ETF dead-time compensation based multiloop control approach for multivariable processes

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Abstract: Due to significant interaction dynamics and time delays in multivariable processes, designing a well-tuned PI controller is always challenging. When tuning multi-loop PI controllers, effective open-loop transfer function should be used instead of the diagonal transfer function element. The complexity of the effective open-loop transfer function (EOTF) increase rapidly for high-dimensional multivariable processes. Therefore, in this manuscript, it is proposed to use equivalent transfer functions (ETFs) under perfect control approximation. Subsequently, a dead-time compensation (DTC) scheme is used for ETFs and PI controllers are tuned. DTC can facilitate eliminating the impact due to time delay from the multivariable process, making the task of tuning PI controllers relatively less complex. The proposed multi-loop control topology minimizes the controller effort, and a good controller design for a multivariable process can be achieved. The PI controller parameters for dead-time compensated ETFs are calculated using simple IMC PI tuning rules, where the filter factor is taken based on the paired element relative gain value. To examine the effectiveness of the proposed controller tuning approach, the proposed control strategy was applied to a wide range of multivariable processes. Compared to state-of-the-art control strategies, the proposed control strategy provides superior closed-loop performance indices, even when there is a significant plant-model mismatch.

Keywords: Multivariable processes, ETF dead-time compensation, IMC PI tuning rules.

1. INTRODUCTION

In process industries, many industrial processes are integrated into a layered architecture, forming a multivariable system to develop a product. The productivity and economy of the industries are highly dependent on the performance of the industrial processes. As per some recent estimates, around 90-95% control loops are still operated by PI/PID controllers, even though advanced controller approaches like state feedback control and model-based predictive controllers are available (Shen et al. (2010)). The primary advantage of the PI/PID controllers stem from their simplicity in design and ease of implementation. They also provide explainability, realizability, and fault-tolerant control structure. Secondly, a well-tuned PI controller provides robustness against any plant-model mismatch. Due to these inherent advantages, PI controllers are more prominent and dominant in industries. Despite such popularity, unlike single-input single-output (SISO) systems, tuning PI controller parameters for multivariable systems is challenging. In multivariable systems, each input potentially affects all the outputs. Therefore, the performance of one control loop will significantly impact the other loops due to the interactions. These interaction dynamics degrade the controller performance and in some cases, it can even make the system unstable if the controllers are not designed properly.

Numerous multi-loop control strategies have been reported in the literature, such as detuning-based control approaches (Luvben (1986); Besta and Chidambaram (2016); Huang et al. (2003)), analytical IMC PI controllers (Khandelwal et al. (2017)), robust multiloop controllers (Mahapatro and Subudhi (2023)), optimization-based methods (Khandelwal and Detroja (2017)) and independent loop design methods (Vu and Lee (2010b,a); Xiong and Cai (2006)). In all these control approaches, a multivariable process is considered as a combination of multiple SISO systems while designing controllers in a multi-loop framework. Subsequently, to eliminate the interaction effect and obtain the desired stability and closed-loop (CL) performance, the obtained PI controller parameters are scaled by a filter factor. This filter factor is estimated from either the theoretical stability bounds or empirically by trial and error.

Transportation delay is common place in industrial multivariable systems. The closed loop control performance will degrade significantly, if the process has a significant dead time. The key issues associated with higher dead time are listed below:

• Higher dead time decreases the cross-over frequencies and critical gains, making the controller's performance vulnerable to noise.

- Controller takes much longer to take corrective control action.
- Higher dead time makes the system's transient behavior slower, which can further lead to an unstable system.

From the above discussions, it is evident that delay and interaction dynamics create complications in designing a promising PI controller for high-dimensional multivariable processes. The multivariable system control design is significantly simplified if the system has no delay. For a SISO system, the delay can be compensated easily by using Smith predictor control structure. To eliminate the dead time from the multivariable process and simplify the controller design, a dead-time compensation (DTC) based topology was introduced (Chen et al. (2011)) earlier. Recently, Jin et al. (2017) designed a centralized control-based DTC approach for multivariable systems, and Giraldo et al. (2018); Chuong et al. (2019) proposed a decoupler-based DTC approach for multivariable systems. Jin et al. (2017)'s only partially eliminates the delay, i.e. minimum delay from each row of the process transfer function is eliminated. Decoupler based design compensates for delay of the diagonal element of the process only. Further, implementing centralized and decoupler-based control schemes for high-dimensional multivariable processes is complex and challenging. Instead of implementing these complex control structure, implementing a decentralized control is much easier and preferred in industry. Therefore, ETF-based dead-time compensation scheme with a decentralized control structure is proposed in this manuscript. Due to its simple structure, the proposed method applies to higher-dimensional multivariable processes as well.

