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Data-Driven Robust Servo Tuning Method Using Fractional-Order PID Controller

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Abstract: This paper proposes a data-driven method using a Fractional-Order Proportional-Integral-Derivative (FOPID) controller. The proposed method simultaneously obtains FOPID controller and reference model parameters to achieve tracking performance and specified robust stability from only one-shot closed-loop input-output data. The proposed control law is designed by solving an optimization problem, subject to the constraint condition of using the maximum value of the sensitivity function. Therefore, the proposed method provides trade-off design between tracking performance for the reference input and robust stability by selecting robust stability. By comparing numerical example results obtained for FOPID and integer-order controllers, it is shown that the use of the FOPID controller is effective in improving tracking performance for reference output.

Keywords: Data-driven control, Fractional-order PID controller, Trade-off, Robust stability, Extended fictious reference iterative tuning

1. INTRODUCTION

Proportional-Integral-Derivative (PID) control is widely used in industry (Alfaro and Vilanova (2016)) because of its simple control structure and high control performance depending on the design. The control performance of PID control depends on the PID parameters, and many tuning methods for PID parameters have been proposed (Åström and Hägglund (2001)). Fractional-Order Proportional-Integral-Derivative (FOPID) control has attracted much attention in recent years (Padula and Visioli (2011); Arrieta et al. (2015); Yamashita et al. (2023); Tepljakov et al. (2018, 2021); Arrieta et al. (2023)). FOPID controllers provide flexibility in controller design by increasing the number of design parameters to five. Thereby, It enables to obtain frequency characteristics that could not be expressed when using an integer-order controller (Yamashita et al. (2023)), and to design of controllers that are resistant to nonlinear elements (Odai and Hori (2000)).

There are two main methods of controller design: model-based method and data-driven method. The model-based method requires a mathematical model of the plant model, so if a mathematical model is not available, it must be obtained. This requires repeated experiments, which are costly and time-consuming. In addition, modeling errors are caused. In the data-driven method, control performance can be optimized directly from controlled data without a mathematical model of the plant model (Sato et al., 2021; Sakai et al., 2022, 2023). Therefore, modeling errors have no effect. The control performance can be optimized directly from controlled data such as iterative feedback tuning (IFT) (Hjalmarsson et al. (1998)) and correlation-based tuning (CbT) (Karimi et al. (2004)), but these require iterative experimental data. Non-iterative data-driven tuning methods such as Virtual Reference Feedback Tuning (VRFT) (Campi et al. (2002); Previdi et al. (2004)), Fictitious Reference Iterative Tuning (FRIT) (Soma et al. (2004); Kaneko (2013)), and Non-iterative Correlation based Tuning (NCbT) (Karimi et al. (2007); Yubai et al. (2009)) have been proposed. In addition, extension method such as Extended Fictious Reference Iterative Tuning (E-FRIT) (Tasaka et al. (2009); Kano et al. (2011)), which simultaneously optimizes the controller and reference model parameters, has also been proposed. Furthermore, robust PID controller design methods using sensitivity function has been proposed for model-based methods (Kurokawa et al. (2017, 2018b)) and data-driven methods (Kurokawa et al. (2018a, 2022)), respectively.

In conventional FOPID controller design methods, model-based methods that consider robust stability (Padula and Visioli (2011); Arrieta et al. (2015)) have been proposed. On the other hand, in data-driven method, although the control performance optimization method (Yamashita et al. (2023)) has been proposed, there is no method that takes robust stability into account. Therefore, the present study proposes the method to obtain FOPID controller and reference model parameters using data-driven method under the constraint condition that satisfies the prescribed stability margin. In the proposed method, the FOPID controller parameters are determined to optimize the tracking performance for reference output.

The structure of this paper is as follows. First, Formulate the problem considered in this study in Section 2. Then, a data-driven constrained optimization problem is designed in Section 3. Finally, the effectiveness of the proposed method is demonstrated through numerical example in Section 4.

