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Estimation of a SOPDT process transfer function for PID tuning

Massimiliano Veronesi * **Robin De Keyser** ** **Ramon Vilanova** *** **Antonio Visioli** ****

* *Yokogawa Italia, Italy*
e-mail: max.veronesi@yokogawa.com
** *DySC research group on Dynamical Systems and Control,*
Ghent University, Belgium, e-mail: Robain.DeKeyser@UGent.be
*** *Departament de Telecomunicació i d'Enginyeria de Sistemes,*
Universitat Autónoma de Barcelona, Spain, e-mail: ramon.vilanova@uab.cat
**** *Dipartimento di Ingegneria Meccanica e Industriale,*
University of Brescia, Italy, e-mail: antonio.visioli@unibs.it

Abstract: The paper deals with the estimation of a Second-Order-Plus-Dead-Time (SOPDT) process transfer function for the tuning of a Proportional-Integral-Derivative (PID) controller. In particular, different methodologies for the estimation of the transfer function parameters based on the open-loop process step response are proposed and compared to an existing one that has been recently presented in the literature. A pole-zero cancellation strategy is then employed to tune the PID controller. Advantages and disadvantages of the different methods are then discussed by considering the obtained results and implementation issues.

Keywords: PID controllers, SOPDT transfer function estimation, tuning, measurement noise.

1. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are widespread in industry because they are able to provide a satisfactory performance despite their relative simplicity. In other words, they provide a cost/benefit ratio that is difficult to improve with other more advanced control techniques. In addition to a simple structure, one of the main advantages of PID controllers is the availability of many tuning rules (O'Dwyer, 2006) that simplify their design. Usually, these tuning rules are based on a simple model of the process that should be obtained also with a simple experiment in order to keep the design effort at a reasonable level.

In this context, the two typical approaches are the closed-loop and the open-loop ones (Liu et al., 2013). In the first case a relay-feedback experiment is employed to estimate characteristic parameter of the process (see, for example, (Yu, 1999)). In the second case, typically, an open-loop step response is evaluated in order to estimate a First-Order-Plus-Dead-Time (FOPDT) or a Second-Order-Plus-Dead-Time (SOPDT) transfer function of the process. Given that the ideal PID controller transfer function has two zeros and a pole at the origin, a SOPDT transfer function is clearly more appropriate to achieve an overall better controller performance. For this reason, different methodologies have been proposed to estimate an accurate SOPDT transfer function by evaluating an open-loop step response. A recent review of these methods can be found in (Maxim and De Keyser, 2022). It is worth stressing that a SOPDT model is capable to describe both processes with an underdamped (Rangaiah and Krishnaswamy, 1996) and an overdamped response. Focusing on the latter case, which is the most common for industrial processes, an approach based on least squares can be employed (Cox et al., 2016). However, it has a significant computational burden, which makes its implementation difficult in edge (single-station) controllers. An alternative is to consider a few points on the process reaction curve and to estimate the transfer function parameters based on them (Huang et al., 2001). The main disadvantage in this case is that the presence of noise can result in a wrong selection of the points, yielding an inaccurate final result. It is therefore more convenient to have a technique based on integrals of signals so that there is an inherent robustness to the measurement noise (De Keyser and Muresan, 2019). This is the underlying idea of the method proposed in (Maxim and De Keyser, 2022), which has, however, the disadvantage of requiring the numerical solution of a fairly complex equation and the selection of the optimal dead time through an iterative procedure, which makes the approach more suitable to be implemented in a Distributed Control System (DCS) architecture rather than in a single-station control device.

Summarizing, it is desirable to have a computationally simple procedure that is robust to measurement noise (thus, based on integrals of signals) and provides an accurate result.

In this paper we aim to make a step forward in this direction. In particular, we propose different methodologies to estimate a SOPDT transfer function (by evaluating the open-loop step reponse for processes with overdamped dynamics) where these issues are at least improved and we discuss the obtained results with respect to the performance achieved by a PID controller tuned by applying a classic pole-zero cancellation method. The pros and cons of each method are given in order to enable the user to select the most appropriate one for a given application in terms of performance and implementation effort.

