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Study of the feasibility regions for a kind of event-based PI controllers ⋆

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Abstract: In this work we analyze the stability regions for PI controllers under a Symmetric-Send-On-Delta sampling strategy. This event-based sampling provides a significant reduction in the data needed for control and certain immunity against noisy signals without affecting significantly the performance. Hitherto, most of the analysis of this kind of event-triggered strategies to determine the apparition of limit cycles are based on the Describing Function technique, which may provide misleading results. Instead, in this work the analysis is based on the Tsypkin's margin, which provides accurate results. Through this robustness measure, the stability regions for PI controllers are obtained as a function of its parameters. Some well-known tuning rules are also evaluated to determine its stability.

Keywords: Event-based PID control, PID tuning and automatic tuning methodologies, Nonlinear PID control.

1. INTRODUCTION

Nowadays, Event Based Control (EBC) is becoming more and more popular as an alternative to classical periodical time driven control systems. A wide variety of this kind of event-triggered control systems have been proposed in the literature since they provide a reduction in the measurement frequency of certain signals without degrading the closed loop performance. In interconnected systems where the usage time of shared resources is critical, for instance, the communication channel in networked control systems or in wireless communicated systems, being able to relieve the charge on this resources is crucial. For example, in (Feeney and Nilsson, 2001) a wireless networked system was studied and it was proven that the reduction in the data flow provided benefits in terms of power consumption, which can lead to an increase of the lifetime of batteries. Because of those benefits, EBC stands out as a promising control approach for networked control systems, being its importance recognized in (Dotoli et al., 2019). An updated and extensive study about the main contributions to EBC during the last twenty years can be found in (Aranda-Escolástico et al., 2020) and references therein.

Several control techniques have been adapted to perform a correct EBC, among them the PID algorithm stays as one of the most relevant techniques. According to a study conducted in (O'Dwyer, 2006), PID control is among the most spread control techniques across the industry, being implemented in more than 95% of the controllers. Similar results were observed in a survey to the members of the industrial committee of IFAC presented in (Samad, 2017).

The data flow in the context of EBC not only depends on the control technique used but also on the event generation mechanism, which should only transmit events when significant changes in the state of the system are produced. Among the different event generation methodologies, the Send-On-Delta (SOD), presented in (Miskowicz, 2006), has proven to reduce considerably the data flow through the network (Dormido et al., 2008; Ploennigs et al., 2010). A variation of the SOD called Symmetric-Send-On-Delta (SSOD), was first proposed in (Beschi et al., 2012). The SSOD sampling transmits new data whenever a change greater than the threshold δ is detected, being the quantized magnitude a multiple of δ. In (Pawlowski et al., 2016), a control loop with a SSOD was used to regulate a greenhouse production process.

Despite the advantages that this EBC provides, it also presents a characteristic non-linear behavior which may induce limit cycle oscillations in the closed loop response. These sustained oscillations reduce the performance of the control system and accelerate the wear out of actuators, so they must be avoided. Some early works focused on characterizing these limit cycles induced by the SSOD and proposing PID controllers for different applications (Beschi, 2014; Chacón et al., 2013; Ruiz et al., 2017).

In literature, some works address the induction of limit cycle oscillations through the study of the Describing Function (DF) technique (Romero et al., 2014; Miguel-Escrig and Romero-Pérez, 2021). This approach allows using the classical control theory concepts, like the Nyquist plot for the analysis and design of these systems, as well as proposing tuning methodologies based on robustness criteria (Romero-Pérez and Llopis, 2016; Romero-Pérez and Llopis, 2017; Miguel-Escrig and Romero-Pérez, 2021). Thanks to the DF characterization, authors have developed and characterized some variants of the SSOD sam-

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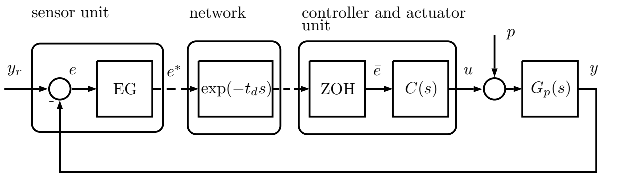


Fig. 1. Networked control system with SSOD sampling strategy in a SSOD-$C(s)$ architecture.

pler, such as the presented in (Romero-Pérez and Llopis, 2017; Miguel-Escrig and Romero-Pérez, 2020; Miguel-Escrig et al., 2022), assessing their effect of the closed loop response.