2. THEORY OF MULTILOOP CONTROLLERS

The schematic of the $n \times n$ multivariable process is represented in Fig. 1.



Fig. 1. Multiloop PI control for the $n \times n$ MIMO process.

It is evident that the dynamics of each loop depend on the controllers placed in the other loops for multivariable processes. This complicates the PI controller parameters tuning and one can not proceed by considering only the PTF.

To overcome this problem, independent loop design approaches have been proposed (Vu and Lee (2010b); Rajapandiyan and Chidambaram (2012)) based on the effective open loop transfer function (EOTF) and/or equivalent transfer function (ETF). The multivariable process is approximated in these control approaches as a combination of multiple SISO systems.

Under perfect control assumption, the ETF model for the MIMO system is derived based on normalized gain arrays such as relative gain (RG) array (Λ), relative normalized gain array (Φ) and relative average residence time array

 (Γ) , mathematically calculated as follows (for proof refer Rajapandiyan and Chidambaram (2012)):

$$\frac{y_k}{u_k} = \widehat{g}_{kk}(s)e^{-\widehat{\theta}_{kk}s} = \frac{k_{kk}}{\lambda_{kk}(\gamma_{kk}T_{kk}s+1)}e^{-\gamma_{kk}\theta_{kk}s}$$
(1)

where k_{kk} , θ_{kk} , and T_{kk} denotes the gain, time delay, and time constant corresponding to $g_{kk}(s)$, and λ_{kk} and γ_{kk} are the relative gain and relative average residence time values corresponding to $\hat{g}_{kk}(s)$. The ETF models for multivariable systems may be realizable even when EOTF is not realizable. When ETF contains higher dead-time $(e^{-\hat{\theta}_{kk}s})$, it isn't easy to tune PI controllers for the system. Therefore, an ETF dead-time compensation framework for multivariable systems is proposed in the next section.

3. PROPOSED ETF DTC-BASED MULTI-LOOP CONTROLLERS

Fig. 2 illustrates the structure of the proposed ETF-DTC-based multiloop control for a $n \times n$ MIMO system.

The k^{th} output for a multivariable system is given by :

$$y_k(s) = g_{kk}(s)u_k(s)e^{-\theta_{kk}s} + \sum_{j=1, j \neq k}^n g_{kj}(s)e^{-\theta_{kj}s}u_j(s)$$
(2)

The error available in k^{th} loop is:

$$e_k(s) = r_k(s) - y_k(s) \tag{3}$$

When controllers are placed in all the loops except $y_k - u_k$ loop, then under perfect control assumption, the output y_k can be approximated as :

$$\tilde{y}_k(s) = \hat{g}_{kk}(s)e^{-\theta_{kk}s}u_k(s) \tag{4}$$

Therefore the error in k^{th} loop in the multi-loop control framework is given by :

$$e_k(s) = r_k(s) - \widehat{g}_{kk}(s)e^{-\theta_{kk}s}u_k(s)$$
(5)

In order to design dead-time compensation, this error should be delay free. It can be achieved by introducing a feedback term $q_k(s)$, such that the dead-time is eliminated from the error.

The error in k^{th} loop with $q_k(s)$ is given by:

$$e_k(s) = r_k(s) - y_k(s) - q_k(s)u_k(s)$$
(6)

Comparing Eq. (5) and Eq. (6), and solving for $q_k(s)$,

$$q_k(s) = \left(\widehat{g}_{kk} - \widehat{g}_{kk}e^{-\widehat{\theta}_{kk}s}\right) \tag{7}$$

Hence, by selecting $q_k(s) = \hat{g}_{kk}(1 - e^{-\hat{\theta}_{kk}s})$, the error in each loop will be independent of the time delays. Thus, the proposed DTC multi-loop structure allows full dead-time compensation for each loop in MIMO processes. Therefore, improved controller freedom is achieved, and a better PI controller design is possible by considering only the minimum phase $(\hat{g}_{kk}(s))$ dynamics of the ETF matrix.



Fig. 2. Proposed ETF dead-time compensation structure for the $n \times n$ MIMO process.