The paper is organized as follows. The problem is formulated in Section 2. The new methodologies for the estimation of the transfer function parameters are presented in Section 3. Illustrative results are shown and discussed in Section 4 and, finally, conclusions are given in Section 5.

2. PROBLEM FORMULATION

We consider the following SOPDT transfer function to model the process dynamics:

$$P(s) = \frac{Y(s)}{U(s)} = \frac{\mu}{(\tau_1 s+1)(\tau_2 s+1)} e^{-\theta s} \quad (1)$$

where, evidently, μ is the gain, τ_1 and τ_2 are the two time constants and θ is the dead time. Without loss of generality, we assume $\tau_1 \geq \tau_2$. Further, we assume $\tau_1 \geq \theta$ as this is the typical case for industrial processes where a PID controller is applied.

We aim to estimate the process parameters by evaluating the process output $y(t)$ when a unit step signal is applied to the input u, that is:

$$y(t) = \mu \left[1 - \frac{\tau_1}{\tau_1 - \tau_2} e^{-\frac{t-\theta}{\tau_1}} + \frac{\tau_2}{\tau_1 - \tau_2} e^{-\frac{t-\theta}{\tau_2}} \right] \quad (2)$$

In this context, the process gain μ can be easily estimated by calculating the ratio between the variation of the steady-state value of the output and the amplitude of the step input. For this reason and for the sake of simplicity, we will not consider the estimation of μ in the rest of the paper.

The purpose of the model is then to tune a PID controller in series form, whose transfer function is

$$C(s) = K_p \frac{T_i s+1}{T_i s} \frac{T_d s+1}{T_f s+1} \quad (3)$$

where K_p is the proportional gain, T_i is the integral time constant, T_d is the derivative time constant and T_f is the time constant of a low-pass filter that is necessary to have a proper controller transfer function. Although many methods exist to tune the PID parameters based on a SOPDT model (1), we decide to apply the Haalman method based on pole-zero cancellation (Åström and Hägglund, 2006) because it is directly related to the estimated model. Thus, the three main PID parameters are selected as

$$T_i = \tau_1 \qquad T_d = \tau_2 \qquad K_p = \frac{2\tau_1}{3\mu\theta} \quad (4)$$

By neglecting the filter, the loop transfer function resulting from the pole-zero cancellation is

$$L(s) = \frac{2}{3\theta s} e^{-\theta s} \quad (5)$$

The filter time constant can then be selected as one tenth of the inverse of the gain crossover frequency of (5). In this way, the additional phase lag introduced by the filter does not significantly influence the dynamics of the closed-loop system. It is worth stressing that the focus of the paper is not on the design of the PID controller (for this reason we do not consider any practical issue such as actuator saturation, etc.). Instead, the performance achieved by the PID control system is used only to assess the methodology to estimate the process parameters.

3. ESTIMATION METHODS

In this section we present three newly developed methods for estimating the parameters of the SOPDT transfer function (1). They will then be compared with the one presented in (Maxim and De Keyser, 2022).

3.1 Iterative method

In the first method we first consider the process $P(s)$ without dead time, that is, $\theta = 0$. Denoting as

$$y_\infty := \lim_{t \to +\infty} y(t) \quad (6)$$

the final value of the step response, we can easily calculate

$$A_1 := \int_0^\infty (y_\infty - y(t))dt = \mu(\tau_1 + \tau_2). \quad (7)$$

Since the gain μ is known, the sum of the two time constants can be employed to define a time interval to compute the (normalized) integral of the process response, that is:

$$A_2 := \int_0^{\tau_1+\tau_2} y(t)dt \quad (8)$$

(see the areas A_1 and A_2 in Figure 1). Now, define the ratio between the two time constants as $r := \frac{\tau_2}{\tau_1}$. The value of A_2/A_1 can be expressed as a function of r (De Keyser and Muresan, 2019):

$$\frac{A_2}{A_1} = \frac{e^{-1}}{1-r^2}(e^{-r} - r^2 e^{-1/r}) \quad (9)$$

It can then be computed for different values of r and then a numerical fitting of the results can be found. It results:

$$r = \frac{p_1}{\frac{A_2}{A_1} + p_2} \quad (10)$$

where $p_1 = 0.01043$ and $p_2 = -0.2594$. Finally, we have that the estimated values of the time constants are

$$\hat{\tau}_1 = \frac{A_1}{\mu(1+r)} \qquad \hat{\tau}_2 = r\hat{\tau}_1. \quad (11)$$