However, the DF technique presents the limitation of being applicable only to those cases where the linear part of the system is capable of filtering the effect of high order harmonics. This makes the DF analysis not suitable for studying processes modeled by First Order Plus Time Delay (FOPTD) models, widely used in industry to describe the dynamic behavior of actual plants and processes.

To overcome this limitation, in (Miguel-Escrig et al., 2018) a measure called Tsypkin's margin (M_T) was presented. This robustness measure is based on the Tsypkin's method (Tsypkin, 1984), which takes into consideration the effect of the harmonics. This measure was applied in (Miguel-Escrig et al., 2020) to develop a tuning procedure for obtaining robust PID controllers under a SSOD sampling strategy.

Due to the predominance of PI over PID controllers in the industry, in this work, the Tsypkin margin M_T will be used to determine the regions defined by the plane K_p-K_i which conduce to a robust controller according to M_T. The regions will be obtained for FOPTD processes. In addition, several well-known tuning rules will be evaluated to determine their applicability in loops with a SSOD, namely, Ziegler-Nichols (Ziegler and Nichols, 1942), AMIGO (Åström and Hägglund, 2004), One-Third rule (Hägglund, 2019) and SIMC (Skogestad, 2003).

The paper is distributed as follows. In Section 2 the problem to be addressed is deeply presented. In Section 3 the approach followed to determine the regions $K_p - K_i$ and its robustness is explained. The obtained results and its discussion is presented in Section 4. The performance in terms of robustness of the evaluated classical methods is presented in Section 5. Finally, in Section 6, the conclusions about this work are drawn.

2. PROBLEM STATEMENT

Consider the networked control system shown in Figure 1. This control loop presents a sensor unit modeled by the reference tracking and the event generation block EG, which for this work is a SSOD. The network effect is modeled by a delay $exp(-t_d s)$ representing the worst case latency scenario, the controller and actuator unit is modeled by a zero-order hold ZOH and the controller $C(s)$ and the process to control is modeled by its transfer function $G_p(s)$. The reference signal y_r, the quantized error $\bar{e}$, the control action u and the system output y are also presented as well as the disturbance input signal p.

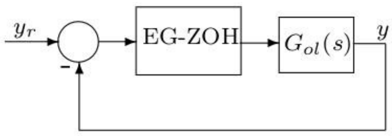


Fig. 2. Hammerstein-Wiener condensation for the analysis of the schema presented in Figure 1.

To evaluate the effect of the elements in the loop on the robustness, the schema presented in Figure 1 has been rearranged since it admits a Hammerstein-Wiener representation, resulting in the schema presented in Figure 2. In this figure, $G_{ol}(s) = C(s)G_p(s)e^{-t_d s}$ represents the open-loop transfer function and gathers all the linear elements: controller, process and network delay. On the other hand, the block EG-ZOH contains the functionality of the SSOD quantization and the ZOH. This representation splits the system in a linear and a non-linear part, which allows applying some analysis techniques in the frequency domain.

As aforementioned, since the DF technique is valid only for certain cases, in this study, the robustness measure M_T, called Tsypkin margin, will be used. The margin M_T represents the minimum distance in the Nyquist plane between any point of the open-loop transfer function and its respective Tsypkin Branch. A Tsypkin Branch is a trajectory in the Nyquist plane defined for a given frequency. If a Tsypkin Branch intercepts the open-loop transfer function at the frequency that defines it, a limit cycle oscillation can take place, and the oscillations will have that frequency. In this case where the intersection takes place, a measure $M_T = 0$ is obtained.

In (Miguel-Escrig et al., 2020) it was concluded that the M_T margin for a FOPTD model that approximates a high order process is lower than the value of M_T for the high order process model. That is to say that assuring a value of M_T for the FOPTD approximation of a high order system will assure, at least, the same or a higher robustness for the original model.

Other works like (Sánchez et al., 2020) suggested an approach which consider a limited amount of harmonics as initial point of departure for evaluating the robustness of the controllers. Nevertheless, as the amount of harmonics was not sufficient, simulations had to be run to validate the obtained controllers.

Hence, in this work the feasibility regions in terms of robustness for a PI controller, determined in the plane K_p-K_i, for FOPTD models will be evaluated using the robustness measure M_T. Therefore, the obtained feasibility regions can be safely applied to evaluate high order processes approximated by these FOPTD models, which will offer at least the same amount of robustness.