3.1 Simple IMC PI tuning rules

The simple IMC based PI tuning settings (Skogestad (2003)) for first order delay free ($\theta_e = 0$) systems are given as follows:

$$K_{ci} = \frac{T_m}{K_m T_c} \tag{8}$$

$$T_{Ii} = T_m \tag{9}$$

where K_m , T_m , and θ_e represent gain, time constant, and time delay of the first order system, respectively.

To ensure better closed-loop stability and set-point tracking with a IMC based PI controller, a filter factor (T_c) is chosen as the factional value of the effective time delay (Rajapandiyan and Chidambaram (2012); Skogestad (2003)). For overdamped and lag-dominated systems, the closed-loop time constant can be considered as the filter factor (Vu and Lee (2010a)). In this manuscript, the filter factor is taken as follows:

- For highly interacting multivariable systems ($\lambda_{ii} \geq$ 2), $T_c = 0.5T_m$ is considered.
- For Moderately interacting multivariable systems, i.e. systems with $(0.5 \leq \lambda_{ii} \leq 1.5)$. For such processes, estimated ETF models have more gain over the actual open loop gain of the process. To obtain a stable output response, $T_c = \frac{0.4}{T_m}$ is considered. • For high dimensional multivariable systems with
- moderate interactions, $0.2T_m$ is considered as the T_c .

The T_c values suggested here are indicative and these values may be tuned and changed for a different industrial application.

The performance of the closed-loop system is assessed in terms of the integral of the absolute error (IAE) and integral of the squared error (ISE), defined as:

$$IAE = \sum_{i=1}^{n} \int_{0}^{t} |e_{i}(t)| dt$$
 (10)

$$ISE = \sum_{i=1}^{n} \int_{0}^{t} e_{i}^{2}(t) dt$$
 (11)

where n is the number of output variables.

In general, industrial systems have uncertainties due to various reasons. Hence, the controller should be robust against unmodeled dynamics and plant-model mismatch.

According to the inverse maximum singular value approach (Maciejowski (1989)), if the PTF (G(s)) with $\Delta_I(s)$ as the input perturbation model, the corresponding input multiplicative uncertainty (IMU) is then $(G(s)[I+\Delta_I(s)])$, the closed-loop system $(H_i(jw))$ is stable, if

$$|\Delta_{I}(jw)| \leq \frac{1}{\overline{\sigma}} \{H_{i}(jw)\} \leq \frac{1}{\overline{\sigma}} [I + G_{ci}(jw)G(jw)]^{-1}G_{ci}(jw)G(jw)\}$$
(12)

Where $\overline{\sigma}$ is the maximum singular value of $(H_i(jw))$.

If $\Delta_o(s)$ is the output perturbation model, $[I + \Delta_o(s)]G(s)$ is the associated output multiplicative uncertainty (OMU), and the closed-loop system $(H_o(jw))$ is stable, if

$$|\Delta_{o}(jw)| \leq \frac{1}{\overline{\sigma}} \{H_{o}(jw)\} \leq \frac{1}{\overline{\sigma}} \{[I + G(jw)G_{ci}(jw)]^{-1}G(jw)G_{ci}(jw)\}$$
(13)

The area under the curve designates the stable region of the closed-loop system. When uncertainty is involved in G(s) also, if the control method regards the same area under the curve, the designed control method is commented as robust.

4. SIMULATION RESULTS AND DISCUSSION

To demonstrate the simplicity and effectiveness of the proposed method, popular industrial process models, such as Wood and Wardle (WW), Industrial Scale Polymerization (ISP) and Heating, Ventilation and Air Conditioning (HVAC) System, are chosen. WW and ISP are 2×2 processes, and HVAC is a 4×4 large-scale multivariable process. The performance of the proposed method is compared against the well known existing multi-loop PI control strategies such as the detuning iterative continuous cycle (Khandelwal and Detroja (2019)) (DICC), Vu and Lee (2010a) method, and one detuning parameter

(Khandelwal and Detroja (2017)) (ODP) method. The PI controller parameters for all the processes are listed in Table 1. The closed loop simulations are performed, where an i^{th} loop is subjected to a unit step while all the other loops are regulated at zero. To verify the robustness of the proposed method, the simulations are carried out for the $\pm 10\%$ of the plant-model mismatch. The obtained performance indices values for these simulations (nominal and plant model mismatch) are tabulated in Table 2.

