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Model Parametrisation and Rule Selection for Problem-tailored PID Autotuning

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Abstract When employing measured input/output data to determine the parameters of a process model and then exploiting that model to tune a controller, the used parametrisation procedure can exert a significant (and often overlooked) influence on the obtained results. In this paper we propose a methodology to select the best combination of model parametrisation procedure and (PID) controller tuning rule depending on the nature of the control problem to address, characterised by conveniently defined quality indices.

Keywords: PID control, Autotuners, Model parametrisation, Industrial controllers.

1. INTRODUCTION

Model-based PID (auto)tuning – MBAT for short – consists of gathering process input/output data and using them to identify a process model, in turn exploited to determine the controller parameters. For applicability-related reasons the tuning rules employed are generally simple, hence the used models are simple as well, with a structure dictated by that of the controller to synthesise. Thus, an MBAT *procedure* is composed of (i) a process experiment followed by (ii) a Model Parametrisation Procedure (MPP) and (iii) a Tuning Rule (TR). The point of this research is that the MPP can heavily influence the tuning results.

A previous paper by Seva et al. (2021), on which this one builds, showed that starting from the same input/output data, considering a set of TRs that share the model structure and ranking them by some Tuning Quality Index (TQI), the said ranking can sometimes be altered by just changing the employed MPP. With respect to the quoted work we reconsider the set of MPPs, extend that of TRs, and most important, start addressing the choice of MPP and TR as a compound. Also, we demonstrate our findings with reference to a standard literature benchmark for PID tuning assessment.

2. BACKGROUND

In this section we introduce the ingredients of the proposed MPP-TR compound selection methodology. For brevity we refer right from now to the particular entities considered in the following but it should be clear that the idea is more general, i.e., it could be applied to other process model structures, other sets of MPPs and TRs, and other TQIs.

2.1 Process model

Most tuning rules for MBAT employ as process model a first- or second-order transfer function with delay. In this study we limit the focus to the First Order Plus Dead Time (FOPDT) structure, that is, to a transfer function in the form

$$P(s) = \mu \frac{e^{-sD}}{1+sT}, \quad T>0, D\geq 0. \tag{1}$$

and as indicated by the parameter bounds in (1) we only consider the asymptotically stable case, that however covers the great majority of the process control domain toward which the presented research is somehow implicitly geared. Extensions – at least to the integrating case – will be addressed in the future.

To lighten the notation we hereafter operate in normalised conditions as for gain and time constant (de-normalising is trivial). We define the normalised delay as

$$\theta = \frac{D}{D+T} \tag{2}$$

and consider as tuning model the transfer function

$$M(s) = \frac{e^{-s\frac{\theta}{1-\theta}}}{1+s}, \quad 0\leq\theta<1. \tag{3}$$

Coming to the tuning rules, some are parameter-free and some are not. In this respect, to keep the treatise acceptably compact, we only consider rules of the first type or of the second type but with one tuning parameter, interpretable as the desired closed-loop dominant time constant (here too, the choice in fact covers the great majority of the overall *scenario*). Denoting the said parameter with λ, we re-convert the closed-loop response speed requirement into an acceleration factor – k_a to name it – with respect to the open-loop process dynamics. The meaning of this choice is that we require the closed-loop settling time to be k_a times smaller than the settling time of the tuning model. With reference to (1) this means

$$5\lambda = \frac{5T+D}{k_a}, \tag{4}$$

which in the normalised terms of (3) corresponds to

$$\lambda = \frac{5-4\theta}{k_a(1-\theta)}. \tag{5}$$

and allows to specify a process model plus requirement couple in the form (θ, k_a).

2.2 Model parametrisation procedures

In this paper we consider two MPPs. The first one (M1) is the Method of Areas (Ohta et al., 1979) or MoA; the second one (M2) is the "Method of Percentage" by Sundaresan and Krishnaswamy (1978) or MoP. This choice differs from that in the ancestor paper by Seva et al. (2021), where the alternatives to MoA were the methods of tangent and moments. The reason is that the former of these is too noise-sensitive for many applications, while the second admits an analytical application only for a limited parameter range, which hinders the comparisons we need. The MPP matter is still being studied.