In this way the two time constants have been obtained by using only integrals of signals, so that the procedure is insensitive to the measurement noise. However, the numerical fitting (10) introduces an inaccuracy in the estimated model.

In order to estimate the parameters of a SOPDT model, an iterative approach like the one proposed in (Maxim and De Keyser, 2022) can be employed. In particular, different values of the dead time can be considered. For each of them the two time constants can be estimated by considering the delay-free response and, eventually, the value of the dead time can be selected as the one that minimizes the sum of square errors between the estimated and the true responses. Formally, given the step response $y(t)$, the following algorithm can be applied.

(1) Select a minimum dead time $\underline{\theta}$ (for example, $\underline{\theta} = 0$) and a maximum one $\overline{\theta}$ (for example, as the time when the step response attains half of its final value).
(2) For each value $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$:
   (a) calculate $A_1 = \int_0^\infty (y_\infty - y(t))dt - \mu\hat{\theta}$.
   (b) calculate $\bar{t} = A_1/\mu - \hat{\theta}$.
   (c) calculate $A_2 = \int_0^{\bar{t}} y(t)dt$.
   (d) calculate r according to (10) and $\hat{\tau}_1$ and $\hat{\tau}_2$ according to (11).
   (e) calculate the step response $\hat{y}(t)$ of $\hat{P}(s) = \frac{\mu}{(\hat{\tau}_1 s+1)(\hat{\tau}_2 s+1)} e^{-\hat{\theta} s}$.
   (f) calculate $SSE = \int_0^\infty (y(t) - \hat{y}(t))^2 dt$.
(3) Select $\hat{\tau}_1$, $\hat{\tau}_2$ and $\hat{\theta}$ as those that provide the minimum value of SSE.

Remark. With respect to the procedure in (Maxim and De Keyser, 2022), the method proposed in this paper is computationally simpler, as it has the advantage to avoid to solve numerically a

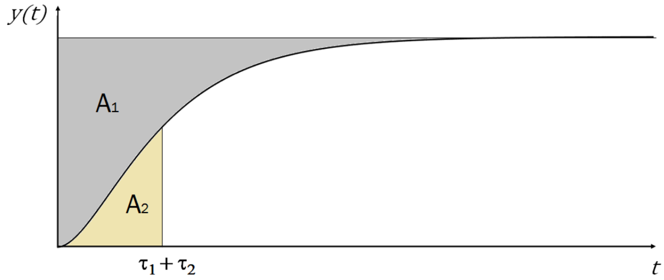

Fig. 1. Depiction of the areas A_1 and A_2 for the iterative method.

transcendental equation and to compute the response of a first-order system.

3.2 Non-iterative method based on a single point

In order to avoid the iteration to determine the dead time, which makes the algorithm unsuitable for edge controllers, an algorithm based on a single point (thus, in principle, sensitive to the measurement noise) can be applied.
In fact, in the presence of dead time, we have that

$$A_1 = \int_0^{\infty} (y_\infty - y(t))dt = \mu(\tau_1 + \tau_2 + \theta) \tag{12}$$

By defining $\bar{t} = \tau_1 + \tau_2 + \theta$, it can be found that

$$y(\bar{t}) = \mu \left(1 - \frac{\tau_2 e^{-\frac{\tau_1+\tau_2}{\tau_2}} + \tau_1 e^{-\frac{\tau_1+\tau_2}{\tau_2}}}{\tau_1 - \tau_2} \right) \tag{13}$$

This means that the value of $y(\bar{t})$ does not depend by θ but only by the ratio r. In fact, by collecting τ_1 and simplifying, it results:

$$y(\bar{t}) = \mu \left(1 - \frac{re^{-\frac{1+r}{r}} + e^{-\frac{1+r}{r}}}{1-r} \right) \tag{14}$$

However, equation (14) cannot be solved analytically to find r once $y(\bar{t})$ is found on the open-loop step response. Therefore, non-linear regression has to be applied for exploiting this dependency as

$$r = (\bar{y} - a)^{4b} \tag{15}$$

where $\bar{y} = \frac{y(\bar{t})}{\mu}$, $a = 1.592$ and $b = 13.98$.