2. PROBLEM FORMULATION

2.1 Control System

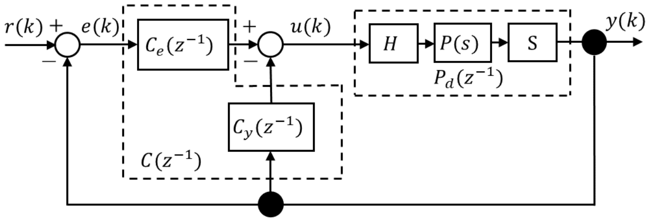


Fig. 1. Block diagram of control system

Consider the block diagram shown in Fig. 1, where $r(k)$ is a reference input, $y(k)$ is a plant output, $u(k)$ is a control input, $e(k)(= r(k) - y(k))$ is the control error, and $P(s)$ is a linear time-invariant controlled plant. The discrete-time signal applied to $P(s)$ is converted to a continuous-time signal by the zeroth-order holder H, and the output continuous-time is converted to a discrete signal by the sampler S. Thus, $P_d(z^{-1})$ is a linear time-invariant discrete-time controlled plant where the dynamics are unknown. In the present study, the following discrete-time control law:

$$u(k) = C_e(z^{-1})e(k) - C_y(z^{-1})y(k) \tag{1}$$

$$C_e(s) = K_p + K_i s^{-\alpha_i} \tag{2}$$

$$C_y(s) = K_d s^{\alpha_d} \tag{3}$$

$$C(z^{-1}) = C_e(z^{-1}) + C_y(z^{-1}) \tag{4}$$

where $C_e(s)$ and $C_y(s)$ are continuous-time controllers and discretize with sampling time T_s, which is discussed later. In addition, α_i is the fractional-order of the integrator and α_d is the fractional-order of the differentiator, both positive, and $\alpha_i = \alpha_d = 1$ is identical to integer-order PID controller. Moreover, K_p, K_i and K_d, are the proportional gain, the integral gain, and the differential gain, respectively. Furthermore, in the present study, the FOPID controller is discretized with the sampling time T_s using the method for approximation to discrete integer order in Tepljakov et al. (2014). The method is given as follows. First, we define the approximation frequency range $[\omega_b, \omega_h]$ rad/s, the order of approximation $\nu \in \mathbb{Z}^+$ and the fractional-order $\alpha \in [-1, 1] \subset \mathbb{R}$. Then, the $(2\nu + 1)$ zeros and $(2\nu + 1)$ poles of the filter are computed as follows:

$$\omega'_k = \omega_b \theta^{\frac{k+\nu+0.5-0.5\alpha}{2\nu+1}}, \quad \omega_k = \omega_b \theta^{\frac{k+\nu+0.5+0.5\alpha}{2\nu+1}} \tag{5}$$

where, $k = \{-\nu, -\nu+1, \ldots, 0, \ldots, \nu-1, \nu\}$ and $\theta = \omega_h/\omega_b$. So, the continuous recursive Oustaloup filter transfer function is obtained as follows

$$s^\alpha \approx \hat{G}(s) = \omega_h^\alpha \frac{(s - \omega'_{-\nu})(s - \omega'_{-\nu+1}) \cdots (s - \omega'_\nu)}{(s - \omega_{-\nu})(s - \omega_{-\nu+1}) \cdots (s - \omega_\nu)} \tag{6}$$