Both the MPPs here considered start from the record $y_{us}(t)$ of an open-loop process (unit) step response. The MoA computes the two integrals (or areas, whence the name)

$$A_0 = \int_0^\infty (y_{us}(\infty) - y_{us}(t))dt, \quad A_1 = \int_0^{A_0/y_{us}(\infty)} y_{us}(t)dt, \tag{6}$$

where

$$y_{us}(\infty) = lim_{t\to\infty} y_{us}(t), \tag{7}$$

and then sets

$$\mu = y(\infty), \quad T = e\frac{A_1}{\mu}, \quad D = \frac{A_0}{\mu} - T. \tag{8}$$

The MoP selects two instants t_1 and t_2 that correspond to the 35.3% and 85.3% of the total process step amplitude, respectively, and then sets

$$\mu = y(\infty), \quad T = \frac{2}{3}(t_2 - t_1), \quad D = 1.3t_1 - 0.29t_2. \tag{9}$$

2.3 Tuning rules

PI controller The considered TRs refer to the 1-dof PI controller

$$C(s) = K\left(1 + \frac{1}{sT_i}\right). \tag{10}$$

The rules selected in this work are

- the IMC-PI formulæ Morari and Zafiriou (1989); Braatz (1996); Leva and Colombo (2004), hereafter denoted with "IMC";
- the SIMC rule (Skogestad, 2005, 2006), denoted with "Sko";
- the improved IMC PI by Rivera et al. (1986b), hereafter "Riv";
- the "direct synthesis for disturbance" method by Chen and Seborg (2002), here "Dsd";
- the formulæ used in the ABB Easy-Tune and reported in Li et al. (February 2006), here "ABB";
- the IAE (Integral of Absolute Error) minimisation rule by Lopez et al. (1967), here "LSM";
- the formulæ by Cox et al. (1997), a specialisation of Vrančić et al. (1996), indicated as "D+C";
- method 31/32 for a closed loop response overshoot below 5% by Mann et al. (2001), here "Mann";
- method 1 from Hägglund and Åström (2002), here "H+A";
- method 1 by Lee et al. (1998), denoted with "Lee";
- method 1 from Isaksson and Graebe (1999), here "I+G";
- method 1 from Smith (2002), termed here "Smith".

In normalised form, evidencing the (θ, k_a) couple as just discussed, these take the form detailed in Table 1.

Table 1. PI tuning rules in normalised form.

	K	T_i
IMC	$\frac{k_a(\theta-1)}{4\theta^3-13\theta^2-k_a\theta+14\theta-5)}$	1
Sko	$\frac{k_a(\theta-1)}{4\theta^3-13\theta^2-k_a\theta+14\theta-5)}$	$\min\left(1, 4\frac{16\theta^3-52\theta^2+(56-4k_a)\theta-20}{k_a(\theta-1)}\right)$
Riv	$\frac{k_a(1-\theta/2)}{(1-\theta)^2(5-4\theta)}$	$\frac{1-\theta/2}{1-\theta}$
Dsd	expressions too long to report	
ABB	$1.164\left(\frac{\theta}{1-\theta}\right)^{0.977}$	$1.484\left(\frac{\theta}{1-\theta}\right)^{0.68}$
LSM	$0.758\left(\frac{1-\theta}{\theta}\right)^{0.861}$	$0.98\frac{1-\theta}{1-1.317\theta}$
D+C	$\frac{1-2\theta+1.5\theta^2-0.333\theta^3}{2\theta(1-\theta+0.667\theta^2)}$	$\frac{1-2\theta+1.5\theta^2-0.333\theta^3}{1-2\theta+1.5\theta^2-0.5\theta^3}$
Mann	$0.51\frac{1-\theta}{\theta}$	1
H+A	$3.571\frac{\theta}{1-\theta/2}$	$3.83\frac{\theta(\theta-1.862)}{1+8\theta+9\theta^2}$
Lee	expressions too long to report	
I+G	$\frac{0.2k_a(1-0.75\theta)}{1-2.8\theta+2.6\theta^2-0.8\theta^3}$	$\frac{1-0.75\theta}{1-\theta}$
Smith	$\frac{1-\theta}{\theta}$	1