Since $\tau_1 + \tau_2 + \theta = \frac{A_1}{\mu}$, $\bar{y}$ can be easily found on the step response but its determination is quite sensitive to the measurement noise. Thus, some filtering can be necessary, for example by computing the mean value of ten samples of the step response $y(t)$ before and after the time instant $\bar{t}$.
At this point equation (10) can be used to find

$$\alpha = \frac{A_2}{A_1^*} = \frac{p_1}{r} - p_2 \tag{16}$$

where $A_1^* = A_1 - \mu\theta$ is the part of A_1 after the dead time θ. So, once the area A_2 is computed by integrating the step response until $\bar{t}$, A_1^* can be obtained as

$$A_1^* = \frac{A_2}{\alpha} \tag{17}$$

Hence, the dead time can be estimated as

$$\hat{\theta} = \frac{A_1 - A_1^*}{\mu} \tag{18}$$

Finally, similarly to equations (11), the estimations of the two time constants can be obtained as

$$\hat{\tau}_1 = \frac{A_1/\mu - \hat{\theta}}{1+r} \qquad \hat{\tau}_2 = r\hat{\tau}_1 \tag{19}$$

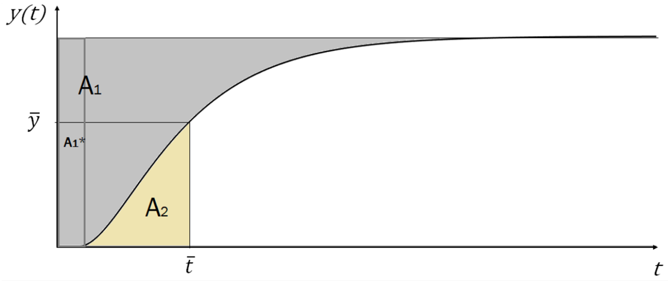

Fig. 2. Depiction of the main parameters for the non-iterative method based on a single point.

A graphical representation of the method is shown in Figure 2. It appears that the computational burden of this method is less than that of the previous one where iterations have to be made. However, being based on a single point of the step response (in addition to the defined areas), it is more sensitive to measurement noise and to modelling uncertainties. Further, two numerical approximations of the solutions of equations (15) and (10) are present.

3.3 Non-iterative method based on integrals only

The third method aims to avoid iterations and to use only integrals of signals. As a first step, the area A_1 is calculated as in (12). Then, we consider the step response $y_F(t)$ of a first-order system whose time constant is $\tau_1 + \tau_2 + \theta$:

$$y_F(t) = \mu \left[1 - e^{-\frac{t}{\tau_1+\tau_2+\theta}} \right] \tag{20}$$

It can be found that the amplitude of the intersection between $y_F(t)$ and the response (2) of the SOPDT system depends on the values of the system parameters but it is in any case close to 0.7μ. Let's denote the time interval when this intersection happens as t_{07}. This can be found by solving the equation

$$1 - e^{-\frac{t_{07}}{\tau_1+\tau_2+\theta}} = 0.7 \tag{21}$$

which yields (see (12))

$$t_{07} = -\frac{A_1}{\mu} \log(0.3) \tag{22}$$

Then, it can be found that there is a linear relationship between the ratio r of the time constants and the ratio of the delay-free intersection time instant and of the highest time constant, that is:

$$\frac{\tau_2}{\tau_1} = m \frac{t_{07} - \theta}{\tau_1} + q \tag{23}$$

Thus, after having calculated A_1 and A_2 (see Figure 3), we have a system of three equations (12), (16) and (23) with three unknowns (τ_1, r and θ). Note that there is an analytical solution for such a system (the resulting expressions are not shown for the sake of brevity). The values of p_1, p_2, m and q have been found through a numerical procedure in order to minimize the approximation error, yielding $p_1 = 0.0110$, $p_2 = -0.2555$, $m = 0.7091$ and $q = 0.8292$.
The advantage of this method is that it is only based on integrals of signals. However, this comes at the expense of a numerical approximation in the selection of the value of the intersection point and in the determination of the coefficients p_1, p_2, m and q. Further, as a single point of the step response plays a key role, the robustness to modelling uncertainties can be critical.

