1.4023	8.3779	4.5084	1.4037

Table 6. Improvement index values for the Example 1.

α	0.10	0.25	0.50	0.75	1.00
Υ_{sp} (%)	4.935	2.880	3.570	5.021	3.279
Υ_{ld} (%)	8.790	7.320	4.926	7.143	5.347
Υ_{erd} (%)	6.331	4.486	4.056	5.778	4.011

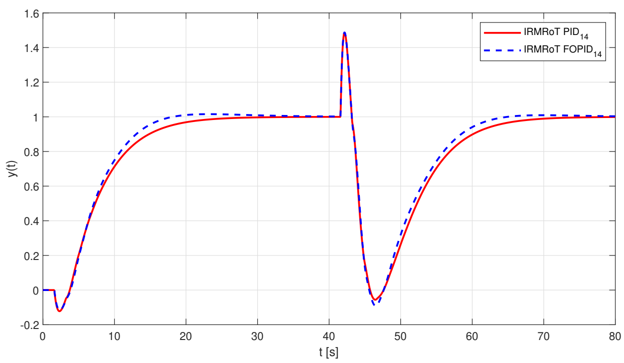

Figure 7. Set-Point and Load-Disturbance response for $P_1(s)$ $\alpha = 0.75$ process example.

Fig. (7) presented the response of the close-loop control system under the conditions described in the $\alpha = 0.75$ case. The controller parameters for this case are $K_p = 0.1986$, $T_i = 1.7582s$, $T_d = 0.6571s$ for the PID and $K_p = 0.1767$, $T_i = 1.4030s$, $T_d = 1.040s$, $\mu = 0.68848$ for FOPID.

As anticipated, the response of the system improves with the fractional order controller, as shown in the graphical analysis and confirmed in 6, where the improvement indices are shown, even obtaining a $\Upsilon_{erd} = 6.331\%$ improvement with respect to the PID controller, considering the errors of both operating modes simultaneously.

4.2 Example 2.

Another example is proposed to evaluate the performance of IRM-RoT controllers with respect to the tuning rule proposed in Kaya and Cengiz (2017). In this instance, the controlled process model of (25) is proposed, where interpolated parameters of the IRM-RoT rule are employed.

$$P_2(s) = \frac{(-0.8s+1)e^{-0.6s}}{(2s+1)^2} \tag{25}$$

For this model, the IRM-RoT PID controller parameters are $K_p = 0.5407$, $T_i = 3.2160s$, $T_d = 1.1020s$ and $K_p = 0.6222$, $T_i = 3.6979s$, $T_d = 1.1139s$, $\mu = 1.1298$ for the FOPID, $K_p = 1.2160$, $T_i = 3.9983s$, $T_d = 1.2262s$ in the case of the Kaya-Cengiz PID.

Table 7. Performance and robustness values evaluation for the Example 2.

Tuning Rule	**Metric Index**			
	J_{sp}	J_{ld}	J_{rd}	M_s
IRM-RoT PID	6.8864	6.7380	13.6244	1.4037
IRM-RoT FOPID	6.3828	6.4535	12.8363	1.4033
Kaya-Cengiz PID	10.0112	6.8361	16.8473	4.2262

Table (7) presents the performance and robustness metrics for the diferent tuning rules and Fig. (8) shows the closed-loop system response implementing the three controllers.

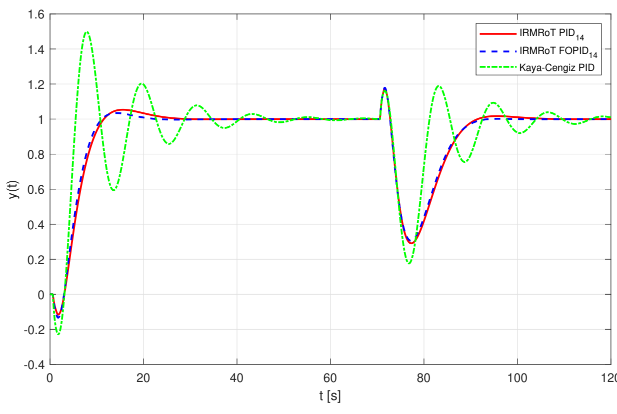

Figure 8. Set-Point and Load-Disturbance step response for $P_2(s)$ process example.

The implementation of IRM-RoT controllers in both control modes shows a significant enhancement of system performance compared to Kaya-Cengiz tuning, as demonstrated in Table (8).

Table 8. Improvement index with Kaya-Cengiz PID as base controller.

Tuning Rule	Υ_{sp} (%)	Υ_{ld} (%)	Υ_{erd} (%)
IRM-RoT PID	31.213	1.435	19.130
IRM-RoT FOPID	36.243	5.597	23.808

Furthermore, when implementing the Kaya-Cengiz rule for IRSOPDT models, it is restricted to using only the time constants ratio $\alpha = 1$. This results in the rule being highly impractical for inverse response processes, unlike what is proposed by IRM-RoT rule.

5. CONCLUSIONS AND FUTURE WORK

The present work proposes the IRM-RoT rule for PID and FOPID controllers, considering a trade-off between the performance and robustness of the control system by implementing an IRSOPDT model to determine the optimal parameters. The study validates the enhancement obtained by implementing a IRM-RoT controller for inverse-response process control by means of concrete examples. Given the complexity of controlling processes with inverse response, controller parameters are obtained by fitting equations with either 4 or 5 constants. This results in a substantial amount of tables needed to implement the rule. Therefore, future work proposes to develop an auto-tuning tool for the IRM-RoT rule.

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