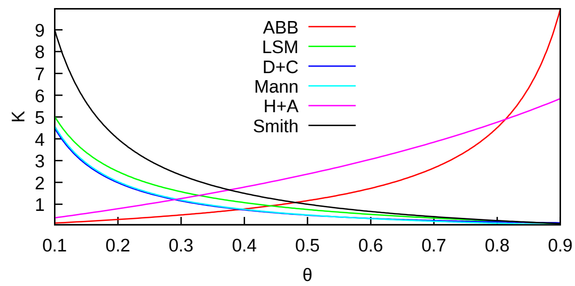

Figure 1. Parameter K as a function of θ for some of the PI TRs in Table 1.

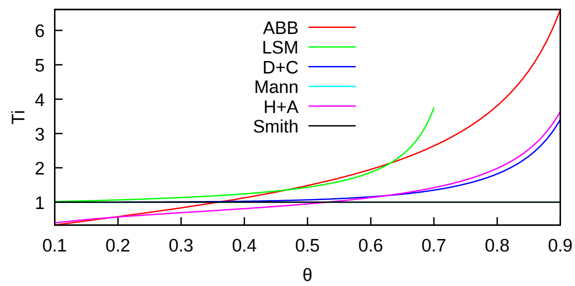

Figure 2. Parameter T_i as a function of θ for some of the PI TRs in Table 1.

To see how differently the rules above behave, Figures 1 and 2 respectively report K and T_i as a function of θ for those not employing the additional parameter λ — or equivalently, k_a. There is not the space for a complete discussion, that we defer to future works, but note for example that some TRs tend to accommodate for a large θ by increasing both K and T_i and some by just decreasing K. Such different strategies make the process/model mismatch manifest itself differently on the tuning quality. The said mismatch depends on the MPP, and this is another motivation for the presented research.

PID controller The PID structure controller we consider is the standard 1-dof real ISA form, that is,

$$C(s) = K\left(1 + \frac{1}{sT_i} + \frac{sT_d}{1 + sT_d/N}\right) \tag{11}$$

The tuning rules are

- the IMC-PID by Rivera et al. (1986a), indicated as "IMC";
- method 2 (approximated 1/4 decay ratio) from Connell (1996), here "Connell";
- method 1 from Moros (1999), attributed to Oppelt and denoted here as "Moros";
- method 2 by Lipták (2001), termed "Liptak";
- the formulæ from Padma Sree and Chidambaram (2004), here "Sree";
- method 1 for minimum IAE from Wang et al. (1995), indicated with "Wang";
- method 2 from Fruehauf et al. (1994), here "Fruehauf";
- method 1 by Rivera et al. (1986b), termed as 'Riv';
- method 1 from Lee et al. (1998), here 'Lee'.

For brevity we omit the normalised parameter expressions as well as a comparison of the ways they depend on θ; suffice to say that the considerations made about the PI TRs apply also to the PID ones.

2.4 Tuning quality indices

The TQIs we selected cover two different control *scenarii*, namely set point tracking (or "servo" tuning) and disturbance rejection (or "regulatory" tuning).

The indices for set point tracking refer to a step set point variation and are

- the ISE (Integrated Squared Error),
- the maximum overshoot,
- and the 99% settling time.

The indices for disturbance rejection refer to a step load disturbance and are

- the ISE (Integrated Squared Error),
- and the peak error magnitude.

With respect to the previous paper Seva et al. (2021) the set is smaller, and was selected having in mind an immediate interpretability also on the part of the typical plant personnel. Other indices from the quoted paper could of course be included, but some (like the controller frequency response magnitude at the cutoff or the T_i/T ratio) are relevant from a control-theoretical standpoint but less intuitive for practitioners. Anyway, further streamlining the sets of TQI will be among the future research directions.