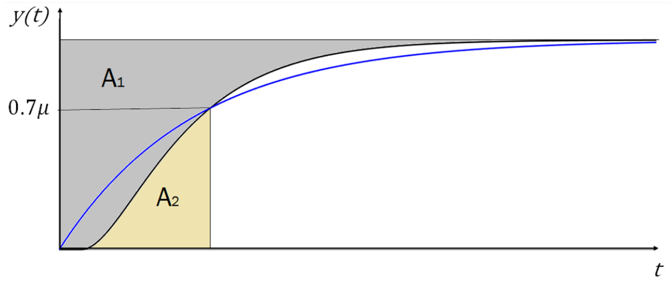

Fig. 3. Depiction of the main parameters for the non-iterative method based on integrals only. The blue line is the first-order system response $y_F(t)$.

4. ILLUSTRATIVE RESULTS

4.1 Example 1

As a first illustrative example, we consider a true SOPDT process

$$P_1(s) = \frac{1}{(10s+1)(5s+1)}e^{-2s}. \tag{24}$$

First, the case without noise is evaluated. By applying the methodologies presented in Section 3 and the one proposed in (Maxim and De Keyser, 2022), we obtain the values of the estimated parameters shown in Table 1. The comparison between the step responses of the true process and the estimated ones is shown in Figure 4, where it appears that the responses are almost completely overlapped. The PID tunings resulting from the application of the tuning rules (4) are shown in Table 2. The unit set-point step responses and the unit load disturbance step responses of the closed-loop systems are shown in Figures 5. It can be observed that the responses are very similar, with the exception of the non-iterative method based on integrals, which provides a more sluggish response. This is due to a smaller value of the proportional gain, as a consequence of the larger estimated value of the dead time of the process.

Subsequently, the case with measurement noise is considered. In particular, at each sampling instant, a random value between -0.1 and 0.1 (that is, 10% of the set-point step amplitude) is added to the true open-loop step response. The resulting process parameters are shown in Table 3 and the comparison between the step responses of the estimated models are shown in Figure 6. Also in this case it appears that they are almost overlapped. However, differently from the noise-free case, the non-iterative method based on single point introduces some uncertainty (as expected, since, despite the filtering, the single point value is sensible to the noise). This can be clearly seen from the PID tuning (see Table 4) and from the closed-loop set-point and load disturbance step responses shown in Figures 7. Indeed, the PID tuning based on the model estimated by the non-iterative method based on single point provides a more aggressive response because of the smaller estimated value of the dead time, which yields a higher value of the proportional gain.

Table 1. Estimated process parameters for Example 1 with no noise.

Method	τ_1	τ_2	θ
Iterative method	10.02	5.00	1.97
Non iterative method - single point	9.89	5.20	1.91
Non iterative method - integrals only	10.77	4.01	2.22
(Maxim and De Keyser, 2022)	9.99	5.01	2.00

Table 2. PID parameters for Example 1 with no noise.