Next, the pole-zero matching equivalents method is used to obtain a discrete-time transfer function that is equivalent to the continuous-time transfer function (Franklin et al. (1997)). Note that assuming that the sampling time $T_s \in \mathbb{R}^+$, the higher frequency bound of approximation (5) may be up to $\omega_h = 2/T_s$. The poles and zeros are mapping as follows:

$$z = e^{sT_s} \tag{7}$$

where s denotes a particular zero or pole. Thus, for each k in (5), we have

$$\sigma'_k = e^{-T_s\omega'_k}, \quad \sigma_k = e^{-T_s\omega_k} \tag{8}$$

In this way, continuous zeros and poles are directly mapping to their discrete-time equivalents. Then, we need to compute the gain K_u of the resulting discrete-time system at the central frequency $\omega_u = \sqrt{\omega_b\omega_h}$ by taking

$$K_u = \left|H(e^{j\omega_u T_s})\right| \tag{9}$$

The correct gain at this frequency is as follows:

$$K_s = \omega_u^\alpha \tag{10}$$

Hence, the gain of the system is obtained as follows:

$$K_c = \frac{K_s}{K_u} \tag{11}$$

Consequently, the transfer function of the discrete-time system is thus described as follows:

$$H(z, \alpha) = K_c \frac{(z - \sigma'_{-\nu})(z - \sigma'_{-\nu+1}) \cdots (z - \sigma'_\nu)}{(z - \sigma_{-\nu})(z - \sigma_{-\nu+1}) \cdots (z - \sigma_\nu)} \tag{12}$$

Using (12), (2) and (3) are discretized as follows:

$$C_e(z^{-1}) = \begin{cases} K_p + K_i\left(\dfrac{T_s}{1 - z^{-1}}\right)H(z, 1 - \alpha_i) & (0 < \alpha_i < 1) \\ K_p + K_i\left(\dfrac{T_s}{1 - z^{-1}}\right)H^{-1}(z, \alpha_i - 1) & (1 \le \alpha_i) \end{cases}$$

$$C_y(z^{-1}) = \begin{cases} K_d\left(\dfrac{1 - z^{-1}}{T_s}\right)H^{-1}(z, 1 - \alpha_d) & (0 < \alpha_d < 1) \\ K_d\left(\dfrac{1 - z^{-1}}{T_s}\right)H(z, \alpha_d - 1) & (1 \le \alpha_d) \end{cases}$$

where, $C_e(z^{-1})$ for $0 < \alpha_i < 1$ is the structure that ensures the control effect at low frequencies. This ensures the effect of an integer-order integrator at low frequencies and is expected to eliminate steady-state error (Sasano et al. (2010)).

2.2 Constrained Optimization Problem

In the present study, the optimal controller and reference model parameters are obtained to minimize the objective function subject to the stability margin constraint to guarantee robust stability. The stability margin is given a prescribed value by the designer and quantitatively expressed using the maximum value of the sensitivity function:

$$S_f(z^{-1}) = \frac{1}{1 + C(z^{-1})P_d(z^{-1})} \tag{13}$$

$$M_s = \max_\omega |S_f(e^{-j\omega})| \tag{14}$$

$$|M_s - M_s^d| = 0 \tag{15}$$

where M_s^d is the stability margin given by the designer, and the recommended range is 1.4 to 2.0 (Åström and Hägglund (2006)). And, the maximum value of the sensitivity function M_s is related to the gain margin g_m and the phase margin ϕ_m as follows (Kurokawa et al. (2017)) :

$$g_m \geq \frac{M_s}{M_s - 1} \tag{16}$$

$$\phi_m \geq 2\text{arcsin}\left(\frac{1}{2M_s}\right) \tag{17}$$

here g_m and ϕ_m for the values of M_s are for $M_s = 1.4$, $g_m \geq 3.5$ and $\phi_m \geq 41°$, and for $M_s = 2.0$, $g_m \geq 2.0$ and $\phi \geq 28°$ (Kurokawa et al. (2017)). Additionally, there is a trade-off relationship between tracking performance for the reference input and stability margin (Kurokawa et al. (2018a)): smaller values of M_s increase stability margin while decreasing tracking performance for the reference input, and larger values of M_s decrease stability margin while increasing tracking performance for the reference input.