2.5 Evaluation benchmark

The process transfer functions we use to set up and evaluate the proposed MPP-TR selection technique come from the work by Åström and Hägglund (2000), as the benchmark proposed therein is well accepted and widely used in the literature. As the presented research is mostly geared to process control we limit the set to the first five process classes in the said benchmark, that is,

$$\begin{aligned}
P_1(s) &= \frac{1}{(1+s)^\alpha} && \alpha \in \{2,3,4,5,6,7,8\} \\
P_2(s) &= \frac{1}{\prod_{i=0}^{3}(1+\alpha^i s)} && \alpha \in [0.05, 0.95], \\
P_3(s) &= \frac{1-\alpha s}{(1+s)^3} && \alpha \in [0.1, 5], \\
P_4(s) &= \frac{e^{-s}}{1+\alpha s} && \alpha \in [0.1, 10], \\
P_5(s) &= \frac{e^{-s}}{(1+\alpha s)^2} && \alpha \in [0.1, 10].
\end{aligned} \tag{12}$$

Class P_1 is quoted as a test case by several controller manufacturers, and for large values of α tends to behave as a delay system, and with respect to (Åström and Hägglund, 2000) here we omit the trivial case $\alpha = 1$; class P_2 has four poles spaced through the parameter α, and with respect to the quoted benchmark here we do not extend the rance to $\alpha = 1$ as this duplicates P_1 with $\alpha = 4$; systems of class P_3 exhibit an inverse response that adversely affects the achievable performance as α increases; class P_4 is the FOPDT itself, to test the procedure in structurally nominal conditions and to have parameter α directly control the lag- or delay-dominated system character; class P_5 is similar to P_4 but with a higher frequency roll off. To avoid validity issues with some TRs we do not consider pure delay processes, i.e., P_4 and P_5 with $\alpha = 0$. This point might be delicate and deserve further research; for the scope of this paper, suffice the practical consideration that such a model parametrisation should hardly ever emerge from a sensible MPP fed with sane process input/output data.

3. THE PROPOSED TECHNIQUE

The proposed evaluation technique is composed of two parts, one offline that prepares selection tables given the process models to consider, the set of MPPs and that of TQIs, and one online, that uses the said tables to perform the MPP-TR compound selection in the case at hand.

3.1 Offline part

The offline part of the technique is articulated in the steps listed below, and illustrated as a flowchart in Figure 3.

(1) Take a process class in the considered set.
(2) Make the class parameter change in the defined range.
(3) Compute the corresponding FOPDT models through the considered MPPs.
(4) Compute for each model the normalised estimated delay θ_e.
(5) Tune the controller for each parametrised model and TR;
(6) Compute the considered TQIs;
(7) List in a table the best MPP-TR compound for each TQI versus θ_e.