Method	K_p	T_i	T_d	T_f
Iterative method	3.39	10.02	5.00	1.25
Non iterative method - single point	3.44	9.89	5.20	1.22
Non iterative method - integrals only	3.23	10.77	4.01	1.42
(Maxim and De Keyser, 2022)	3.34	9.99	5.01	1.27

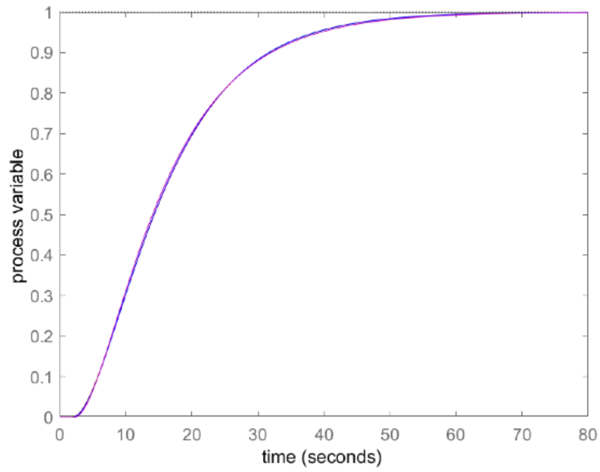

Fig. 4. Open-loop step responses for Example 1 with no noise. Black line: true process. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

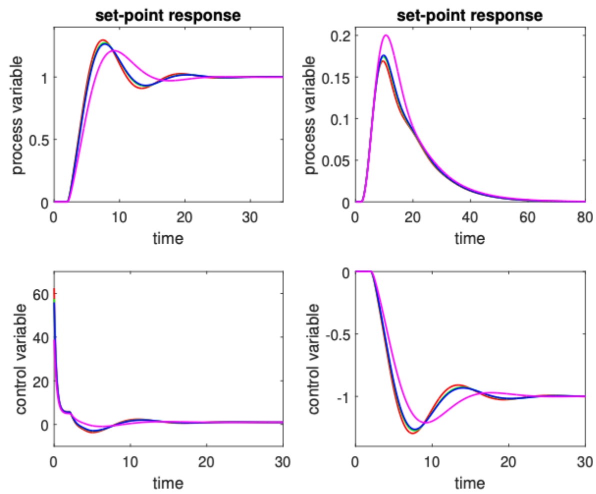

Fig. 5. Set-point and load disturbance step responses for Example 1 with no noise. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

Table 3. Estimated process parameters for Example 1 with noise.

Method	τ_1	τ_2	θ
Iterative method	10.02	5.03	1.97
Non iterative method - single point	9.02	6.33	1.65
Non iterative method - integrals only	10.77	4.01	2.23
(Maxim and De Keyser, 2022)	9.99	5.01	2.00

Table 4. PID parameters for Example 1 with noise.

Method	K_p	T_i	T_d	T_f
Iterative method	3.39	10.00	5.03	1.25
Non iterative method - single point	3.65	9.02	6.33	1.05
Non iterative method - integrals only	3.22	10.77	4.01	1.42
(Maxim and De Keyser, 2022)	3.34	9.99	5.01	1.27

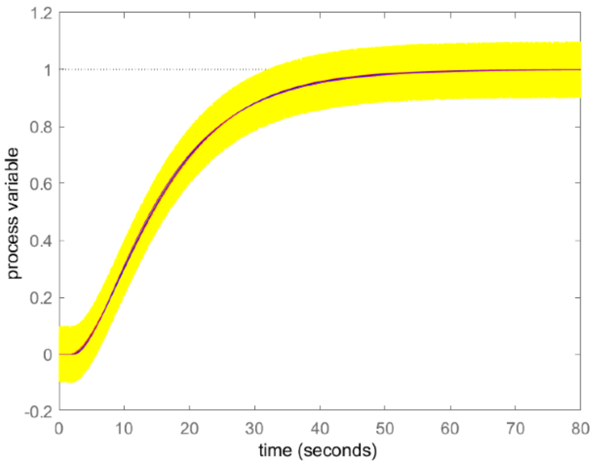

Fig. 6. Open-loop step responses for Example 1 with noise. Yellow line: true process. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