In the present study, the objective function is defined as follows:

$$J = \frac{1}{N}\sum_{k=1}^{N}\epsilon(k)^2 \tag{18}$$

$$\epsilon(k) = (G_{yr}(z^{-1}) - M_d(z^{-1}))r(k)$$

where N is the number of data, $G_{yr}(z^{-1})$ is the transfer function from $r(k)$ to $y(k)$ of the closed-loop system, and $M_d(z^{-1})$ is a reference model from $r(k)$ to $y(k)$. In the present study, $M_d(z^{-1})$ is defined as follows:

$$M_d(s) = \frac{\omega_o^2}{s^2 + 2\omega_o s + \omega_o^2}e^{-L_o s} \tag{19}$$

Consequently, the constrained optimization problem is defined as follows:

$$\min_{K_p, K_i, K_d, \alpha_i, \alpha_d} J \tag{20}$$

$$\text{subject to } \left|M_s - M_s^d\right| = 0$$

The problem considered in the present study is to obtain the controller parameters by solving (20).

3. DATA-DRIVEN DESIGN

3.1 Constraint Condition

Since unknown controlled plant $P_d(z^{-1})$ is required in the constraint condition, the frequency characteristics of $P_d(z^{-1})$ are estimated using the initial input-output data $u_0(k)$ and $y_0(k)$ based on Matsui et al. (2010). In order to Discrete Fourier Transform (DFT) the waveform is absolutely integrable smooth and to eliminate the discontinuity between the two ends of the data, the bandpass filter is used as follows:

$$F(s) = \frac{T_1 s}{(T_1 s + 1)(T_2 s + 1)} \tag{21}$$

$$T_1 = 100T_s$$

$$T_2 = 10T_s$$

The filtered data $u_f(k), y_f(k)$ using the bandpass filter is given as follows:

$$u_f(k) = u_0(k) * f(k) \tag{22}$$

$$y_f(k) = y_0(k) * f(k) \tag{23}$$

where $f(k)$ is the impulse response of the bandpass filter discretized at sampling time T_s. The filtered data $u_f(k)$ and $y_f(k)$ are DFT to obtain $U_f(\omega)$ and $Y_f(\omega)$, which are used to estimate the frequency response of the controlled plant, as follows:

$$\hat{P}(e^{-jw}) = \frac{Y_f(\omega)}{U_f(\omega)} \tag{24}$$

(14) and (15) can be rewritten using the estimated frequency response of the controlled plant as follows:

$$\hat{M}_s = \max_{\omega}|\hat{S}_f(e^{-j\omega})| \tag{25}$$

$$|\hat{M}_s - M_s^d| = 0 \tag{26}$$

$$\hat{S}_f(e^{-j\omega}) = \frac{1}{1 + C(e^{-j\omega})\hat{P}(e^{-j\omega})} \tag{27}$$

3.2 Optimal Tuning of tracking performance using E-FRIT

Obtain the optimal controller parameters from initial data u_0, y_0 using E-FRIT. Using the initial data u_0, y_0, the fictitious reference signal $\tilde{r}(k)$ is defined as follows:

$$\tilde{r}(k) = C_e(z^{-1})^{-1}(u_0(k) + C(z^{-1})y_0(k)) \tag{28}$$

E-FRIT obtains the parameters by minimizing the error between the reference output $M_d(z^{-1})r(k)$ for the fictitious reference signal $\tilde{r}(k)$ and the initial output data y_0. Thus, the new objective function is given by:

$$J^* = \frac{1}{N}\sum_{k=1}^{N}\epsilon^*(k)^2 \tag{29}$$

$$\epsilon^*(k) = y_0(k) - M_d(z^{-1})\tilde{r}(k)$$

The parameters that minimizes (29) correspond to the parameters that minimizes (18) (Soma et al. (2004)). In addition, E-FRIT also obtains the reference model parameters ω_o, L_o simultaneously with the controller parameters by minimizing (29).