4.1 Highly interacting multivariable System

The MIMO processes, which have $\lambda_{ii} \geq 2$, are highly interacting multivariable systems.

Wood and Wardle (WW) process: WW (Luyben (1986)) is a two-dimensional distillation column.

$$\mathbf{G}(\mathbf{s}) = \begin{bmatrix} \frac{0.126e^{-6s}}{(60s+1)} & \frac{-0.101e^{-12s}}{(48s+1)(45s+1)}\\ \frac{0.094e^{-8s}}{(38s+1)} & \frac{-0.12e^{-8s}}{(35s+1)} \end{bmatrix}$$
(14)

WW process $\mathbf{G}(\mathbf{s})$, has significant dead-time, and RGA value is 2.6875.

The ETF between paired input-output $(y_i - u_i, i \in \{1, 2\})$ are given in (15)

$$\widehat{g}_{11}e^{-\widehat{\theta}_{11}s} = \frac{0.0469e^{-3.7170s}}{37.1702s+1}; \widehat{g}_{22}e^{-\widehat{\theta}_{22}s} = \frac{-0.0447e^{-4.9560s}}{21.6826s+1}$$
(15)

As seen from the closed-loop simulation results in Fig. 3, while the Vu & Lee method provides oscillatory output responses, the proposed method provides better set-point tracking and regulatory responses.



Fig. 3. closed-loop set-point responses for the WW process.

Stability regions of the IMU and OMU of the WW process (see Fig. 4) indicated that the proposed controller provides approximately the same area under the curve with nominal operating conditions and for the plant with model mismatch. These results concluded that the proposed control method is more robust.

4.2 Moderately interacting multivariable system

The MIMO systems with $(0.5 \leq \lambda_{ii} \leq 1.5)$ can be considered as systems with moderate interactions.



Fig. 4. Stability regions of the WW process for input and output uncertainties.

| Table 1. | \mathbf{PI} | Controller | parameters | for | various |
|----------|---------------|--------------|---------------|-----|---------|
| | r | nultivariabl | le processes. | | |
| | | | | | |

| G(s) | Tuning Method | K _{ci} | T_{Ii} | | |
|--------|---------------|--|----------|--|--|
| | Proposed | 42.64 | 37.170 | | |
| | rioposeu | -44.74 | 21.682 | | |
| - | DICC | 40.351 | 22.77 | | |
| 11/11/ | DICC | $\begin{array}{c cccc} K_{ci} & T_{Ii} \\ \hline 42.64 & 37.170 \\ -44.74 & 21.682 \\ \hline 40.351 & 22.77 \\ -19.636 & 25.901 \\ \hline 67.06 & 66.04 \\ -40.42 & 41.52 \\ \hline 41.55 & 60 \\ -19.089 & 35 \\ \hline 0.968 & 3.536 \\ \hline 0.592 & 1.393 \\ \hline 0.294 & 3.68 \\ \hline 0.305 & 2.56 \\ \hline 0.43 & 3.95 \\ \hline 0.13 & 1.18 \\ \hline 0.219 & 4.572 \\ \hline 0.1703 & 1.801 \\ -65.26 & 113.83 \\ -66.30 & 121.31 \\ -54.40 & 113.90 \\ -51.48 & 123.55 \\ -40.803 & 142.97 \\ -48.95 & 136.34 \\ -48.71 & 104.5 \\ -44.25 & 117.94 \\ -31.328 & 94.668 \\ \hline 0.375 & 47 & 90.875 \\ \hline \end{array}$ | 25.901 | | |
| ~~~~ | Vn & Loo | 67.06 | 66.04 | | |
| | vu & Lee | -40.42 | 41.52 | | |
| - | Viene | 41.55 | 60 | | |
| | Along | -19.089 | 35 | | |
| | Proposed | 0.968 | 3.536 | | |
| | rioposeu | 0.592 | 1.393 | | |
| - | DICC | 0.294 | 3.68 | | |
| ISD | DICC | 0.305 | 2.56 | | |
| 151 - | VII & Lee | 0.43 | 3.95 | | |
| | Vu & Lee | 0.13 | 1.18 | | |
| | Viong | 0.219 | 4.572 | | |
| | Along | 0.1703 | 1.801 | | |
| | | -65.26 | 113.83 | | |
| | Proposed | -66.30 | 121.31 | | |
| | i ioposed | -54.40 | 113.90 | | |
| | | -51.48 | 123.55 | | |
| | | -40.803 | 142.97 | | |
| HVAC | DICC | -48.95 | 136.34 | | |
| IIVAC | DICC | -48.71 | 104.5 | | |
| | | -44.25 | 117.94 | | |
| - | | -31.328 | 94.668 | | |
| | ODP | -37.547 | 89.875 | | |
| | UD1 | -30.894 | 89.328 | | |
| | | -28.188 | 100.489 | | |

Industrial Scale Polymerization (ISP) System: ISP (Chien et al. (1999)) is a binary polymerization reactor.