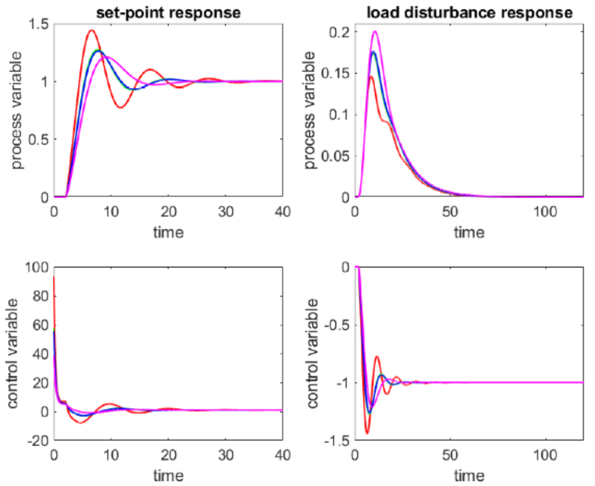

Fig. 7. Set-point and load disturbance step responses for Example 1 with noise. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

4.2 Example 2

As a second illustrative example, we consider a high-order process

$$P_2(s) = \frac{1}{(s+1)^8}. \tag{25}$$

In case there is no noise, the resulting estimated parameters obtained with the proposed methodologies are those shown in Table 5. It can be observed that the non-iterative methods yield an estimated process where a time constant is much higher

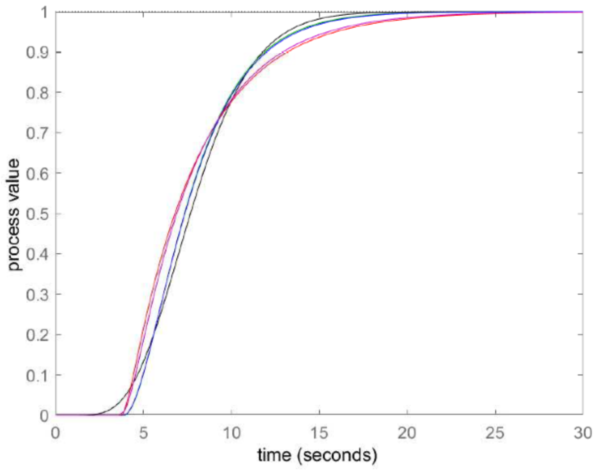

Fig. 8. Open-loop step responses for Example 2 with no noise. Black line: true process. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

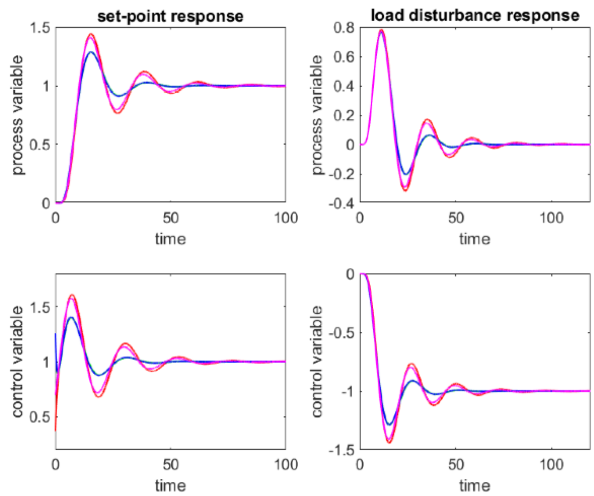

Fig. 9. Set-point step responses for Example 2 with no noise. Green line: iterative method. Red line: non-iterative method based a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

than the other one, so that the resulting model is, in practice, a FOPDT one. This is also confirmed by the comparison between the step responses of the true process and the estimated ones shown in Figure 8. Further, the non-iterative methods yield a smaller dead time, which results in a more aggressive PID controller, as it can be seen in Table 6 and in Figure 9, where the unit set-point step responses and the unit load disturbance step responses of the closed-loop systems are plotted.
Similar considerations can be done regarding the case with noise (again, the estimation of the model is performed when a random value between -0.1 and 0.1 is added to the true open-loop step response). Results are shown in Tables 7 and 8 and in Figures 10 and 11. They clarify that the noise issue is less relevant in case there are significant modelling uncertainties.

Table 5. Estimated process parameters for Example 2 with no noise.