As a result, the constrained optimization problem is reformulated as follows:

$$\min_{K_p, K_i, K_d, \alpha_i, \alpha_d, \omega_o, L_o} J^* \tag{30}$$

$$\text{subject to } \left|\hat{M}_s - M_s^d\right| = 0$$

By solving the constrained optimization problem, FOPID controller and reference model parameters are obtained that satisfy the prescribed stability margin and optimize tracking performance for reference output.

4. NUMERICAL EXAMPLE

In the present study, to demonstrate the effectiveness of the proposed method, we compare the FOPID controller with the integer-order PID controller through a numerical example. In this section, we present the tracking performance for the reference output in Subsection 4.1, and the tracking performance for the reference input and robustness of the trade-off design in Subsections 4.2 and 4.3, respectively.

4.1 Tracking Performance for the reference output

The following transfer function as the controlled plant is given:

$$P(s) = \frac{1}{1.1s+1}e^{-0.6s} \tag{31}$$

where the dynamics is assumed to be unknown. The sampling time is $T_s = 0.01$ [sec] and the reference input is $r(k) = 1$. The discrete-time Oustaloup filter sets order of approximation $\nu = 3$ and approximation frequency range $\omega = [10^{-4}, 10^{2}]$ rad/s. The initial controller parameters are determined by trial and error so that the output of the system is stabilized. In order to obtain the initial data u_0 and y_0, set $K_p^0 = 0.05$, $K_i^0 = 0.10$, $K_d^0 = 0.01$, $\alpha_i^0 = 1.00$, and $\alpha_d^0 = 1.00$. The obtained initial data is plotted as dotted lines and the filtered data as solid lines in Fig. 2(a). The frequency response of the controlled plant estimated using the bandpass filter is plotted with the solid lines and the true values with the dotted lines in Fig. 2(b). Fig. 2(b) shows that the gain and phase characteristics of the controlled plant are well estimated. In the present study, optimization calculations are performed using *fmincon* function (MathWorks MATLAB R2023a), and controller parameters are obtained for $M_s^d = 1.4$, 1.6, 1.8, and 2.0. The initial values used in the optimization calculations are shown in Table 1(a), and the search range of parameters are shown in Table 1(b). The optimization calculations for the integer-order PID are performed using the same initial values as for the FOPID by fixing $\alpha_i = \alpha_d = 1$. The optimized controller, reference model parameters for FOPID and integer-order PID controls, respectively, are shown in Table 2. The control results using the optimized controller and reference model parameters for $M_s^d = 1.4$, 1.6, 1.8, and 2.0, respectively, along with the initial data, are shown in Figs. 3(a) and 3(b). The obtained objective function J in (18) and M_s are shown in Table 3(a). It can be seen from Table 3(a) that for all M_s^d, the values of J are smaller with FOPID than with integer-order PID. Therefore, the tracking performance for the reference output is improved by using FOPID controller compared to integer-order PID controller. In addition, M_s is within $\pm 0.2\%$ of the error with respect to M_s^d, so it can be said that each controller has sufficient stability margin desired by the designer.

Table 1. Conditions of Constraint optimization calculation

(a) Initial values used in the optimization calculation

M_s^d	K_p	K_i	K_d	ω_o	L_o	α_i	α_d
1.4	1.000	0.8000	0.2000	2.600	0.6000	1.000	1.100
1.6	1.300	0.9000	0.2500	3.700	0.6000	1.000	1.100
1.8	1.500	1.000	0.3000	4.900	0.6000	1.000	1.100
2.0	1.700	1.100	0.3500	5.800	0.6000	1.000	1.100

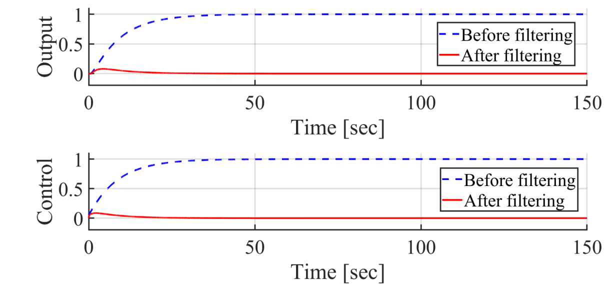
(b) Parameter search range

	K_p	K_i	K_d	ω_o	L_o	α_i	α_d
Upper limit	10	10	10	10	10	2.0	2.0
Lower limit	0.0001	0.0001	0.0001	0.0001	0.0001	0.1	0.1