ISP is a slow process with moderate interactions (RGA value of 0.7087).

The obtained ETF between paired input-output $(y_i - u_i, i \in \{1, 2\})$ for ISP process are given in (16)

$$\widehat{g}_{11}e^{-\widehat{\theta}_{11}s} = \frac{32.3003e^{-0.1547s}}{3.5368s+1}; \widehat{g}_{22}e^{-\widehat{\theta}_{22}s} = \frac{8.1844e^{-0.3094s}}{1.3932s+1}$$
(16)

ISP has higher ETF gains, therefore $0.4/T_m$ is considered as the filter factor for the ISP process to yield good stability.

The closed loop simulations (see in Fig. 5) clearly show that the proposed method outperforms and yields better set-point tracking over the DICC and Xiong's methods.



Fig. 5. closed-loop set-point responses for the ISP process.

4.3 High dimensional moderate interacting multivariable System

Handling interaction dynamic effects for each loop in a high-dimensional MIMO process and design controller in a multiloop control framework is always a complex task.

Heating, Ventilation and Air Conditioning (HVAC) System: HVAC (Shen et al. (2010)) is a large-scale fourroom temperature control multivariable process. It is a slow process with significant time delays.

The RGA values of each loop for the HVAC process are 1.2207, 1.2198, 1.1095, and 1.1124. The ETF between each paired input-output are given as:

$$\widehat{g}_{11}(s) = \frac{-0.0803}{113.8299s+1}; \quad \widehat{g}_{22}(s) = \frac{-0.0754}{121.3735s+1}$$

$$\widehat{g}_{33}(s) = \frac{-0.0919}{113.9022s+1}; \quad \widehat{g}_{44}(s) = \frac{-0.0971}{123.5480s+1}$$

The dead time associated with the ETF are 15.8615, 14.9383, 15.4444 & 17.3739.

The $0.2T_m$ value is chosen as the filter factor for the HVAC process. As shown in Fig. 6, the proposed method provides stable output response, quick settling time, and set point tracking compared to the DICC and ODP control methods. The proposed method provide better performance indices values (See Table 2) compared to the DICC and ODP methods for closed-loop simulations of nominal and plant model mismatch scenarios.

5. CONCLUSION

A novel ETF dead-time compensation-based multiloop control topology for multivariable processes is proposed in this manuscript. Once the dead-time is compensated, the delay free ETFs are used to tune PI controllers based on IMC rules. Controllers tuned based on the proposed controller design method provide satisfactory closed-loop performance and robust stability for a wide range of industrial multivariable systems. Eliminating effective dead time from the MIMO processes simplifies the controller parameters tuning. Due to simplicity of the proposed method, the proposed control approach is suitable for large-scale industrial systems.

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| Fig. | 6. | closed-loop | set-point | responses | \mathbf{for} | the | HVAC process. | |
|------|----|-------------|-----------|-----------|----------------|-----|---------------|--|
|------|----|-------------|-----------|-----------|----------------|-----|---------------|--|

| Table 2 | Performance | indices f | for various | multivariable | nrocesses |
|----------|--------------|-----------|-------------|----------------|------------|
| Table 2. | 1 enformance | mulces i | ior various | munitivariable | processes. |