Method	τ_1	τ_2	θ
Iterative method	2.10	2.01	3.88
Non iterative method - single point	3.97	0.29	3.73
Non iterative method - integrals only	3.74	0.57	3.68
(Maxim and De Keyser, 2022)	2.37	1.79	3.87

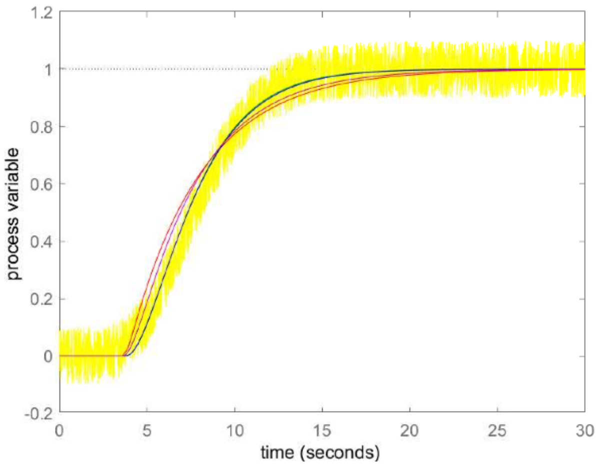

Fig. 10. Open-loop step responses for Example 2 with noise. Yellow line: true process. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

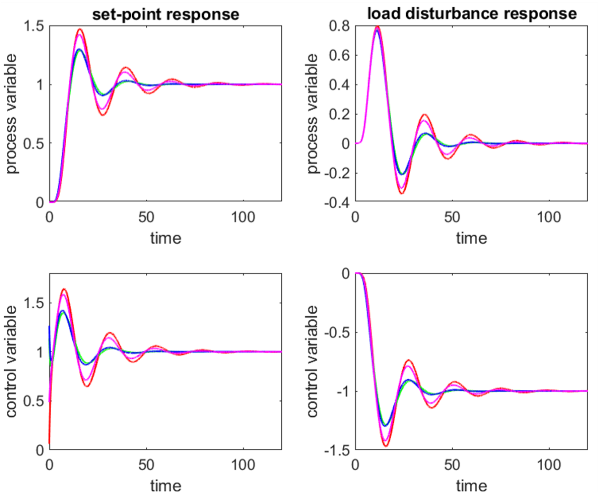

Fig. 11. Set-point step responses for Example 2 with noise. Green line: iterative method. Red line: non-iterative method based on a single point. Magenta line: non-iterative method based on integrals only. Blue line: (Maxim and De Keyser, 2022).

Table 6. PID parameters for Example 2 with no noise.

Method	K_p	T_i	T_d	T_f
Iterative method	0.36	2.10	2.01	2.47
Non iterative method - single point	0.71	3.97	0.29	2.38
Non iterative method - integrals only	0.68	3.74	0.57	2.35
(Maxim and De Keyser, 2022)	0.41	2.37	1.79	2.46

Table 7. Estimated process parameters for Example 2 with noise.

Method	τ_1	τ_2	θ
Iterative method	2.08	1.90	3.93
Non iterative method - single point	4.08	0.04	3.79
Non iterative method - integrals only	3.77	0.41	3.74
(Maxim and De Keyser, 2022)	2.35	1.77	3.82

Table 8. PID parameters for Example 2 with noise.

Method	K_p	T_i	T_d	T_f
Iterative method	0.35	2.08	1.90	2.50
Non iterative method - single point	0.72	4.08	0.04	2.41
Non iterative method - integrals only	0.67	3.77	0.41	2.38
(Maxim and De Keyser, 2022)	0.41	2.35	1.77	2.43

5. CONCLUSIONS

In this paper we have proposed new estimation methods for SOPDT transfer functions, which can play a key role in the achieved control system performance if PID tuning rules are employed. Advantages and disadvantages of each method are provided, so that the user can select the most suitable one for a given application. In particular, the noise level, the computational capabilities of the controller device and the control specifications have to be considered.

In general, simple non-iterative methods can be on-line implemented into real-time controllers, without affecting their computational load. However, potential robustness issues arise for methods based on a single point. For higher order processes, more accurate estimations can be obtained by iterative methods requiring several step response simulations. Thus, they have to be implemented into upper layer supervisory operator workstations, not in charge of real-time control algorithms.

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