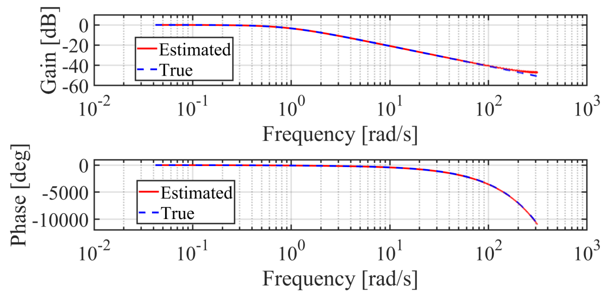
4.2 Tracking Performance for the reference input

The tracking performance for the reference input is evaluated by the following equation:

$$J_T = \frac{1}{N}\sum_{k=1}^{N}(r(k) - y(k))^2 \tag{32}$$



(a) Closed-loop experimental data and filtered data



(b) Estimated frequency response

Fig. 2. Estimated frequency response of controlled plant

Table 2. Obtained controller and reference model parameters

(a) Fractional-order PID

M_s^d	K_p	K_i	K_d	ω_o	L_o	α_i	α_d
1.4	1.000	0.7912	0.1998	2.605	0.5967	0.9722	1.096
1.6	1.355	0.9118	0.2559	3.773	0.5814	0.9953	1.102
1.8	1.497	0.9957	0.3017	4.918	0.6251	0.9851	1.103
2.0	1.707	1.136	0.3491	5.746	0.6000	0.9719	1.097

(b) Integer-order PID

M_s^d	K_p	K_i	K_d	ω_o	L_o
1.4	1.019	0.6969	0.1966	2.492	0.6034
1.6	1.346	1.014	0.2538	3.610	0.4435
1.8	1.525	0.9583	0.3746	4.455	0.6462
2.0	1.809	1.052	0.3439	5.853	0.6164

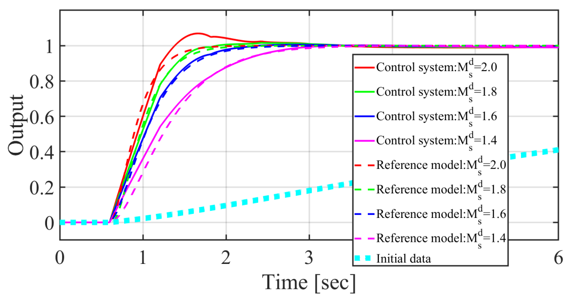
The obtained tracking performance for the reference input results are shown in Table 3(b). Table 3(b) shows that the larger the value of M_s^d, the smaller the value of J_T, which indicates that the tracking performance for the reference input is higher.

4.3 Verification of robust stability

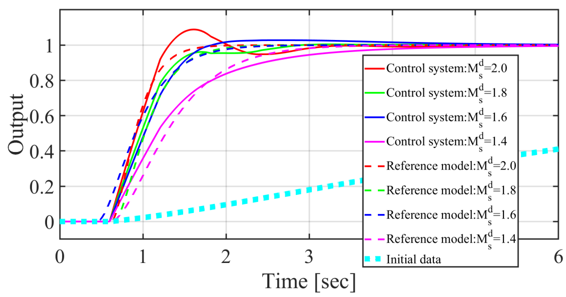
To verify robust stability, consider the situation where the plant dynamics changes to $P'(s)$ after 75 [sec].