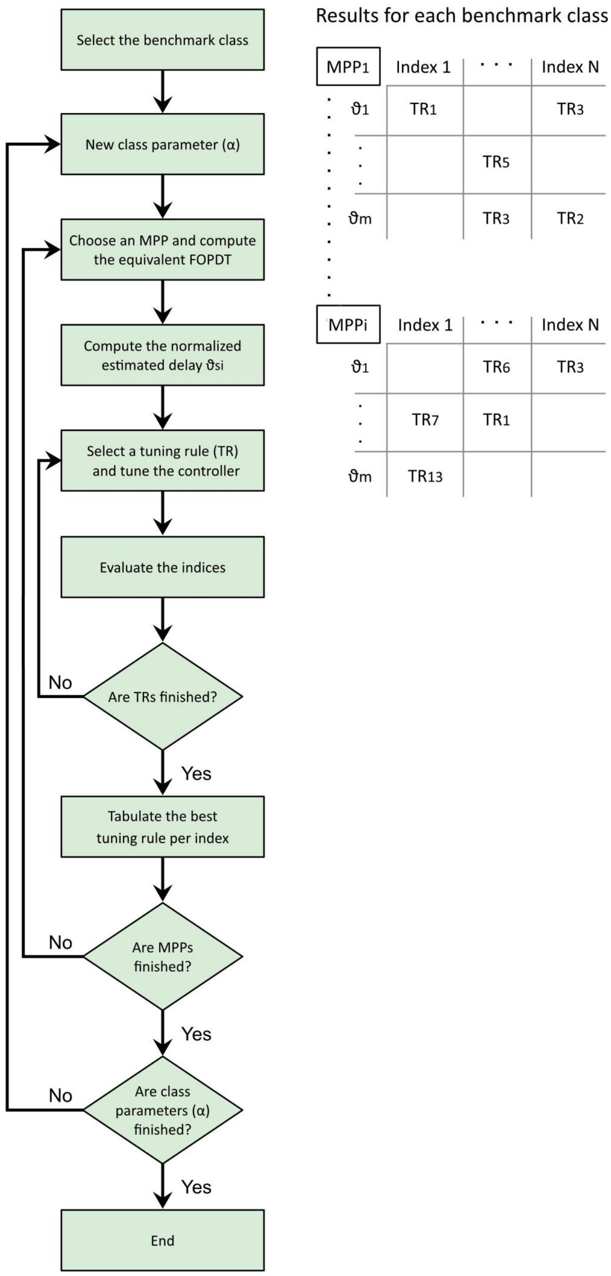

Figure 3. Offline part of the proposed technique

The online technique, given an unknown process, is composed of the following steps, and summarised as flowchart in figure 4.

(1) Decide the TQI to optimise.
(2) Perform a step response and record the process output.
(3) Compute the equivalent FOPDT and θ_e with the considered MPPs.
(4) Identify in which benchmark class the process falls. At present we do this manually, but it should not be too difficult to decide based on some pattern recognition technique applied either to the step response, as done in Leva and Piroddi (1996), or to the difference between that response and the one from the FOPDT model. We shall include this into the future activities, but since the paper just quoted guaranteed feasibility we do not foresee any criticality in this respect.

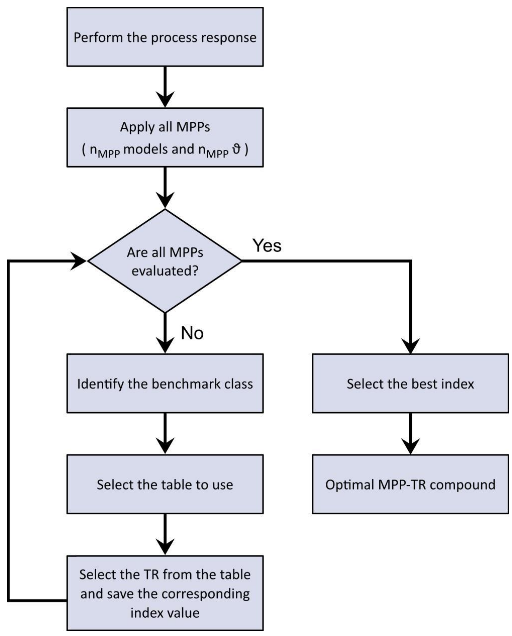

Figure 4. Online part of the proposed technique

(5) For each MPP take the table corresponding to the chosen class and with the corresponding θ_e select the best TR.
(6) Among the so formed MPP-TR compounds, select the one with the best TQI.

As a final note, in the description above the technique appears structured for a single controller structure — i.e., in the procedure as shown one has to decide *a priori* whether a PI or a PID shall be used, and the best MPP-TR compound will be suggested. In principle one could ask for the best compound for both controllers, and then select the one with the best TQI so that also the decision about introducing derivative action or not be automated. This most be done carefully, however, because sometimes that action can for example reduce the set point ISE, but at the cost of a significantly increased settling time. This matter will be further studied in the future; for the moment, we still leave the controller structure selection to the user.

4. BENCHMARK EVALUATION

To assess the operation of the proposed technique, we start by showing some MPP-TR selection tables as coming from the offline part. As we have two MPPS (M_1 and M_2) we obtain two tables for each class and each TQI. Figures 5 and 6 report two examples, referring respectively to classes P_2 and P_3. The labels "Max $|e|$", "ISE_{dr}", "ISE_{sp}" and "Max Ovr", stand respectively for peak error magnitude of the load disturbance response, ISE for the load disturbance step response, ISE for the set point step response, maximum overshoot of the set point step response; "99% settling time" refers to the set point response and is self-explanatory. Rules R1 to R21 (bottom to top) refer to the 12 PI and the 9 PID TRs in the order they were presented in Section 2.5.