3. ANALYSIS STUDY

As explained in the previous section, the study will be focused on FOPTD models since the obtained results can be extended to higher order processes. Thus, the models that will be evaluated will present the following structure:

$$G_p(s) = \frac{K\, e^{-Ls}}{1 + Ts}.$$

Consider that the network delay $e^{-t_d s}$ is included within the delay of the process model G_p and that the system is regulated by a PI controller:

$$C(s) = K_p + \frac{K_i}{s}.$$

Note that by considering the parallel form, I controllers are also included in the study.

The margin M_T admits a dimensionless analysis (Miguel-Escrig et al., 2020), i.e. the same value of M_T will be obtained for a given system and for its equivalent dimensionless process. This simplifies the analysis by reducing the number of cases to evaluate, defined by the combinations of parameters of process and controller. The parameters to evaluate following the dimensionless approach are KK_p and KK_iL, and the obtained results will hold for different processes as long as the same ratios KK_p and KK_iL are maintained. For this study an unitary gain K and time constant T have been considered.

The different dynamics of FOPTD processes can be gathered through a single normalized parameter:

$$\tau = \frac{L}{L+T}.$$

For small values of τ almost first order systems are found. On the other hand, for values of τ close to 1 we find pure delay systems. In this analysis study a range of $\tau \in [0.01, 1[$ has been considered. A small delay has been kept to model the network behavior.

Regarding to the evaluated controller's parameters' ranges it has been considered $K_p \in [0, 1/\gamma_{cp}[$ where γ_{cp} is the gain margin of $G_p(s)e^{-t_d s}$ and $K_i \in [0, 1/\gamma'_{cp}[$ where γ'_{cp} is the gain margin of $G_p(s)e^{-t_d s}/s$. These ranges for K_p and K_i, which are different for each process model, conform a grid that includes P, I and PI controllers.

4. RESULTS

Using the previously described set up, the boundaries of the region which assures robustness against limit cycle oscillations has been obtained, i.e. the boundaries of the region that assures $M_T > 0$. These boundaries are presented in Figure 3. In these figures the region is presented from two points of view and with linear and logarithmic KK_p axis to increase the visibility for low values of τ.

From these figures it can be seen that for lower values of τ, i.e. for processes with low influence of the delay with regard to its dynamic, the regions of admissible values for the controller parameters are more spread than for delay dominant processes.

A detailed view of the feasibility region is given in Figures in 4 for several values of τ. These figures are flattened extracts from the regions presented in Figure 3a and can be used to evaluate the robustness of a given controller. In this figures it can be appreciated that the range of the parameter KK_p is reduced as τ increases but the maximum KK_iL is maintained.

In Figure 5 it has been presented the contour of the regions that assure different degrees of robustness according to M_T for three processes defined by τ. Figure 5a presents the robustness regions for a process whose delay is negligible with regard to its dynamics. In Figure 5b a balanced process, whose delay is comparable to its dynamics is presented. And in Figure 5c a process where the delay dominates the dynamic is presented.

In all those cases the resultant regions reduce their size with the increase of M_T and are subsets of the regions with lower M_T. The resultant areas tend to group towards the origin of the presented axis, meaning that for a given process, a reduction in K_p and/or K_i must be performed to reach higher values of M_T.

To provide a comparison between Tsypkin and DF based analysis methods, in Figure 6, the inverse negative of the DF traces for a SSOD sampler, in red, have been presented in the Nyquist diagram together with the open-loop transfer function of the processes and controllers placed at the limit of stability, i.e. the closest stable cases to the $M_T = 0$ borderline. In this figure it can be seen that some processes intersect the traces of the inverse negative of the DF and others are placed far from these traces. However, according to the robustness measure M_T, they are all close to present limit cycle oscillations, which remarks the limited applicability of DF's theory for this kind of systems.

The presented regions can be used to asses the robustness against limit cycle oscillations induced by the SSOD of a given controller, specially using Figures in 4. This usage of the regions is detailed in the following example.