$$P'(s) = \frac{1.5}{0.72s+1}e^{-0.7s} \tag{33}$$

The simulation results using the optimized FOPID controller parameters shown in Table 2(a) are shown in Fig. 4(a). It can be seen from Fig. 4(a) that the output diverges with $M_s^d = 2.0$. Fig. 4(b) shows an enlarged view of Fig. 4(a) excluding the output with $M_s^d = 2.0$ and the initial data for better visibility. Fig. 4(b) shows that the smaller the value of M_s^d, the smaller the effect of the plant dynamics change, i.e., the higher the robust stability. Therefore,



(a) Fractional-order PID control



(b) Integer-order PID control

Fig. 3. Obtained plant outputs and reference outputs

Table 3. Obtained control results

(a) Objective function J and M_s

	Fractional-order PID		Integer-order PID	
M_s^d	J	M_s	J	M_s
1.4	3.043×10^{-5}	1.401	5.800×10^{-5}	1.400
1.6	5.063×10^{-6}	1.602	7.119×10^{-5}	1.600
1.8	1.180×10^{-5}	1.803	4.092×10^{-5}	1.800
2.0	3.865×10^{-5}	2.004	5.514×10^{-5}	2.000

(b) Tracking performance for the reference input J_T

M_s^d	Fractional-order PID	Integer-order PID
1.4	6.709×10^{-3}	6.825×10^{-3}
1.6	5.962×10^{-3}	5.931×10^{-3}
1.8	5.749×10^{-3}	5.741×10^{-3}
2.0	5.527×10^{-3}	5.477×10^{-3}

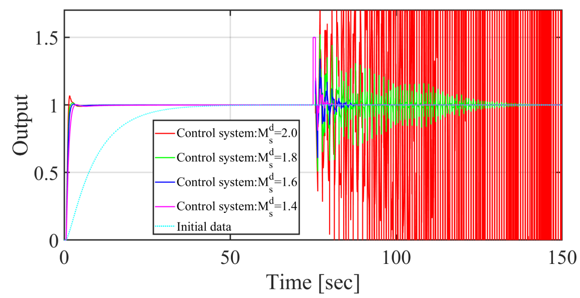
the trade-off design between tracking performance for the reference input and robust stability is confirmed.

4.4 Discussion

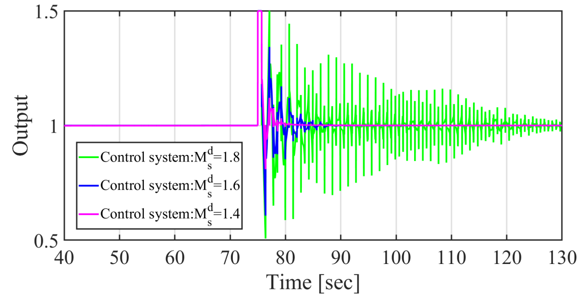
The values of J in Table 3(a) show that for $M_s^d = 1.6$, the FOPID control well improves the tracking performance for the reference output, but it is less pronounced for the other M_s^d. Additionally, the method solves a nonlinear optimization problem, so the obtained solution is a local optimal solution, which is affected by the initial values. Therefore, it is necessary to consider a control law that reduces these effects as a future study.

5. CONCLUSIONS

In this paper, a data-driven tracking performance for reference output optimization method is proposed using a FOPID controller. In the proposed method, the parameters of the FOPID controller and reference model are obtained by minimizing the objective function of E-FRIT



(a) Outputs for plant dynamics changes after 75 [sec]



(b) Enlarged view of Fig. 4(a) excluding $M_s^d = 2.0$ and initial data

Fig. 4. Verification result of robust stability

using a set of input-output data, subject to the constraint condition on the stability margin. In the proposed method, a trade-off design of tracking performance for the reference input and robust stability is accomplished by selecting robust stability. Finally, the numerical example results show that the tracking performance for reference output is improved by using the FOPID controller.

The proposed method focuses on servo performance, and we will investigate regulator performance in the future study.

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