Based on these figures, and on the entire set of tables that we omit for obvious reasons, we can now make some considerations. First, no MPP-TR compound uniformly

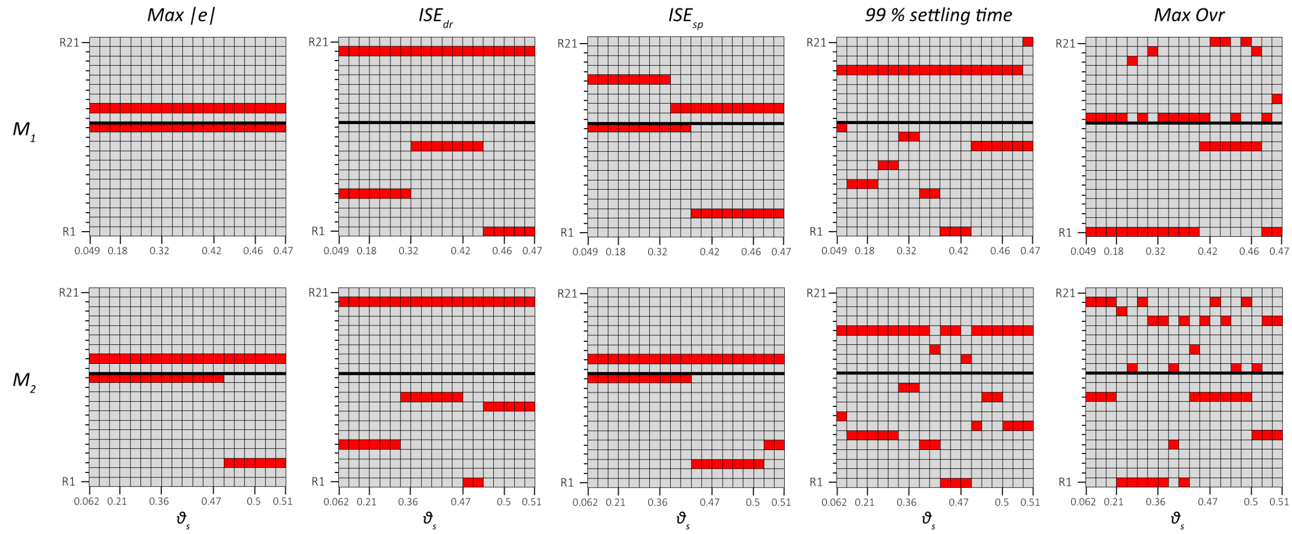

Figure 5. Best PI and PID TR per TQI, benchmark class P_2 – the red colour indicates the best TR, the others are gray.