Example 1. Consider a FOPTD model with $\tau = 0.1$ whose transfer function is defined by:

$$G(s) = \frac{e^{-0.1s}}{1+0.9s},$$

and two PI controllers whose transfer function is given by:

$$C_1(s) = 9.2 + \frac{5}{s}, \qquad C_2(s) = 10 + \frac{5}{s}.$$

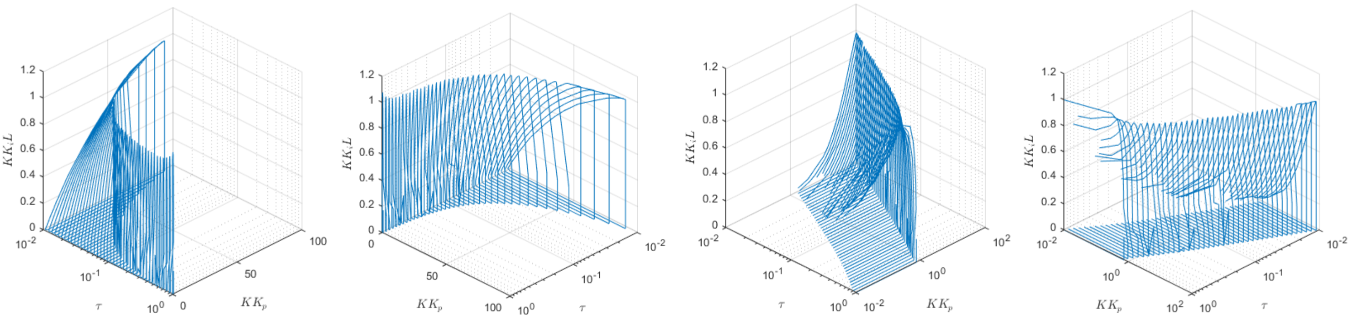
The feasibility region for FOPTD models with $\tau = 0.1$ is presented in Figure 7, in which the controllers C_1 and C_2 have been marked with a red dot.

As it can be seen, controller C_1 is placed inside the stable region ($M_T \geq 0$), but controller C_2 is placed outside, therefore, the apparition of limit cycle oscillations is expected only for the controller C_2 case.

To visualize both cases, the analysis using Tsypkin's method has been performed. In Figure 8 it has been presented each of the open-loop transfer functions together with their critical Tsypkin branch, i.e. the trajectory that defines M_T.

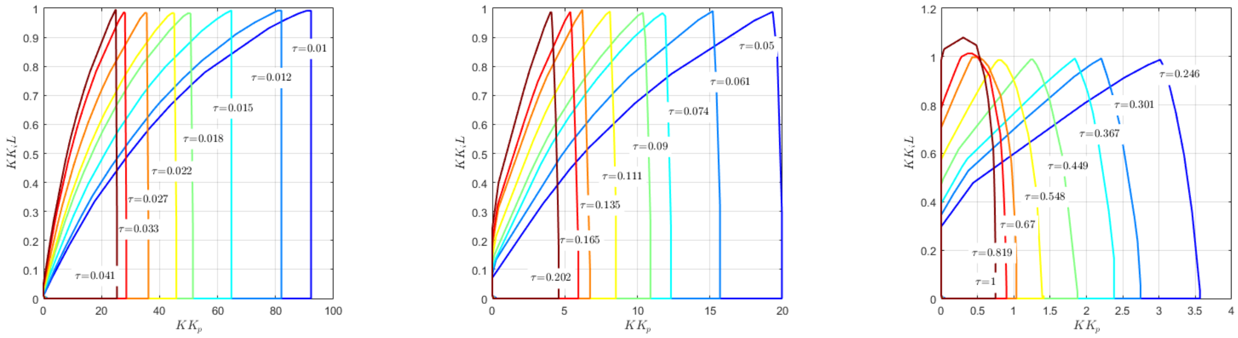
Despite the similar shape of both open-loop transfer functions, which present similar gain, phase and sensitivity margins, the usage of the measure M_T reveals that for GC_1 we have $M_T = 0.01 > 0$, avoiding limit cycle oscillations, and for GC_2 we obtain $M_T = 0$, presenting oscillations.

Both controllers have been tested in simulation. In Figure 9 the closed-loop temporal response for both cases to a unitary step change at the reference input and a 2δ step change at the disturbance input at $t = 10$s is presented. As it can be seen, both systems manage to stabilize the response when the reference change is produced, however,