| | Mathad | Nominal | | | | +10% plant-model variation | | | | -10% plant-model variation | | | |
|--------|----------|---------|---------|--------|----------|----------------------------|---------|--------|--------|----------------------------|---------|--------|--------|
| | Method | IAE_i | ISE_i | IAE | ISE | IAE _i | ISE_i | IAE | ISE | IAE_i | ISE_i | IAE | ISE |
| | Proposed | 48.12 | 21.02 | 04.67 | 26 1 2 | 58.55 | 27.60 | 110.90 | 46.40 | 33.97 | 10.95 | 68.30 | 18 19 |
| | | 46.55 | 15.11 | 94.07 | 30.13 | 51.75 | 18.80 | 110.30 | 40.40 | 34.33 | 7.17 | | 10.12 |
| SSS | DICC | 58.71 | 31.45 | 101 10 | 46.23 | 72.03 | 41.3 | 114.16 | 57.23 | 51.51 | 25.24 | 94.67 | 39.42 |
| 000 | | 42.39 | 14.78 | 101.10 | | 42.13 | 15.93 | | | 43.16 | 14.18 | | |
| Pr | VI & Loo | 112.13 | 64.72 | 176 35 | 9E 90 | 264.91 | 184.11 | 387.23 | 227 73 | 72.28 | 35.06 | 195 54 | 50.38 |
| \geq | VU & Lee | 64.22 | 21.17 | 110.55 | 00.00 | 122.32 | 43.62 | | 221.15 | 53.26 | 15.32 | 120.04 | |
| \geq | Viena | 64.09 | 24.56 | 128 50 | 41.57 | 65.02 | 27.92 | 125.58 | 45.02 | 66.58 | 22.49 | 137 02 | 39.96 |
| | Along | 64.50 | 17.01 | 120.03 | 41.01 | 60.56 | 17.10 | | 40.02 | 71.34 | 17.47 | 101.32 | |
| | Proposed | 1.26 | 0.46 | 3.51 | 1 64 | 1.58 | 0.55 | 1 13 | 2.00 | 1.06 | 0.41 | 2.89 | 1.42 |
| ß | Tioposea | 2.25 | 1.18 | 0.01 | 1.01 | 2.85 | 1.45 | 1.10 | | 1.83 | 1.01 | | |
| ces | DICC | 1.46 | 0.50 | 4 27 | 2.00 | 3.44 | 0.65 | 5.23 | 2.45 | 1.20 | 0.51 | 3.56 | 1.73 |
| ro | | 2.81 | 1.50 | 1.21 | | 1.79 | 1.80 | | 2.10 | 2.36 | 1.22 | | |
| ISP P | Vu & Lee | 1.35 | 0.52 | 3.51 | 1.70 | 1.52 | 0.57 | 3.93 | 1.84 | 1.22 | 0.47 | 3.25 | 1.59 |
| | | 2.16 | 1.18 | 0.01 | | 2.41 | 1.27 | | | 2.03 | 1.12 | | |
| | Xiong | 1.51 | 0.63 | 4.31 | 2.17 | 1.67 | 0.68 | 4.66 | 2.33 | 1.39 | 0.60 | 4.25 | 2.06 |
| | | 2.80 | 1.54 | | | 2.99 | 1.65 | | | 2.86 | 1.46 | | |
| | Proposed | 61.16 | 28.73 | | 115.27 | 61.44 | 30.55 | 236.69 | 121.40 | 64.96 | 26.82 | 255.15 | 110.22 |
| | | 61.40 | 28.49 | 236.75 | | 61.21 | 30.05 | | | 66.39 | 27.30 | | |
| | | 54.78 | 27.52 | | | 54.50 | 28.81 | | | 59.36 | 26.60 | | |
| SSS | | 59.41 | 30.53 | | | 59.54 | 31.99 | | | 64.44 | 29.50 | | |
| 000 | DICC | 74.84 | 31.43 | 292.59 | | 88.52 | 34.22 | 371.01 | 134.61 | 73.31 | 29.52 | 252.4 | 109.4 |
| Pr | | 74.55 | 30.06 | | 119.9 | 92.09 | 33.15 | | | 69.04 | 27.92 | | |
| Ŋ | | 69.27 | 27.63 | | 115.5 | 92.72 | 31.99 | | | 52.88 | 24.56 | | |
| V. | | 73.93 | 30.73 | | | 97.68 | 35.25 | | | 57.17 | 27.40 | | |
| Ξ | ODP | 67.03 | 33.64 | | | 75.57 | 35.78 | 296.61 | 139.03 | 63.96 | 31.40 | 243.51 | 123.21 |
| | | 68.22 | 32.20 | 259.97 | 7 130.42 | 78.35 | 34.56 | | | 63.07 | 30.40 | | |
| | | 59.78 | 30.46 | | | 68.45 | 32.46 | | | 54.81 | 28.90 | | |
| | | 64.94 | 34.12 | | | 74.24 | 36.23 | | | 61.67 | 32.51 | | |

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