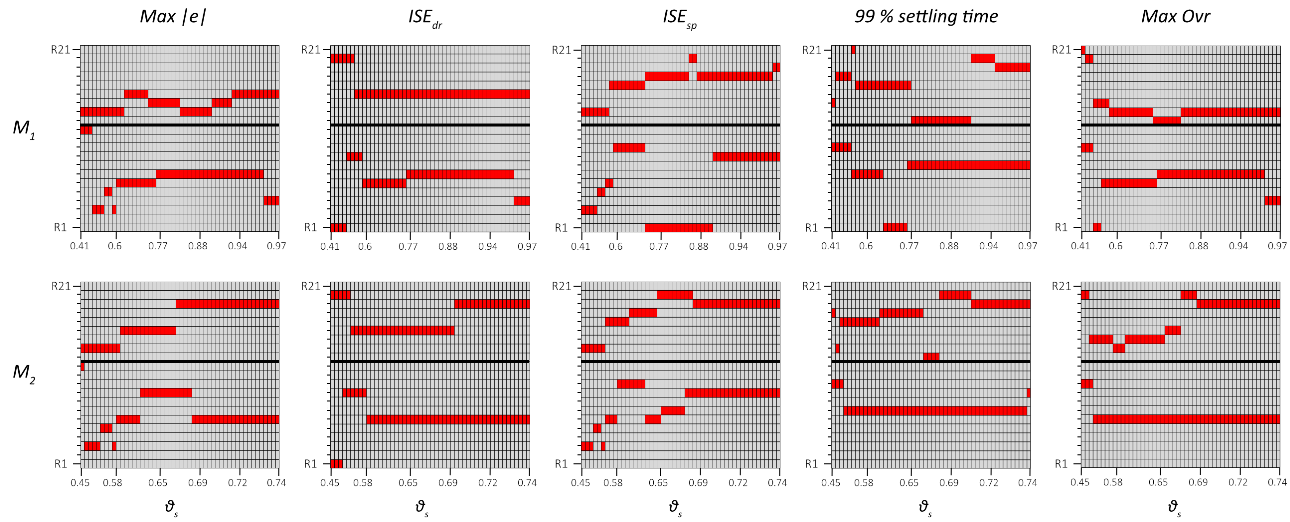

Figure 6. Best PI and PID TR per TQI, benchmark class P_3 – the red colour indicates the best TR, the others are gray.

dominates the others, but this is almost true for some class and some TQI (see for example Max $|e|$ in Figure 5); thus, accounting for the class – or said otherwise, the dynamic character of the process – is important.

Then, in some cases (see for example Max Ovr in Figure 5, especially with M2) the selected compound varies a lot with θ, most likely indicating a "too close to call" situation — an aspect on which further research is in order to decide how to discriminate (assuming this is necessary, as in several of these cases an inspection of the responses show that they are in fact very similar). In many other cases there are quite evident θ intervals in the favour of a certain compound, conversely, hence a reasonable decision can most often be reached.

Finally, the results for M1 and M2 are very often different, which further (and *a posteriori*) confirms that one has not to select the best TR but the best MPP-TR compound.

To end this section, we show just one example to demonstrate that choosing the best MPP-TR compound is useful also from a very practical standpoint, that is, as seen by observing closed-loop responses in the time domain. To this end, Figure 7 compares the MPP-TR compound suggested by the proposed technique to another random one: as can be seen, a proper selection does help.

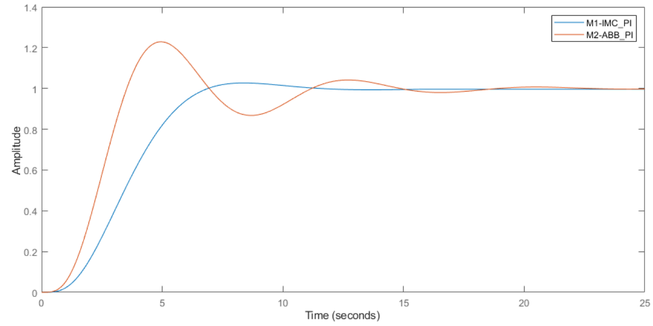

Figure 7. Selection for optimum (minimum) overshoot of the set point response, class P_2 – chosen MPP-TR compound in blue, another random one in orange.

5. CONCLUSIONS AND FUTURE WORK

We presented a technique to choose the best compound of model parametrisation procedure and tuning rule given a tuning quality index to optimise and a recorded process response. We did this referring to the PI(D) controller structure and using an open-loop step response, but the idea can certainly be generalised about the former and maybe also about the latter. We showed a benchmark evaluation to assess the correct operation and the practical usefulness of the proposal.

Numerous activities are in order in the future, as anticipated. These shall first include addressing robustness besides performance as well as automating the process class selection along the sketched path, and then revisiting the set of TQIs (and possibly of some meaningful combinations of them) for a balance between system-theoretical expressiveness and practitioner interpretability, further investigating the role of integral indices as (part of) TQIs, extending the proposal to the integrating case, carrying out further benchmark testing, and porting the technique onto control hardware for testing in the laboratory and possibly on some real plant.

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