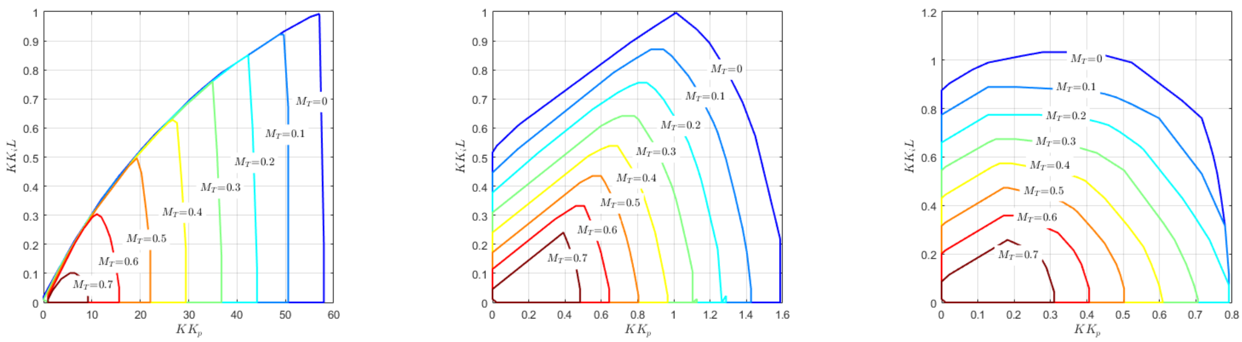
(a) Linear KK_p axis. (b) Logarithmic KK_p axis.

Fig. 3. Robust regions ($M_T > 0$) for PI controllers with the normalized axis.



(a) $\tau \in [0.045, 0.01]$ (b) $\tau \in [0.05, 0.202]$ (c) $\tau \in [0.246, 1]$

Fig. 4. Feasibility regions, ($M_T > 0$) for different values of τ.



(a) $\tau = 0.017$ (b) $\tau = 0.498$ (c) $\tau = 0.89$

Fig. 5. Robustness within the feasible regions for different values of τ.

a limit cycle is induced by the SSOD sampler when controller C_2 is used after the disturbance change.

5. EVALUATION OF CONTINUOUS TUNING RULES

A common practice in this type of control structures is to tune the controller according to continuous tuning rules, usually those that are well settled and provide a reasonable amount of robustness regarding to classical margins. In this section several tuning rules will be evaluated using the previously presented regions, namely, Ziegler-Nichols (Ziegler and Nichols, 1942), AMIGO (Åström and Hägglund, 2004), One-Third rule (Hägglund, 2019) and SIMC (Skogestad, 2003). These design methods are well-known and its applicability has been proven to control numerous linear processes.

The parameters K_p and K_i have been calculated for the previous tuning rules for the studied range of τ. Each of them is presented as a line in Figure 10, Ziegler-Nichols in blue, AMIGO in green, One-Third in red and SIMC in orange. As the influence of the delay is lower on the studied process, i.e. the parameter τ becomes lower, some tuning rules provide values of K_p and K_i outside the feasible region, being One-Third tuning rule the unique method that provides robust controllers in all the evaluated range. For the other methods the application range has been found. Ziegler-Nichols can be applied safely for processes with $\tau \geq 0.25$, AMIGO for $\tau \geq 0.098$ and SIMC for $\tau \geq 0.087$.

Despite the ranges of application for the different tuning rules, it is worth remarking that none of them has been

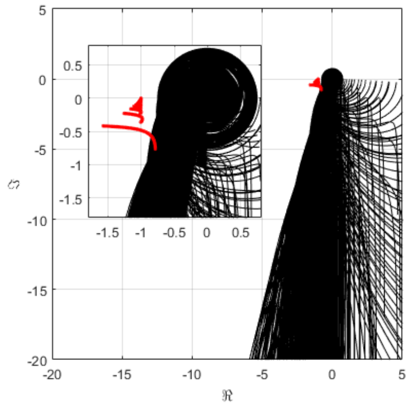


Fig. 6. Representation in the Nyquist diagram of $G_{ol}(j\omega)$ for all the limiting cases.

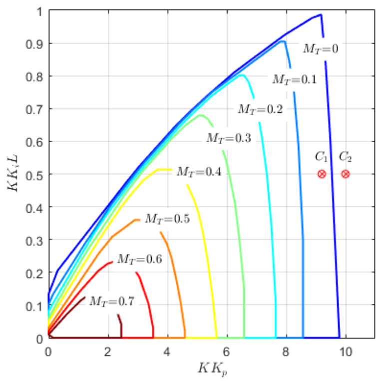


Fig. 7. Feasibility region for $\tau = 0.1$ and placement of controllers C_1 and C_2 within it.

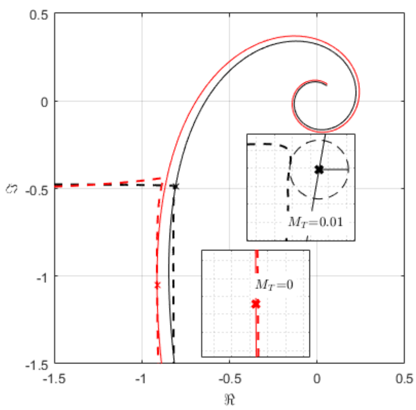


Fig. 8. Nyquist diagram of GC_1 (solid black line) and GC_2 (solid red line) with its respective critical Tsypkin branches (dashed lines) to which M_T is calculated.

conceived to be applied in this kind of control loop, and therefore, their robustness in terms of M_T varies in all the evaluated range. Hence, to contribute with more information about the robustness that these methods provide, M_T has been calculated. The results have been presented in Figure 11, where it can be seen the variation in M_T when applying the studied methods. As it can be seen One-Third tuning rule provide robust controllers in

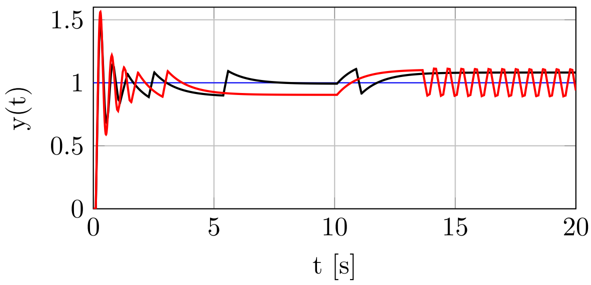


Fig. 9. Closed loop temporal response of GC_1 (black) and GC_2 (red) to step change at the reference input (blue) and a 2δ step disturbance at $t = 10$s.

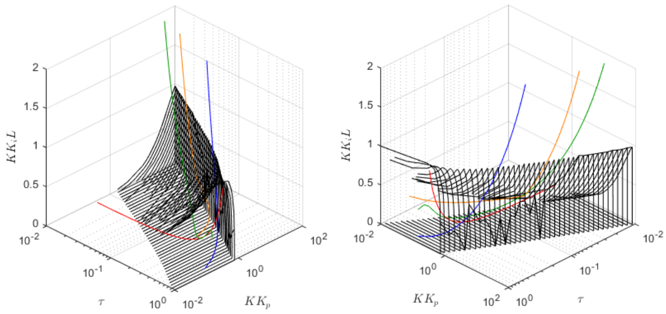


Fig. 10. Feasible region with Ziegler-Nichols (blue), AMIGO (green), One-Third (red) and SIMC (orange) tuning rules.

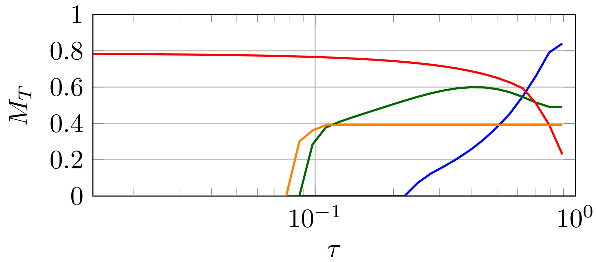


Fig. 11. M_T for the studied tuning rules: Ziegler-Nichols (blue), AMIGO (green), One-Third (red) and SIMC (orange).

terms of M_T for all the studied range of τ, providing the higher level of robustness among the methods for most of the range. The other methods present a critical value of τ until which $M_T = 0$, from that critical value of τ the methods provide robust controllers in terms of M_T. SIMC tuning rule presents a very peculiar behavior, which is the stabilization in a given level of robustness $M_T = 0.4$, which is due to the characteristics of the tuning method and the dimensionless properties of M_T.

6. CONCLUSIONS

In this work the feasible regions for PI controllers applied to loops under SSOD sampling have been presented. The regions have been evaluated for First Order Plus Time Delay models using the robustness margin M_T, derived from the Tsypkin method, which takes into account the contribution of high order harmonics in the apparition of limit cycle oscillations unlike other methods based on the Describing Function approach.

It has been observed that the regions, defined by the dimensionless parameters KK_p and KK_iL, are more extensive for processes with low influence of the delay, and narrower for delay dominated processes. This difference in the extension is provided mostly by the variability that KK_p can admit, as KK_iL remains unaltered in all the evaluated range.

In addition, several tuning rules have been tested, namely Ziegler-Nichols, AMIGO, One-Third and SIMC. The study reveals that only One-Third tuning rule can be applied safely through all the evaluated range, providing different levels of robustness. The other tuning rules present a critical value of τ, which defines the dynamic of a FOPTD process, from which they can be applied safely. Below that critical value, their temporal responses can present limit cycle oscillations.

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